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News Shocks under Financial Frictions*

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Abstract

We examine the dynamic effects and empirical role of TFP news shocks in the context of frictions in financial markets. We document two new facts using VAR methods. First, a (positive) shock to future TFP generates a significant decline in various credit spread indicators considered in the macro-finance literature. The decline in the credit spread indicators is associated with a robust improvement in credit supply indicators, along with a broad based expansion in economic activity. Second, VAR methods also establish a tight link between TFP news shocks and shocks that explain the majority of un-forecastable movements in credit spread indicators. These two facts provide robust evidence on the importance of movements in credit spreads for the propagation of news shocks. A DSGE model enriched with a financial sector generates very similar quantitative dynamics and shows that strong linkages between leveraged equity and excess premiums, which vary inversely with balance sheet conditions, are critical for the amplification of TFP news shocks. The consistent assessment from both methodologies provides support for the traditional ‘news view’ of aggregate fluctuations.

Keywords: TFP News shocks, Business cycles, DSGE, VAR, Bayesian estimation.

JEL Classification: E2, E3.

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1 Introduction

The news driven business cycle hypothesis formalized in Beaudry and Portier (2004) and restated in Jaimovich and Rebelo (2009) posits that changes in expectations of future fundamentals are an important source of business cycle fluctuations. Movements in financial markets encapsulate changes in expectations about the future and are a powerful mechanism that triggers changes in economic activity. A vast body of research finds that financial markets are characterized by frictions that lead to credit spreads—differences in yields between private debt instruments and government bonds of comparable maturities—whose movements contain important information on the evolution of the real economy and encompass predictive content for future economic activity.\(^1\)

In this paper we quantify the empirical significance and dynamic effects of total factor productivity (TFP) news shocks in light of propagation through financial frictions. We investigate the issue using two widely-used methods (VAR and DSGE) that provide complementary readings on the significance and dynamics of news shocks. We use a vector autoregression (VAR) model enriched with credit spread indicators and measures of credit supply conditions to isolate two novel stylized facts.

First, a TFP news shock identified from the VAR model as the shock that explains the majority of the variance in TFP in a long horizon, generates an immediate and significant decline of key credit spread indicators along with a broad based increase in economic activity in anticipation of the future improvement in TFP. The decline of the credit spread indicators is a robust finding that holds across alternative specifications of the VAR model and different identification methods.\(^2\) We focus on the dynamics of the highly informative credit spread indicator introduced by Gilchrist and Zakrajsek (2012) (GZ spread), and its two components, namely, the expected default component, and excess bond premium component. We find that the decline in the GZ spread is primarily driven by a decline in the excess bond premium, not a fall in the expected default component of the GZ spread, which exhibits an insignificant

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\(^1\)See Gilchrist and Zakrajsek (2012) and Philippon (2009).

\(^2\)Our baseline identification scheme follows the approach in Francis et al. (2014). We discuss robustness to alternative identification approaches in section 2.3.
response. The excess bond premium is interpreted by Gilchrist and Zakrajsek (2012) as an indicator of the capacity of intermediaries to extend loans or more generally the overall credit supply conditions in the economy.

Second, we independently apply an agnostic methodology proposed by Uhlig (2003) to identify a single shock that explains the majority of the unpredictable movements in the excess bond premium. This exercise reveals a striking fact: the single shock, identified from this procedure, generates dynamics that resemble qualitatively and quantitatively those produced by a TFP news shock. Specifically, it generates a broad based increase in economic activity, a delayed build-up of TFP towards a new permanently higher level, and an immediate and strong decline in the excess bond premium. Moreover, the robust decline in inflation helps to clearly distinguish this shock from a conventional financial shock. The shock we recover from this agnostic identification explains approximately two thirds of the forecast error variance in the excess bond premium over business cycle frequencies. The two novel stylized facts we document provide robust evidence on the importance of movements in credit spread indicators for the propagation of news shocks and motivate our modelling approach in the second part of the paper.

We investigate the link between credit spread indicators and news shocks using a two-sector dynamic stochastic general equilibrium (DSGE) model whose micro-foundations enable the underpinning of the mechanisms for the propagation of news shocks. To this end, we introduce financial frictions in the supply side of finance via leveraged banks similar to Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). Furthermore, we introduce frictions in the adjustment of financial claims that fund capital acquisitions. These financial claims are held by banks in the form of debt, and by households in the form of corporate equity. This approach is motivated by earlier work in corporate investment and finance (see Gomes (2001), Altinkilic and Hansen (2000), Hennessy and Whited (2007) among others),

3An important motivation for considering a two-sector economy is the recent evidence in Basu et al. (2013), which suggests that sector-specific technological changes have different macroeconomic effects. The consumption- and investment-goods-producing sectors are therefore subject to sector-specific TFP technologies, in line with this recent evidence.
that identifies significant issuance costs for equity and debt. We apply the DSGE model directly to post-1984 U.S. real and financial data to estimate the model’s parameters with Bayesian methods. We produce dynamic responses and business cycle statistics that suggest TFP news shocks are important drivers of business cycle fluctuations, accounting for approximately 52% and 50% of the variance in output and hours respectively. The DSGE model provides a compelling structural narrative for the propagation mechanism and the empirical relevance of TFP news shocks and allows to assess the strength of the financial amplification channel by switching it off. The presence of leveraged financial intermediaries delivers a strong amplification of news shocks due to the feedback loop between leveraged bank equity and corporate bond prices. Financial intermediaries hold claims to productive capital in their portfolios in the form of corporate bonds. When the price of corporate bonds increases, their equity value increases and their leverage constraint eases, making the excess premium on holding debt to fall and their balance sheet to expand. This dynamic generates a further rise in the demand for bonds and a further rise in the price of bonds. The demand for bonds is thus amplified by leverage, bidding up the bond prices relative to a standard New Keynesian model without financial frictions. The amplification delivers a strong lending and investment phase and a strong economy-wide boom. In contrast, in the standard DSGE model without financial frictions, amplification is weak. It predicts that TFP news shocks account for a maximum of 14% and 18% of the variance in output and hours worked, respectively, much in line with the existing estimated DSGE literature.

To formally assess whether the financial channel conforms the dynamic responses of the variables to TFP news shocks in the DSGE and VAR methods, we perform a Monte Carlo experiment. We compare the impulse responses to an aggregate TFP news shock from the empirical VAR model with those estimated from the same VAR model on artificial data generated using posterior estimates of the DSGE model. We find that empirical VAR responses of key macroeconomic aggregates (including corporate bond spreads) are consistent with the VAR responses estimated from artificial model data. The experiment shows that accounting for financial frictions leads the two methodologies independently implemented to
reach similar conclusions on the dynamic effects of TFP news shocks.

To appraise the quantitative relevance of news shocks between the two methods, we undertake a comparison in the shares of the forecast error variance of key macro aggregates. The VAR and DSGE methodologies provide a very consistent picture on the importance of TFP news shocks: for example, at business cycle frequencies (6 to 32 quarters), the VAR model establishes that TFP news shocks account for between 44% to 69% of the variance in output and between 36% to 45% of the variance in hours worked. The DSGE model finds the same shocks account for between 33% to 51% of the variance in output and between 33% to 46% of the variance in hours worked. Taken together, these findings suggest that both methodologies find TFP news shocks an important source of business cycles in the Great moderation era and hence provide support for the traditional ‘news view’ of aggregate fluctuations.

Our study is related to the large research agenda on the role of news shocks for macroeconomic fluctuations. The literature shows substantial disagreement over the propagation mechanism and empirical plausibility of TFP news shocks. In the context of the VAR methodology, e.g. Beaudry and Portier (2006), Beaudry and Lucke (2010), Beaudry et al. (2012) and Görtz et al. (2020a) find that TFP news shocks account for a major fraction of macroeconomic fluctuations whereas Barsky and Sims (2011) and Forni et al. (2014) detect a limited role of TFP news shocks to aggregate fluctuations. More recently, Ben Zeev and Khan (2015) identify investment-specific news shocks as a major driver of U.S. business cycles, a finding supportive of the technology news interpretation of aggregate fluctuations. In the context of the DSGE methodology, Schmitt-Grohe and Uribe (2012) estimate a real business cycle model and find that TFP news shocks are unimportant drivers of business cycle fluctuations, but suggest alternative non-structural news shocks, such as wage mark-up news shocks, are important drivers of fluctuations. Fujiwara et al. (2011) and Khan and Tsoukalas (2012) reach a similar conclusion in models with nominal rigidities. Christiano et al. (2014) estimate a DSGE model that emphasizes borrowers’ credit frictions and find an empirical

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4The review article by Beaudry and Portier (2014) provides an extensive discussion on the literature.

Our contribution to this literature is twofold. First, using VAR methods, we document new facts that speak to the relevance and importance of credit supply frictions for the propagation of news shocks. We establish a tight link between TFP news shocks and shocks (identified independently from news shocks) that drive the majority of unpredictable movements in credit spread indicators suggesting the latter are important asset prices that reflect future economic news. Second, our DSGE estimation, offers a quantification of financial frictions by estimating parameters that control rigidities in the adjustment of debt and equity, and a parameter which controls the elasticity between the corporate bond spread and the leverage constraint of banks. This is crucial as the model relies on frictions in financial markets as key amplification mechanisms to assign significant empirical relevance to TFP news shocks. Our model with financial frictions is consistent with the VAR narrative and therefore a very good first step in understanding the propagation of news shocks. By focusing on financial frictions our study therefore suggests that different methodologies can result in consistent readings and provide a unified view for the macroeconomic effects of TFP news shocks.

The remainder of the paper is organized as follows. Sections 2 and 3 describe the VAR and DSGE analysis, respectively. Section 4 reconciles the differences between the DSGE and the VAR findings and section 5 concludes.

2 VAR analysis

This section describes the VAR model, the data and the methodology used for the estimation and the results from the VAR analysis.
2.1 The VAR model

Consider the following reduced form VAR($p$) model,

$$ y_t = A(L)u_t, \quad (1) $$

where $y_t$ is an $n \times 1$ vector of variables of interest, $A(L) = I + A_1L + A_2L^2 + \ldots + A_pL^p$ is a lag polynomial, $A_1, A_2, \ldots, A_p$ are $n \times n$ matrices of coefficients and, finally, $u_t$ is an error term with $n \times n$ covariance matrix $\Sigma$. Define a linear mapping between reduced form, $u_t$, and structural errors, $\varepsilon_t$,

$$ u_t = B_0\varepsilon_t, \quad (2) $$

We can then write the structural moving average representation as

$$ y_t = C(L)\varepsilon_t, \quad (3) $$

where $C(L) = A(L)B_0$, $\varepsilon_t = B_0^{-1}u_t$, and the matrix $B_0$ satisfies $B_0B_0' = \Sigma$. The $B_0$ matrix may also be written as $B_0 = \tilde{B}_0D$, where $\tilde{B}_0$ is any arbitrary orthogonalization of $\Sigma$ and $D$ is an orthonormal matrix ($DD' = I$).

The $h$ step ahead forecast error is,

$$ y_{t+h} - E_{t-1}y_{t+h} = \sum_{\tau=0}^{h} A_{\tau}\tilde{B}_0D\varepsilon_{t+h-\tau}. \quad (4) $$

The share of the forecast error variance of variable $i$ attributable to shock $j$ at horizon $h$ is then

$$ V_{i,j}(h) = \frac{e_i'\left(\sum_{\tau=0}^{h} A_{\tau}\tilde{B}_0D e_j A_{\tau}'\right)e_i}{e_i'\left(\sum_{\tau=0}^{h} A_{\tau}\Sigma A_{\tau}'\right)e_i} = \frac{\sum_{\tau=0}^{h} A_{i,\tau}\tilde{B}_0\gamma'\tilde{B}_0'A_{i,\tau}'}{\sum_{\tau=0}^{h} A_{i,\tau}\Sigma A_{i,\tau}'}, \quad (5) $$

where $e_i$ denotes selection vectors with one in the $i$-th position and zeros elsewhere. The $e_j$ vectors pick out the $j$-th column of $D$, denoted by $\gamma$. $\tilde{B}_0\gamma$ is an $n \times 1$ vector corresponding to the $j$-th column of a possible orthogonalization and can be interpreted as an impulse response vector. In the following section, we discuss the estimation and identification methodology that yields an estimate for the TFP news shock from the VAR model.
2.2 VAR estimation

We estimate the VAR model using quarterly U.S. data on a Great moderation sample for the period 1984:Q1–2017:Q1.\textsuperscript{5} To estimate the VAR model we use five lags with a Minnesota prior and compute confidence bands by drawing from the posterior—details are given in the online Appendix. A key input is an observable measure of TFP and for this purpose we use the utilization-adjusted aggregate TFP measure provided by John Fernald of the San Francisco Fed. The methodology used to compute the TFP measure is based on the growth accounting methodology in Basu et al. (2006) and corrects for unobserved capacity utilization, described in Fernald (2014). The time series included in the VAR enter in levels, consistent with the treatment in the empirical VAR literature (e.g. Barsky and Sims (2011) and Beaudry and Portier (2004, 2006, 2014)). Details about the data are provided in the online Appendix. The data used in the VAR and DSGE analysis are from the St.Louis Federal Reserve Bank (Federal Reserve Bank of St.Louis (2017)), the Federal Reserve Board (Favara et al. (2016)), the Wharton Research Data Services (Wharton Research Data Services (2018)), Shiller (2017), and Fernald (2014).

To identify the TFP news shock from the VAR model, we adopt the identification scheme of Francis et al. (2014) (referred to as the Max Share method). The Max Share method recovers the news shock by maximizing the variance of TFP at a specific long but finite horizon (we set the horizon to 40 quarters) and imposes a zero impact restriction on TFP conditional on the news shock.

2.3 Results from the VAR model

TFP news shock and credit market indicators. We begin our exploration with a VAR specification that estimates responses to a TFP news shock. Our set of observables allows

\textsuperscript{5}Galí and Gambetti (2009), among others, document significant changes in the co-movement properties of important macro-aggregates before and after the mid-1980s, and Jermann and Quadrini (2009) highlight changes in moments of financial sector variables in the mid- and late 1980s. We report robustness of our findings to end date (excluding the Great recession period) of the sample in the online Appendix available at Görtz et al. (2020b).
us to examine responses to the GZ spread constructed by Gilchrist and Zakrajsek (2012).\textsuperscript{6} The GZ spread indicator uses firm level information from corporate senior unsecured bonds traded in the secondary market, controls for the maturity mismatch between corporate and treasuries, and spans the entire spectrum of issuer credit quality (from investment grade to below investment grade).

Figure 1: TFP news shock. Impulse responses to a TFP news shock from a seven-variable VAR. The shaded gray areas are the 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.

Figure 1 displays Impulse Response Functions (IRFs) from a VAR featuring aggregate TFP, output, consumption, hours, GZ spread, the S&P 500, and inflation (log change in GDP deflator). Several interesting findings emerge. First, TFP rises in a delayed fashion, and it becomes significantly different from zero after approximately three years. This pattern shows that the identification scheme produces empirically plausible news shocks, as discussed in Beaudry and Portier (2014). Second, the VAR-identified TFP news shock creates a boom today: output, consumption, and hours increase significantly on impact, and they display hump-shaped dynamics. Third, the GZ spread declines significantly, suggesting that corporate bond markets anticipate movements in future TFP, which is consistent with an

\textsuperscript{6}We have also examined the popular BAA spread (difference between the yield of a BAA rated corporate bond and a ten year Treasury) and found results that are very similar to the ones reported in the main body of the paper.
economic expansion induced by an increase in lending. The behavior of the GZ spread is a novel stylized fact that, to the best of our knowledge, no previous studies have documented. Further, the S&P 500 also increases in anticipation of the future rise in TFP, consistent with the work by Beaudry and Portier (2006) that finds the stock market captures changes in agents’ expectations of future economic outlook. Finally, the news shocks is associated with a short-lived decline in inflation. The decline in inflation is a very robust finding in the empirical news shock literature with VAR methods (see Barsky and Sims (2011), Barsky et al. (2015), Cascaldi-Garcia (2019)) and at first pass it may appear puzzling, given the ”demand-like” nature of news shocks, i.e. a broad based increase in activity in the absence of a productivity improvement in the short term. We discuss this finding in section 4 and note that the New Keynesian model we estimate in section 3 may partly rationalize the behavior of inflation in response to a news shock.

**TFP news shock, excess bond premium and balance sheet conditions.** Evidence by Gilchrist and Zakrajsek (2012) strongly suggests that the GZ spread is superior, relative to conventional indicators such as the BAA spread, in terms of forecasting future economic activity. The GZ spread can be usefully decomposed into a component capturing cyclical changes in default risk (i.e. expected defaults), and a component that measures cyclical changes in the relationship between default risk and credit spreads, the ‘excess bond premium’ (EBP). Importantly, Gilchrist and Zakrajsek (2012) provide evidence to indicate that over the sample 1985-2010, the excess bond premium contains most of the predictive content of the GZ spread for various measures of economic activity. We further examine the role of balance sheet conditions of intermediaries for the propagation of news shocks using two indicators. First, the market value of U.S. commercial bank’s equity (henceforth bank equity), and, second, the Senior Loan Officer Opinion Survey of Bank Lending Practices (SLOOS).\(^7\) We examine the behaviour of the excess bond premium, default risk, market

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\(^7\)The market value of equity is aggregated from all publicly listed financial institutions provided by the Center for Research in Securities Prices (CRSP)(Appendix B provides details on the data). The SLOOS measures the net percentage of domestic respondents tightening standards for commercial and industry loans. We use the net percentage applicable for loans to medium and large firms. Specifically the net percentage measures the fraction of banks that reported having tightened (“tightened considerably” or “tightened some-
value of bank equity and indicator of lending standards by replacing each of these indicators in the VAR specification discussed above in place of the GZ spread. Figure 2 displays the results. Our novel finding is that the excess bond premium declines significantly on impact and, similarly to the behaviour of the GZ spread, ahead of the future rise in TFP. Notice that the forecasting ability of the excess bond premium as emphasized by Gilchrist and Zakrajsek (2012) is implicitly reflected in the shape of the dynamic responses, given the hump shaped dynamics of the real activity variables (as shown in Figure 1). Interestingly, the default risk component of the GZ spread is, in contrast to the excess bond premium, not reacting significantly in response to the news shock. This observation suggests that the variation in the GZ credit spread conditional on the news shock is driven by factors mostly related to credit supply conditions. We provide more evidence for this link below.

The dynamic responses displayed in Figure 2 suggest an immediate, strong and significant positive response of bank equity. The response of bank equity is consistent with the notion that it reflects increased profitability and/or higher asset valuation in the balance sheet of intermediaries. The response of the SLOOS variable suggests an immediate and significant relaxation of lending standards, which persists for about two years. Both sets of findings related to the joint response of the excess bond premium, bank equity and lending standards are consistent with the evidence reported in Gilchrist and Zakrajsek (2012), where higher profitability of the U.S. financial corporate sector is associated with a reduction in the excess bond premium. Taken together, these findings support the hypothesis that balance sheet and more generally credit supply conditions are an important transmission channel for TFP news shocks.

What are the shocks that move credit spread indicators? The preceding evidence suggests that credit spread indicators may be capturing a transmission mechanism for news shocks that is grounded on credit market frictions. To provide further evidence for the link
between news shocks and the excess bond premium we proceed to independently identify shocks that explain the majority of the un-forecastable movements in the excess bond premium. Specifically, we proceed to identify, in an agnostic manner, following the methodology proposed by Uhlig (2003), a single shock that maximizes the forecast error variance (FEV) of the excess bond premium (we term it the “max FEV EBP shock”) at cyclical frequencies (horizons 6 to 32 quarters). This exercise is similar in spirit to the analysis in Beaudry and Portier (2006) who focus on shocks that explain short run movements in stock prices and then establish a link between those shocks and TFP news shocks.

Here the goal is to establish the link, if any, between movements in asset prices from the corporate debt market and news shocks. Consider a VAR specification featuring the excess bond premium, output, hours, consumption, TFP, inflation, and the S&P 500 indicator. We find that the max FEV EBP shock identified from this VAR specification, explains approximately two thirds of the forecast error variance (median shares) in the EBP in forecast horizons from six to thirty-two quarters. We then compare the IRFs induced by the max FEV EBP shock with the IRFs induced by the TFP news shock using the same VAR specification. Figure 3 displays the two sets of IRFs. The comparison reveals a striking new finding. The two shocks, independently identified, exhibit very similar dynamic paths, both qualitatively and quantitatively. Both shocks are associated with an immediate increase in

Figure 2: TFP news shock. Impulse responses to a TFP news shock from seven-variable VARs. The estimated VARs include the variables shown in Figure 1 where we replace the GZ spread with the shown variables one at a time and re-estimate the VAR. The shaded gray areas are the 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.
activity, and a countercyclical response of the excess bond premium.\textsuperscript{9} The similarity in the dynamics of the excess bond premium across the two independent identification exercises is, we think, an important finding since, according to the arguments and evidence in Gilchrist and Zakrajsek (2012), the excess bond premium captures cyclical variations in credit market supply conditions. Adopting this interpretation, a favourable TFP news shock is associated with a reduction in the excess bond premium and a relaxation of credit market supply conditions that coincides with a boom in activity, leading to the hypothesis we advance in this paper: balance sheet conditions of financial intermediaries matter for the propagation of news shocks. Importantly, the max FEV EBP shock is a relevant business cycle shock in a quantitative sense as this shock explains more than 64\% of the FEV in output and hours (median shares).\textsuperscript{10, 11}

\textsuperscript{9}Notice that in the VAR with the agnostic identification that seeks for the max FEV EBP shock, there is no zero impact restriction associated with the IRF of TFP, hence TFP can freely move on impact of this shock. Nevertheless, the IRF confidence bands for TFP in this identification suggest that this positive impact response in not significantly different from zero. In fact TFP rises significantly above zero at approximately 20 quarters.

\textsuperscript{10}To conserve space the contribution of the max FEV EBP shock to the FEV of all variables included in the VAR is shown in the online Appendix.

\textsuperscript{11}Our findings are robust in a number of dimensions. In the online Appendix we show responses based
It is interesting to note that recent work in Queralto (2019) and Moran and Queralto (2018) emphasize demand driven factors behind medium term dynamics in TFP. Under this interpretation financial shocks influence business innovation activities and consequently future TFP. To address a concern that our identification strategy confounds TFP news with financial shocks we proceed to identify, within the same VAR framework above, additional to a TFP news shock, a financial shock as the innovation to the EBP. This analysis can distinguish a TFP news shock that moves future TFP, from a financial shock that moves both current and future TFP. To conserve space, we report these dynamic responses in the online Appendix: following a positive financial shock that generates a decline in EBP, in the short run, activity increases, and TFP rises with a long delay in the future—indeed very similar to the IRFs displayed by the max EBP shock in Figure 3. The important insight of this analysis is the fact that the behavior of inflation is critical to be able to clearly distinguish a financial shock from a news TFP shock. Conditional on a financial shock, inflation co-moves with activity.\footnote{The dynamics following a financial shock are therefore consistent with the empirical VAR analysis in Gilchrist and Zakrajsek (2012).} In contrast, as discussed above, conditional on a TFP news shock, inflation declines concurrently with an increase in activity.

3 DSGE analysis

This section discusses the DSGE model, the data, the methodology used for the estimation and the results from the DSGE analysis.
3.1 The model

Below, we describe the parts of the model related to the goods-producing sectors, households, the financial sector, the exogenous disturbances, and the arrival of information. The online Appendix provides a description of the complete model.

3.1.1 Intermediate and final goods production

A monopolist produces consumption and investment-specific intermediate goods according to the production technologies

\[
C_t(i) = \max \left[ a_c L_{C,t}(i)^{1-a_c} (K_{C,t}(i))^{a_c} - A_t V_t^{\frac{a_c}{1-a_c}} F_C, 0 \right]
\]

and

\[
I_t(i) = \max \left[ v_i L_{I,t}(i)^{1-a_i} (K_{I,t}(i))^{a_i} - V_t^{\frac{1-a_i}{1-a_i}} F_I, 0 \right],
\]

respectively. The variables \(K_{x,t}(i)\) and \(L_{x,t}(i)\) denote the amount of capital and labor services rented by firm \(i\) in sector \(x = C, I\), and the parameters \((a_c, a_i) \in (0, 1)\) denote capital shares in production. The variables \(A_t\) and \(V_t\) denote the (non-stationary) level of TFP in the consumption and investment sector, respectively, and the variables \(z_t = \ln(A_t/A_{t-1})\) and \(v_t = \ln(V_t/V_{t-1})\) denote (stationary) stochastic growth rates of TFP in the consumption and investment sector, respectively. The variables \(a_{lt}, v_{lt}\), denote the stationary level of TFP in the consumption and investment sector, respectively. To facilitate the exposition, subsection 3.1.5 describes the processes for the exogenous disturbances. Intermediate goods producers set prices according to Calvo (1983) contracts.

Perfectly competitive firms manufacture final goods, \(C_t\) and \(I_t\), in the consumption and investment sector by combining a continuum of intermediate goods in each sector, \(C_t(i)\) and

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13As in Christiano et al. (2005), the presence of fixed costs in production in both sectors (i.e. \(F_C > 0\) and \(F_I > 0\)) leads to zero profits along the non-stochastic balanced growth path thereby the analysis abstracts from entry and exit of intermediate good producers. Fixed costs grow at the same rate of sectoral output to retain relevance for the firms’ profit decisions.
\[ I_t(i) \text{, respectively, according to the production technologies} \]

\[
 I_t = \left[ \int_0^1 (I_t(i))^{\frac{1}{1+\lambda_{p,t}}} \, di \right]^{1+\lambda_{p,t}} \\
 C_t = \left[ \int_0^1 (C_t(i))^{\frac{1}{1+\lambda_{p,t}}} \, di \right]^{1+\lambda_{p,t}},
\]

where the exogenous elasticities \( \lambda_{p,t} \) and \( \lambda_{p,t} \) across intermediate goods in each sector determine the (sectoral) price markup over marginal cost. Similar to the standard New Keynesian framework, prices of final goods in each sector \( P_{C,t} \) and \( P_{I,t} \) are CES aggregates of intermediate goods prices. The online Appendix provides details on price-setting decisions of the intermediate goods producers.

### 3.1.2 Households

As in Gertler and Karadi (2011), households comprise two types of members, workers of size \( 1 - f \) and bankers of size \( f \). Each worker \( j \) supplies diversified labor in return for a wage. Effectively, households own the intermediaries managed by bankers, but they do not own the deposits held by the financial intermediaries. Perfect risk sharing exists within each household. The proportion of workers and bankers remains constant over time. However, members of the households are allowed to switch occupations to avoid bankers having to fund investments from their own capital without having to access credit. Bankers become workers in the next period with probability \( (1 - \theta_B) \) and transfer the retained earnings to households. Households supply start-up funds to workers who become bankers. We moreover enrich this setup to allow workers in each family to save in financial claims that finance capital acquisitions from capital services producers. To make this operational, we introduce fictitious perfectly competitive money market funds that collect savings from households and buy financial claims from a large number of firms in each sector. Each money market fund specializes in buying claims from the consumption or investment sector only. At the end of each period, money market funds return the proceeds from the claims back to households and a new round begins. Each household maximizes the utility function

\[
 E_0 \sum_{t=0}^{\infty} \beta^t b_t \left[ \ln(C_t - hC_{t-1}) - \varphi \left( L_{C,t}(j) + L_{I,t}(j) \right)^{1+\nu} \right],
\]
where $E_0$ is the conditional expectation operator at the beginning of period 0, $\beta \in (0,1)$ is the discount factor, and $h \in (0,1)$ is the degree of external habit formation. The inverse Frisch labor supply elasticity is denoted by $\nu > 0$, and the parameter $\varphi > 0$ enables the model to replicate the steady state level of total labor supply in the data. The variable $b_t$ denotes an intertemporal preference shock. Each household faces the following budget constraint expressed in consumption units

$$C_t + \frac{B_t}{P_{C,t}} + \frac{S_{C,t}^h + S_{I,t}^h}{P_{C,t}} \leq \frac{W_t(j)(L_{C,t}(j) + L_{I,t}(j)) + R_{t-1}}{P_{C,t}}$$

$$+ \frac{R_{C,t-1}^h S_{C,t-1}^h}{P_{C,t}} + \frac{R_{I,t-1}^h S_{I,t-1}^h}{P_{C,t}} - \frac{T_t}{P_{C,t}} + \frac{\Psi_t(j)}{P_{C,t}} + \frac{\Pi_t}{P_{C,t}},$$

(6)

where $S_{C,t}^h$ and $S_{I,t}^h$ are financial (equity) claims in the consumption and investment sectors respectively purchased from households through the sector-specialized money market funds that pay a nominal return per unit of equity equal to $R_{C,t}^h$ and $R_{I,t}^h$, respectively. The variable $B_t$ denotes holdings of risk-free bank deposits, $\Psi_t$ is the net cash flow from the household’s portfolio of state contingent securities, $T_t$ is lump-sum taxes, $R_t$ is the (gross) nominal interest rate paid on deposits, $\Pi_t$ is the net profit accruing to households from ownership of all firms, and $P_{C,t}$ is the consumption deflator. The wage rate, $W_t$, is identical across sectors due to perfect labor mobility.

The households first order condition for the purchase of financial claims from capital services producing firms in sector $x = C, I$ is

$$1 = E_t \beta \Lambda_{t+1} \frac{R_{x,t}^h P_{C,t}}{\Lambda_{t} P_{C,t+1}}.$$  

(7)

Household’s return, $R_{x,t}^h$, related to the acquisition of financial claims from capital services producers will be formalized in the following section.

As in Erceg et al. (2000), each household sets the wage according to Calvo contracts. The desired markup of wages over the household’s marginal rate of substitution (or wage mark-up), $\lambda_{w,t}$, follows an exogenous stochastic process.

14Note that consumption is not indexed by (j) because perfect risk sharing leads to similar asset holding across members of the household.
3.1.3 Production of capital goods

Production of physical capital. We assume that significant reallocation costs between sectors lead to immobile sector-specific capital.\textsuperscript{15} Capital producers in each sector $x = C, I$ manufacture capital goods using a fraction of investment goods from final-goods producers and undepreciated capital from capital-services producers, subject to investment adjustment costs (IAC), similar to Christiano et al. (2005). Solving the optimization problem of capital producers yields the standard capital accumulation equation

$$
\ddot{K}_{x,t} = (1 - \delta_x)\dot{K}_{x,t-1} + \mu_t \left( 1 - S\left( \frac{I_{x,t}}{I_{x,t-1}} \right) \right) I_{x,t},
$$

for $x = C, I$. The parameter $\delta_x$ denotes the sectoral depreciation rate, the function $S(I_{x,t}/I_{x,t-1})$ captures IAC and has standard properties — i.e. $S(\cdot)$ satisfies the following conditions: $S(1) = S'(1) = 0$ and $S''(1) = \kappa > 0$. Finally, the variable $\mu_t$ denotes the marginal efficiency of investment (MEI) shock, as in Justiniano et al. (2010).

Production of capital services and finance sources. The producers of capital services purchase capital from physical capital producers and choose the utilization rate to convert it into capital services. This purchase is financed by issuing claims on physical capital and producers have two sources of finance. As in Gertler and Karadi (2011), capital services producers issue claims to financial intermediaries to finance the purchase of capital at the end of each period, as described in the next subsection. Moreover — following the description of households — the producers can issue claims on physical capital to households and these purchases are facilitated through the money market funds. Capital services producers rent capital services to intermediate-goods producers that operate in a perfectly competitive market for a rental rate equal to $R_{x,t}^K/P_{C,t}$ per unit of capital. At the end of period $t + 1$, they sell the undepreciated portion of capital to physical capital producers. The utilization rate, $u_{x,t}$, transforms physical capital into capital services according to

$$
K_{x,t} = u_{x,t}\ddot{K}_{x,t-1},
$$

\textsuperscript{15}Ramey and Shapiro (2001) find strong evidence of large reallocation costs between sectors. Boldrin et al. (2001), Ireland and Schuh (2008), Huffman and Wynne (1999) and Papanikolaou (2011) establish that constrained factor mobility improves the performance of theoretical models of the business cycle to replicate movements in aggregate fluctuations.
for \( x = C, I \) and subjects to a cost \( a_x(u_{x,t}) \) per unit of capital. The function \( a_x(u_{x,t}) \) has standard properties—i.e. in steady state, \( u = 1 \), \( a_x(1) = 0 \) and \( \chi_x \equiv (a''_x(1)/a'_x(1)) \) denotes the cost elasticity.

Producers of capital services adjust capital acquisitions by adjusting financial claims to households and financial intermediaries, \( S^h_{x,t} \) and \( S^s_{x,t} \), respectively, at the nominal price \( Q^h_{x,t} \) and \( Q^s_{x,t} \), respectively. The total value of capital acquired, \( Q^T_{x,t} \tilde{K}_{x,t} \), equals the total value of financial claims held by households and financial intermediaries against this capital

\[
Q^T_{x,t} \tilde{K}_{x,t} = Q^h_{x,t} S^h_{x,t} + Q^s_{x,t} S^s_{x,t}.
\]

(9)

Capital services producers in sector \( x = C, I \) choose utilization and quantity of financial claims to households and financial intermediaries to maximize expected profits,

\[
\max_{u_{x,t},S^h_{x,t},S^s_{x,t}} E_0 \sum_{t=0}^{\infty} \beta^t A_t \left\{ \frac{R^K_{x,t}}{P_{C,t}} u_{x,t} \tilde{K}_{x,t-1} - a_x(u_{x,t}) \tilde{K}_{x,t-1} - \gamma^h S^h_{x,t} + \gamma S^s_{x,t} \right\} A_t V_t^{\frac{1}{1-a_t}}
\]

\[
- \Gamma \left[ \left( \frac{S^h_{x,t}}{S^h_{x,t} V_t^{1-a_t}} - e^{(1-a_t)g_v} \right), \left( \frac{S^s_{x,t}}{S^s_{x,t} V_t^{1-a_t}} - e^{(1-a_t)g_v} \right) \right] A_t V_t^{\frac{1}{1-a_t}},
\]

subject to the constraint (9).

Adjusting the level of financial claims is costly. First, adjustment entails fixed costs per unit of financial claim, \( S^h_{x,t} \) and \( S^s_{x,t} \), controlled by parameters \( \gamma^h \) and \( \gamma \) respectively (these parameters will be pinned down by steady state relationships as described in Appendix ??). Second, it also involves adjustment costs captured by the additively separable function \( \Gamma(\cdot) \).\(^{16}\)

Our approach is largely inspired by Gomes (2001), Cooley and Quadrini (2001) and Hennessy and Whited (2007) who specify fixed, linear and quadratic issuance costs for equity and by Altinkilic and Hansen (2000) who show that debt and equity issuance costs have fixed and convex cost components. Moreover, Leary and Roberts (2005) provide evidence to suggest capital structure choice is subject to adjustment costs.

Formally, the function has the following properties: \( \Gamma(0, 0) = 0 \), \( \Gamma_{S^h_{x,t}}(0, 0) = \Gamma_{S^s_{x,t}}(0, 0) = 0 \),

\(^{16}\)In this function, \( s^h_x = S^h_x / V_t^{1-a_t} \) and \( s^s_x = S^s_x / V_t^{1-a_t} \) denote the stationarized steady state expressions for claims on capital and \( g_v \) is the steady state growth rate of \( V_t \) (also the rate of growth of investment and capital). This implies that in the stationarized economy the function \( \Gamma(0, 0) \) equals zero in the steady state. The stationary economy is described in detail in Appendix ??.
\( \Gamma_{s^h, s^B}(0, 0) = \kappa^h > 0, \Gamma_{s^h, s^h}(0, 0) = \kappa^B > 0 \) and \( \Gamma_{s^B, s^B}(0, 0) = \Gamma_{s^h, s^h}(0, 0) = 0 \), where subscripts denote the marginal cost of \( \Gamma(\cdot) \). Intuitively, all capital acquisitions by capital services firms are financed from either banks or households. Any adjustment in the financing mix by deviating from the steady state levels of debt or equity entails costs.\(^{17}\) Note, that the key parameters that capture the (marginal) rigidities in the adjustment of sources of funds are denoted \( \kappa^h \) and \( \kappa^B \) and they are meant to capture the fact that firms often adjust equity and debt only slowly — one reason may be the well documented phenomenon of dividend smoothing (see Leary and Michaely (2011) and references therein). Our approach to modelling financial frictions is parsimonious. We do not explicitly model agency costs associated with the choice of debt and equity which is beyond the scope of the paper. Our approach is informed by and is similar to Jermann and Quadrini (2012) who capture equity payout costs in a reduced form way in a general equilibrium model.

The first order conditions of this problem are

\[
\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \frac{P_{x,t+1}^K}{P_{C,t+1}} - a_x(u_{x,t+1}) A_{t+1} \right] + \frac{Q_{x,t}^h}{Q_{x,t}^T} - \left( s_{x,t} \frac{s_{x,t} V_{t-1}^{\gamma_x}}{s_{x,t} V_{t-1}^{\gamma_x}} \right) = 0, \tag{10}
\]

\[
\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \frac{R_{x,t+1}^K}{P_{C,t+1}} - a_x(u_{x,t+1}) A_{t+1} \right] + \frac{Q_{x,t}^h}{Q_{x,t}^T} - \left( s_{x,t} \frac{s_{x,t} V_{t-1}^{\gamma_x}}{s_{x,t} V_{t-1}^{\gamma_x}} \right) = 0, \tag{11}
\]

\[
\frac{R_{x,t}}{P_{C,t}} - \frac{\gamma_A}{A_{t+1}} A_{t+1} = 0. \tag{12}
\]

Equations (10) and (11) equates the marginal benefit (e.g. how much the issuance of an additional claim contributes to larger capital production) to the marginal cost of adjustment (i.e. how much the issuance of an additional claim requires larger capital utilization and entails adjustment costs). In the stationary log-linear versions of these equations the

\(^{17}\)Note that since our model abstracts from bankruptcy and distress costs associated with debt we specify the function \( \Gamma(\cdot) \) to treat issuance costs with respect to equity and debt symmetrically.
adjustment costs for adjusting claims will be captured by the parameters $\gamma$, $\gamma^h$, $\kappa^B$ and $\kappa^h$. Equation (12) is the optimal condition for capital utilization.

As in Gertler and Karadi (2011) the stochastic return earned by financial intermediaries from financing capital acquisition is equal to

$$R^B_{x,t+1} = \frac{R^K_x u_{x,t+1} + Q_{x,t+1}(1 - \delta_x) - a_x(u_{x,t+1})A_{t+1}^{\alpha-1}}{Q_{x,t}}. \quad (13)$$

The analogous return that accrues to household’s is given by

$$P^h_{x,t} = \frac{R^K_x u_{x,t+1} + Q^h_{x,t+1}(1 - \delta_x) - a_x(u_{x,t+1})A_{t+1}^{\alpha-1}}{Q^h_{x,t}}. \quad (14)$$

### 3.1.4 Financial sector

Financial intermediaries fund the acquisitions of physical capital from capital-services producers using their own equity capital and deposits from households. They lend in specific islands (sectors) and cannot switch between them. Intuitively, this can be justified by appealing to financial market segmentation, where it may be costly to switch markets once you have developed expertise lending to your market.\(^{18}\) The financial sector in the model follows closely Gertler and Karadi (2011), and we therefore limit the exposition to the key equations and the online Appendix provides the complete set of equations. Three equations encapsulate the key dynamics in the financial sector: the balance sheet identity, the demand for assets that links equity capital with the value of physical capital (i.e. the leverage constraint) and the evolution of equity capital. We describe each of them in turn.

The nominal **balance sheet identity** of a branch that lends to sector $x = C, I$ is,

$$Q_{x,t} P_{C,t} S_{x,t} = N_{x,t} P_{C,t} + B_{x,t}, \quad (15)$$

where the variable $S_{x,t}$ denotes the quantity of financial claims on capital services that the

\(^{18}\) Alternatively we can interpret the financial sector as a single intermediary with two branches, each specializing in providing financing to one sector only, where the probability of lending specialization is equal across sectors and independent across time. Each branch maximizes equity from financing the specific sector. For example, within an intermediary, there are divisions specializing in consumer or corporate finance. The financial sector can be interpreted as a special case of Gertler and Kiyotaki (2010).
producers held by the intermediary, and $Q_{x,t}$ denotes the price per unit of claim. The variable $N_{x,t}$ denotes equity capital (i.e. wealth) at the end of period $t$, $B_{x,t}$ are households deposits, and $P_{C,t}$ is the price level in the consumption sector.

Financial intermediaries maximize the discounted sum of future equity capital (i.e. the expected terminal wealth). Bankers may abduct funds and transfer them to the household. This moral hazard/costly enforcement problem limits the capacity of financial intermediaries to borrow funds from the households and generates an endogenous leverage constraint that limits the bank’s ability to acquire assets. Thus, the equation for the demand of assets is

$$Q_{x,t}S_{x,t} = \varrho_{x,t}N_{x,t},$$

(16)

where the value of assets that the intermediary acquires ($Q_{x,t}S_{x,t}$) depends on equity capital, $N_{x,t}$, and the leverage ratio, $\varrho_{x,t} = \frac{\eta_{x,t}}{\lambda_B - \nu_{x,t}}$. In the expression above, $\lambda_B$ (fraction of assets bankers can divert for personal gain) is the key financial parameter that captures the agency problem between banks and depositors and we will estimate it in section 3.2. Note that when $\varrho_{x,t} > 1$, the leverage constraint (16) magnifies the changes in equity capital on the demand for assets. This amplification turns out to be the critical mechanism to attach an important role to news shocks in the estimated model.

The evolution of equity capital is described by the law of motion,

$$N_{x,t+1} = \theta_B [(R_{x,t+1}^B + R_t)\varrho_{x,t} + R_t] \frac{N_{x,t}}{\pi_{C,t+1}} + \varpi Q_{x,t+1}S_{x,t+1}\varsigma_{x,t},$$

(17)

where $\theta_B$ is the survival rate of bankers, $\varpi$ denotes the fraction of assets transferred to new bankers, $\pi_{C,t+1}$ denotes the gross inflation rate in the consumption sector and $\varsigma_{x,t}$ is an exogenous shock to the bank’s equity capital. Gerali et al. (2010) introduce bank equity shocks in a similar way in a credit and banking model of the Euro Area, but do not estimate the parameters associated with the shocks. Equation (17) shows that equity capital is a function of the excess (leveraged) real returns earned on equity capital of surviving bankers and the value of assets owned by news bankers. Banks earn expected (nominal) returns on

---

19 As shown in the online Appendix, the leverage ratio (i.e. the bank’s intermediated assets-to-equity ratio) is a function of the marginal gains of increasing assets, $\nu_{x,t}$ (holding equity constant), increasing equity, $\eta_{x,t}$ (holding assets constant), and the gain from diverting assets, $\lambda_B$. 

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assets (i.e. the risk premium) equal to

\[ R_{x,t}^S = R_{x,t+1}^B \pi_{C,t+1} - R_t, \quad (18) \]

for \( x = C, I \). The leverage constraint (16) entails non-negative excess returns that vary over time with movements in the equity capital of intermediaries. As in Gertler and Karadi (2011), there are no frictions in the process of intermediation between nonfinancial firms, and banks and therefore we can interpret the financial claims as one-period, state-contingent bonds in order to interpret the excess returns in equation (18) as a corporate bond spread.

### 3.1.5 Exogenous disturbances and arrival of information

The model embeds the following exogenous disturbances: sectoral shocks to the growth rate of TFP \((z_t, v_t)\), sectoral shocks to the level of TFP \((a_t, v_t)\), sectoral price mark-up shocks \((\lambda_{pt,t}^C, \lambda_{pt,t}^I)\), wage mark-up shock \((\lambda_{wt,t})\), preference shock \((b_t)\), monetary policy shock \((\eta_{mp,t})\), government spending shock \((g_t)\), MEI \((\mu_t)\) shock, and sectoral shocks to financial intermediaries’ equity capital \((\varsigma_{C,t}, \varsigma_{I,t})\). Each exogenous disturbance is expressed in log deviations from the steady state as a first-order autoregressive (AR(1)) process whose stochastic innovation is uncorrelated with other shocks, has a zero-mean, and is normally distributed. For the monetary policy shock \((\eta_{mp,t})\), the first order autoregressive parameter is set equal to zero. The online Appendix provides details on the exogenous disturbances.

The model embeds news shocks to sectoral productivity growth. The productivity growth processes in the consumption and investment sector follow the law of motions

\[ z_t = (1 - \rho_z) g_a + \rho_z z_{t-1} + \varepsilon^z_t, \quad \text{and} \quad v_t = (1 - \rho_v) g_v + \rho_v v_{t-1} + \varepsilon^v_t, \quad (19) \]

where the parameters \( g_a \) and \( g_v \) are the steady-state growth rates of the two TFP processes above, and \( \rho_z, \rho_v \in (0, 1) \) determine their persistence.

The representation of news shocks is standard and follows, for example, Schmitt-Grohe and Uribe (2012), and Khan and Tsoukalas (2012). The stochastic innovations in the exogenous disturbances in (19) are defined as

\[ \varepsilon^z_t = \varepsilon^z_{t,0} + \varepsilon^z_{t-4,4} + \varepsilon^z_{t-8,8} + \varepsilon^z_{t-12,12}, \quad \text{and} \quad \varepsilon^v_t = \varepsilon^v_{t,0} + \varepsilon^v_{t-4,4} + \varepsilon^v_{t-8,8} + \varepsilon^v_{t-12,12}. \]
where the first component, $\varepsilon_{t,0}^x$, is unanticipated (with $x = z, v$) whereas the components $\varepsilon_{t-4,4}^x$, $\varepsilon_{t-8,8}^x$, and $\varepsilon_{t-12,12}^x$ are anticipated and represent news about period $t$ that arrives four, eight and twelve quarters ahead, respectively. As conventional in the literature, the anticipated and unanticipated components for sector $x = C, I$ and horizon $h = 0, 1, \ldots, H$ are i.i.d. with distributions $N(0, \sigma_{z,t-h}^2)$ and $N(0, \sigma_{v,t-h}^2)$ that are uncorrelated across sector, horizon and time. Our choice to consider four, eight, and twelve quarter ahead sector-specific TFP news is guided by the desire to limit the size of the state space of the model while being flexible enough to allow the news processes to accommodate revisions in expectations.

### 3.1.6 Model summary

The model builds on Görtz and Tsoukalas (2017), one of the few existing DSGE models that can generate empirical relevance of TFP news shocks, with several notable innovations. These innovations allow us to quantify the overall degree of financial frictions through the lens of Bayesian estimation of the model.\(^{20}\)

Our choice to use a two sector model is three-fold. First, the methodology to measure aggregate TFP described in Fernald (2014) is based on sectoral TFP data. The equation is

\[ dTFP_{agg,t} = w_{i,t}dTFP_{i,t} + (1 - w_{i,t})dTFP_{c,t}, \]

where the variables $dTFP_{agg}$, $dTFP_{i}$, and $dTFP_{c}$ denote (utilization-adjusted) TFP growth rates in aggregate, investment- and consumption-specific sectors, respectively, and the coefficient $w_{i}$ denotes the share of the investment sector, expressed in value added. Equation

\(^{20}\)First, we extend the model to allow households to directly finance capital acquisition by capital services producers. These claims can be interpreted as corporate equity. Covas and Den Haan (2011) emphasize the importance of equity finance over the business cycle. Thus capital services firms have two sources of financing capital acquisitions available to them, one from banks in the form of corporate bonds (debt), and one directly from households in the form of corporate equity. We allow them to optimally choose the use between bonds and equity subject to rigidities in the adjustment of financial claims and estimate adjustment cost parameters that determine the degree of rigidities. Second, we also estimate the parameter that captures the limited enforcement problem between banks and depositors in the Gertler and Karadi (2011) set-up. Third, and consistent with the modelling innovation we introduce, we use a larger set of observables, including the relative price of investment and corporate equity, and estimate the model over a longer time horizon beginning from the onset of the Great Moderation. Fourth, we introduce financial shocks that compete with news shocks in the estimation. All these additional features allow for a more precise comparison with state-of-the-art estimated DSGE models and previous findings in the literature on the sources of business cycles.
shows that the aggregate TFP growth rate is an expenditure share-weighted average of sectoral TFP growth rates. The correlation between $dTFP_i$ and $dTFP_c$ is equal to 0.31, pointing to a weak co-movement between the two series and therefore suggesting that changes in aggregate TFP cannot be interpreted as a single homogeneous technological indicator. It is precisely the sectoral structure that allows us to reconstruct a TFP series from the model consistent with the empirical counterpart in order to be able to conduct the comparison exercise in section 4.1. Second, a two sector model allows a more precise decomposition of the data variation into shocks, compared to a one sector model. Last, Görts and Tsoukalas (2017) show that a two sector model, has a better fit with the data compared to a one sector model.

3.2 DSGE estimation

We estimate the DSGE model using quarterly U.S. data for the period 1984:Q1—2017:Q1, the same sample period as for the VAR model. We estimate the model using the following vector of observables: $[\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta \log W_t, \pi_{C,t}, \Delta (\frac{P_I}{P_C}), \log L_t, R_t, R^S_t, \Delta \log S^h_t, \Delta \log N_t]$, which comprises output ($Y_t$), consumption ($C_t$), investment ($I_t$), real wage ($W_t$), consumption sector inflation ($\pi_{C,t}$), relative price of investment ($\frac{P_I}{P_C}$), hours worked ($L_t$), nominal interest rate ($R_t$), corporate bond spread ($R^S_t$), corporate equity ($S^h_t$), and bank equity ($N_t$).
respectively, and the term $\Delta$ denotes the first-difference operator. Variables for aggregate quantities are expressed in real, per-capita terms using civilian noninstitutional population. We demean the data prior to estimation.\textsuperscript{24} We use these variables to keep the analysis as close as possible to related studies such as Smets and Wouters (2007), Justiniano et al. (2010) and Khan and Tsoukalas (2012) while incorporating important financial variables. 

The online Appendix provides a detailed description of data sources. The financial variables consist of the corporate bond spread as provided (and updated) by Gilchrist and Zakrajsek (2012), a measure of market value of intermediaries’ equity capital, and a measure of corporate equity for the non financial corporate sector available from the Flow of Funds Accounts of the Federal Reserve Board (Z.1 Financial Accounts of the United States). The market value of commercial bank’s equity we use is computed as the product of price and shares outstanding using monthly data from CRSP.

In the DSGE model, TFP news shocks compete with other shocks to account for the variation in the data. The cross equation restrictions implied by the equilibrium conditions of the model identify the different shocks. We estimate a subset of parameters using Bayesian methods and calibrate the remaining parameters as described in Table 12 of the online Appendix. The prior distributions conform to the assumptions in Justiniano et al. (2010) and Khan and Tsoukalas (2012), as reported in Table 1 and posterior estimates of parameters common with these studies are broadly in line with them so we do not discuss them in detail.\textsuperscript{25}

We discuss the parameter estimates that control the degree of financial frictions, namely, the marginal adjustment costs parameters, $\kappa_h$ and $\kappa_B$, which control the degree to which marginal costs are affected by the speed at which firms are adjusting equity and debt, and

\textsuperscript{24}Removing sample means from the data prevents the possibility that counterfactual implications of the model for the low frequencies may distort inference on business cycle dynamics. For example, in the sample, consumption has grown by approximately 0.32% on average per quarter, while output has grown by 0.20% on average per quarter respectively. However, the model predicts that they grow at the same rate. Thus, if we hardwire a counterfactual common trend growth rate in the two series, we may distort inference on business cycle implications that is of interest to us.

\textsuperscript{25}We have examined the identification of the model parameters using various metrics: evidence on prior and posterior densities, marginal likelihood comparisons between the baseline model and a model estimated without news shocks, and the tests of Iskrev (2010) and Koop et al. (2013). These results are available upon request.
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<tr>
<th>Parameter</th>
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<th>Prior Distribution</th>
<th>Posterior Distribution</th>
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<td>Consumption habit</td>
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<td>$\kappa_B$</td>
<td>Bank financing adj. cost</td>
<td>Gamma 0.10 0.10</td>
<td>Mean 0.0679 0.0568 0.0783</td>
</tr>
<tr>
<td>$\lambda_B$</td>
<td>Fraction of funds bankers can divert</td>
<td>Beta 0.60 0.02</td>
<td>Mean 0.6235 0.5790 0.6597</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>C-sector TFP growth</td>
<td>Beta 0.40 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>I-sector TFP growth</td>
<td>Beta 0.40 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_\phi$</td>
<td>Preference</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_\phi$</td>
<td>Marginal efficiency of investment</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Government spending</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3C}$</td>
<td>C-sector price markup</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3I}$</td>
<td>I-sector price markup</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Wage markup</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3C}$</td>
<td>C-sector stationary TFP</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3I}$</td>
<td>I-sector stationary TFP</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3C}$</td>
<td>C-sector bank equity</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
<tr>
<td>$\rho_{3I}$</td>
<td>I-sector bank equity</td>
<td>Beta 0.60 0.20</td>
<td>Mean 0.9529 0.9398 0.9668</td>
</tr>
</tbody>
</table>

**Notes.** The posterior distribution of parameters is evaluated numerically using the random walk Metropolis-Hastings algorithm. We simulate the posterior using a sample of 500,000 draws and discard the first 100,000 of the draws.
the limited enforcement parameter, $\lambda_B$. We set the prior means for these adjustment cost parameters equal to 0.1, broadly guided by marginal equity flotation costs and indirect distress costs associated with bond issuance estimated in Hennessy and Whited (2007). For the limited enforcement parameter, $\lambda_B$, we set a relative tight prior with a mean of 0.6, broadly consistent with the average (quarterly) GZ spread in the data of 0.5% and an average leverage ratio in our sample of 3.34. Its interesting to comment on the posterior estimates for $\kappa^h$ and $\kappa^B$. Information from the data implies posterior estimates which are shifted to the left of the prior means and are equal to 0.068 and 0.026 respectively. Note, that these estimates are still considerably different from zero (which would imply no rigidities) and imply quantitatively relevant rigidities in the adjustment of financial claims. Moreover, the estimates suggest that marginal equity adjustment costs are higher than corresponding marginal debt adjustment costs. The posterior estimate for $\lambda_B$ is equal to 0.62, and it implies a steady state leverage ratio close to its counterpart in the data as discussed above.

3.3 Results from the DSGE model

In this section we discuss key findings from the DSGE model on the empirical significance and the dynamic propagation of news shocks. We also provide a comparison with findings from standard models in the literature that abstract from financial frictions.

Table 2 reports the variance decomposition of the estimated DSGE model for each news shock and the sum of the unanticipated shocks. The entries show that the estimation assigns significant importance to TFP news shocks as a source of fluctuations. In their totality, TFP news shocks account for 52.3%, 50.8%, 42.6%, 50.1% of the variance in output, consumption, investment and hours worked, respectively, at business cycle frequencies. Consumption-specific news shocks play a major role in this total, accounting for 47.3%, 40.3%, 36.3%, 46.3% of the variance in the same macro aggregates. The estimation finds strong links between financial variables and real aggregates as sectoral news shocks explain a sizeable share

26The leverage ratio, most consistent with the model concept, is computed as the ratio of commercial and industrial loans and securities in bank credit (numerator) to equity (denominator) for U.S. commercial banks (H8 release).
Table 2: Variance decomposition at posterior estimates—business cycle frequencies (6-32 quarters)

<table>
<thead>
<tr>
<th></th>
<th>MEI</th>
<th>Bank equity shocks</th>
<th>all other shocks</th>
<th>Sum of TFP growth shock contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>unanticipated</td>
</tr>
<tr>
<td>Output</td>
<td>7.6</td>
<td>0.2</td>
<td>20.1</td>
<td>19.8</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.0</td>
<td>0.0</td>
<td>34.2</td>
<td>15.0</td>
</tr>
<tr>
<td>Investment</td>
<td>6.4</td>
<td>0.2</td>
<td>37.1</td>
<td>13.8</td>
</tr>
<tr>
<td>Total Hours</td>
<td>4.9</td>
<td>0.1</td>
<td>34.8</td>
<td>10.0</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.0</td>
<td>0.0</td>
<td>40.7</td>
<td>10.2</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>4.5</td>
<td>0.1</td>
<td>56.4</td>
<td>3.0</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.0</td>
<td>0.0</td>
<td>95.4</td>
<td>0.6</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>12.0</td>
<td>7.8</td>
<td>38.4</td>
<td>4.5</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.3</td>
<td>69.9</td>
<td>2.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>8.0</td>
<td>11.9</td>
<td>66.0</td>
<td>3.3</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>0.0</td>
<td>0.0</td>
<td>65.7</td>
<td>13.3</td>
</tr>
</tbody>
</table>

MEI = marginal efficiency of investment shock; bank equity shocks are the sum of the consumption sector and investment sector bank equity shock. Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

The propagation of news shocks and the co-movement of aggregate variables hinge on the countercyclical markups, as outlined in Göritz and Tsoukalas (2017) in the context of a two-sector model with nominal rigidities and news shocks. In the aftermath of a positive news shock, countercyclical markups move labour demand and supply curves rightwards offsetting the negative wealth effect on labour supply, thereby generating co-movement in aggregate variables.
the variation in credit spreads is not driven by default risk).

These findings are in sharp contrast to the results from a DSGE model that abstracts from financial frictions. To isolate the contribution of the financial channel in our model, we estimate a restricted version of the two-sector model that abstracts from financial frictions. Table 3 compares the variance decomposition across the different models and shows that version of the model that abstracts from financial frictions finds a limited empirical role to news shocks. In this constrained version of the baseline model, the totality of TFP news shocks account for approximately 14% of the variation in output. This finding is consistent with related results in the DSGE literature that attribute a limited role to TFP news shocks (see, for example, Fujiwara et al. (2011), Khan and Tsoukalas (2012) and Schmitt-Grohe and Uribe (2012), among others). It is worth reporting that the estimated DSGE model can successfully replicate the significant predictive ability of the credit spread for economic activity consistent with the findings in Gilchrist and Zakrajsek (2012). The details of this forecasting exercise are described in the online Appendix.

We examine IRFs in order to gain intuition on the propagation of TFP news shocks and isolate the mechanism that enhances their empirical relevance in the baseline model with financial frictions. Figure 4 plots the response of selected variables to a three-year ahead consumption-specific TFP news shock in the baseline model (solid line) together with those for the estimated two-sector model without financial frictions (dashed line). We normalize the shock to be of equal size across simulations.

From Figure 4 it is notable that the amplification of the news shock is significantly stronger in the model with the financial channel. In this model the impact of the consumption-specific news shock is amplified by the effect of corporate bond prices on intermediaries’ equity. A positive news shock raises bond prices, which in turn boost bank equity. Better capitalized banks expand demand for capital assets, and the process further increases bond

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28 This model turns off the financial channel, i.e. the balance sheet identity (15), the leverage constraint (16), the evolution of equity capital (17), and the financial constraint (9) that describe the financial sector as well as equations (7), (10) and (11) that allow capital services producers to raise funds from households. The only other difference is in the set of shocks. The restricted version has the same set of shocks except bank equity shocks which are specific to the baseline model.
Table 3: Variance decomposition – business cycle frequencies (6-32 quarters)

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>Two sector model without financial frictions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all TFP</td>
<td>all TFP</td>
<td>MEI</td>
</tr>
<tr>
<td></td>
<td>unanticipated</td>
<td>news</td>
<td>shocks</td>
</tr>
<tr>
<td>Output</td>
<td>19.8</td>
<td>52.3</td>
<td>7.6</td>
</tr>
<tr>
<td>Consumption</td>
<td>15.0</td>
<td>50.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Investment</td>
<td>13.8</td>
<td>42.6</td>
<td>6.4</td>
</tr>
<tr>
<td>Total Hours</td>
<td>10.0</td>
<td>50.1</td>
<td>4.9</td>
</tr>
<tr>
<td>Real Wage</td>
<td>10.2</td>
<td>49.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>3.0</td>
<td>36.0</td>
<td>4.5</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.6</td>
<td>4.0</td>
<td>0.0</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>4.5</td>
<td>37.3</td>
<td>12.0</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>3.9</td>
<td>23.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>3.3</td>
<td>10.7</td>
<td>8.0</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>13.3</td>
<td>21.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

Figure 4: Responses to a one std. deviation TFP news shock (anticipated 12-quarters ahead) in the consumption sector. Baseline model with financial intermediation (solid line), and estimated model without financial intermediation (dashed line) (baseline shock persistence and standard deviation). The horizontal axes refer to quarters and the units of the vertical axes are percentage deviations.
prices, leading to a strong investment boom and a decline in the excess premiums on holding bonds, noted as C-Sector spread and I-Sector spread in the figure. Although in equilibrium there is no default of intermediaries, higher equity implies that depositors are better protected from the costly enforcement/inefficient liquidation problem and hence they are willing to place deposits in banks that earn a lower excess premium. The response of the excess bond premium we have documented in section 2.3 is hence consistent with the narrative from the model. Figure 4 shows that the responses of bond prices are qualitatively different between the two models. In the baseline model with financial frictions, bond prices rise sharply due to the amplification effect of financial intermediaries on the demand for capital. As the stock of capital increases and accumulates, agents expect returns from capital to decline. Other things equal, the surge in bond prices creates a strong incentive to build new capital before the improvement in technology materializes, which in turn stimulates a strong rise in current hours worked and investment. In contrast, in the model without financial frictions, the shadow values (i.e. Tobin’s q) of capital increase moderately on impact and rise further in the future, which suppresses—relative to the baseline—current investment spending in anticipation of future increase in the returns to capital. Its is also noteworthy to report that both bond prices and corporate equity prices in the model rise strongly in response to the news shock (see middle row in the figure) consistent with the VAR evidence.

Our study provides relevant insights on the significance of the marginal efficiency of investment (MEI) shock, which recent studies that estimate DSGE models with and without news shocks (Khan and Tsoukalas (2012) and Justiniano et al. (2010), respectively), find considerably more important than TFP shocks to explain business cycles fluctuations. We corroborate these findings in the estimated version of the model that abstracts from the financial channel (see Table 3). For instance, in the two-sector model without financial fric-

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29Strictly speaking, the comparison in the Figure is between the shadow value of capital in the model without the financial channel to the bond price, which represents the price of a claim to capital, in the baseline model.

30We include the MEI shock in the estimation for comparison purposes with the literature. The MEI shock differs from the investment-specific shock in that the latter is a permanent shock and affects only the productivity of the investment sector. By contrast, the MEI impacts the transformation of investment goods to installed capital and affects both sectors.
tions, MEI shocks explain the bulk of movements in the variance of output (52%), investment (66%), and hours worked (47%). In contrast, in the baseline model with the financial sector, MEI shocks account for approximately, 8%, 6%, and 5% in the variance of the same set of macroeconomic aggregates. The key reason for the reduced role of MEI shocks in the presence of financial frictions is related to the fact that an exogenous increase in MEI generates a fall in the price of installed capital by increasing the transformation rate of investment goods to installed capital. The decline in capital prices severs the financial channel that stimulates equity capital gains for the financial intermediaries in response to an increase in investment demand and capital prices. Thus, a decline in capital prices induces a fall in bank equity and restricts the facilitation of lending and investment spending. The same logic operates in the case of investment-specific shocks of the unanticipated or anticipated type.

4 Reconciling DSGE and VAR results

4.1 The DSGE as the data generating process

In this section, we compare the dynamics responses to TFP news shocks across the DSGE and VAR analysis. We perform a Monte Carlo experiment and generate 300 samples of artificial data from the DSGE model, drawing parameter values from the posterior distribution. We compare the empirical IRF from the VAR model against those estimated with identical VAR specifications (along with posterior bands) on the artificial data samples. This exercise is similar in spirit to Barsky et al. (2015) who compare empirical VAR and model implied VAR responses produced by a standard calibrated New Keynesian model. Following the methodology in Fernald (2014), we extract a model-based aggregate TFP measure by weighting (using GDP shares) together the two model-based sectoral TFP growth components as in equation (20) referred to in section 3.

31 We show in the online Appendix that the results of the comparison between the baseline model and a two-sector model without financial frictions also extend to a one-sector model without the financial channel. In comparison to the baseline setup the role of news shocks is much more limited in the one-sector model and MEI shocks are more relevant for explaining variations in macroeconomic aggregates.
Figure 5: TFP news shock. The solid black line is the impulse response to TFP news shock from a six variable VAR featuring aggregate TFP, corporate bond spread (GZ spread), consumption, output, hours, CPI inflation, estimated with 5 lags. The blue line with diamonds is the median impulse response to an aggregate TFP news shock estimated from a VAR on 300 samples, generated from the model. The black dashed (blue dot-dashed) lines are the corresponding 16% and 84% confidence bands. Units of the vertical axes are percentage deviations.

Figure 5 compares the IRF from the empirical VAR model (black lines) with those from the Monte Carlo experiment (blue lines). Several features are noteworthy. First, the model based VAR responses exhibit the delayed response of TFP along with an immediate strong and significant increase in the activity variables that is also present in the empirical VAR responses. Second, we also observe an immediate and significant decline in the credit spread in the model based VAR responses consistent with the empirical VAR responses. Overall, the empirical and model implied responses are qualitatively consistent, indicating a broad based expansion ahead of the future increase in TFP. Finally, we turn to discuss the response of inflation. As the reader can observe, at least qualitatively, the model implied responses also produce an immediate (median) decline in inflation as in the empirical VAR responses. Moreover, the model implied VAR response predicts an inflation response path, where the initial, compared to the empirical VAR response, decline is smaller and it is not significantly different from zero (as is the entire path in the model implied responses). Therefore, the
model, statistically speaking, does not deliver a strong and robust decline in inflation as in the data. In the model, which is built around a New Keynesian core, current inflation is a function of future real marginal costs and the latter decline when higher TFP materializes. But the impact response of inflation depends on the entire path of real marginal costs. We know from Figure 4 that a TFP news shock generates an immediate and strong boom in activity and this comes hand in hand with an increase in real marginal costs in the short term, before any future realization of TFP raises productivity. Thus inflation can increase or decrease on impact depending on whether the short term increase of real marginal cost due to the initial outburst of activity in anticipation of the future increase in productivity outstrips or falls short of the medium to long term decline in real marginal cost after productivity has improved. In other words the anticipation horizon matters. The model contains one, two, and three year ahead future TFP growth shocks. Figure 6 shows the responses of inflation and marginal costs to these shocks. Future real marginal costs decline very sharply and outstrip the initial short term increase in response to the one year ahead future TFP shock, and inflation declines. A longer horizon of anticipation in the future TFP increase however, produces the opposite inflation dynamic, i.e. an increase on impact, because the short term rise in real marginal cost counterbalances the fall in real marginal costs when productivity improves. Among the key parameters of the model which play a role in how real marginal costs respond and ultimately determine the response of inflation are the degree of wage rigidities, the parameters of the policy rule, and the parameters that govern the process for TFP. The new finding from this analysis is the dependence of the short term inflation response on the timing of the news shock, i.e., the anticipation horizon for the future rise in TFP. At the estimated parameters of the model, as shown in Figure 6, inflation falls on impact for the one year ahead shock and rises for the longer anticipated news shocks.

Qualitatively the similarity of the dynamic VAR responses in Figure 5 is a success of the model considering there are key differences in the estimation methodologies between the DSGE and VAR methods. The two methods identify the news shock from different empirical moments, they use a different set of observables and consequently entertain a
different number of shocks. The qualitative similarity of responses suggests that accounting for financial frictions can go some way to reconcile existing and often conflicting results in the literature using the DSGE and VAR methodologies. In the following section, we undertake a variance decomposition exercise to suggest that using this metric there is also a good degree of quantitative similarity in the role the two methodologies assign to TFP news shocks.

4.2 A quantitative evaluation

To evaluate the quantitative differences between the VAR and DSGE methods, we compare the forecast error variance decompositions (FEVD) for the totality of TFP news shocks obtained from the VAR and DSGE models at business cycle frequencies (6-32 quarters). Table 4 shows the FEVD of the common variables in the VAR model (top panel), the baseline DSGE model with financial frictions (center panel), and the DSGE model without financial frictions (bottom panel).

Table 4 shows that in general the median shares of the FEVD accounted for by TFP news shocks in the DSGE model with financial frictions are close and, in the vast majority of cases, fall within the posterior bands of the median shares predicted by the VAR model.
The model that abstracts from financial frictions predicts instead a considerably smaller role that news shocks play in explaining movements in macroeconomic variables. An obvious shortcoming of the model without financial frictions, relative to the baseline model, is its inability to account for the variance in the corporate bond spread indicator. We see this exercise as a useful and informative test to show that accounting for financial frictions, two important methodologies — VAR and DSGE — independently provide a consistent reading on the importance of TFP news shocks. This is despite the differences in the two methods as discussed in the previous section.

4.3 TFP news and financial shocks

In this section, we report results from a streamlined version of the baseline model that encompasses a richer menu of financial shocks. It introduces, in addition to bank equity shocks, shocks that perturb the excess return to capital, equation (18). These shocks can be interpreted as “risk appetite” shocks: ceteris paribus, a positive shock of this type increases the demand for assets by financial intermediaries, and consequently the supply of credit.32 Our goal is to focus on a relative quantitative comparison between disturbances that emanate in the real economy and disturbances that emanate in the financial sector. For this purpose we economize on disturbances that do not admit a straightforward structural interpretation or have very limited contribution in accounting for the variance in the data.33 We estimate this version of the model, and show the variance decomposition in Table 5 below—the full decomposition is reported in the online Appendix. There are two findings to report.34 First, the empirical significance of TFP news continues to be substantial, similar to the baseline model. Second, “risk appetite” shocks explain a sizeable fraction of fluctuations, accounting

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32 We remain agnostic on the timing of arrival of “risk appetite” shocks and incorporate news components— as well as an unanticipated component—consistent with the work of Christiano et al. (2014) who emphasized the importance of financial risk news shocks.

33 To this end we have removed the preference, MEI, government spending, stationary sectoral TFP, and mark-up shocks in the investment sector. Details are provided in the online Appendix.

34 In the online Appendix we also report results from estimation of the baseline model with risk appetite shocks. As in the streamlined version, in the extended version the empirical significance of TFP news is substantial and similar to the baseline.
Table 4: Share of variance explained by TFP news shocks

<table>
<thead>
<tr>
<th>horizon (quarters)</th>
<th>VAR (medians and 16% and 84% posterior bands in brackets)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output*</td>
</tr>
<tr>
<td></td>
<td>[44 55 64 67 69]</td>
</tr>
<tr>
<td></td>
<td>Consumption*</td>
</tr>
<tr>
<td></td>
<td>[48 60 69 72 75]</td>
</tr>
<tr>
<td></td>
<td>Investment†</td>
</tr>
<tr>
<td></td>
<td>[32 47 47 47 47]</td>
</tr>
<tr>
<td></td>
<td>Total Hours*</td>
</tr>
<tr>
<td></td>
<td>[36 46 49 47 45]</td>
</tr>
<tr>
<td></td>
<td>GZ Spread*</td>
</tr>
<tr>
<td></td>
<td>[34 34 34 35 38]</td>
</tr>
<tr>
<td></td>
<td>Excess bond premium♭</td>
</tr>
<tr>
<td></td>
<td>[39 37 38 38 39]</td>
</tr>
<tr>
<td></td>
<td>Bank equity†</td>
</tr>
<tr>
<td></td>
<td>[86 88 88 88 86]</td>
</tr>
<tr>
<td></td>
<td>S&amp;P 500*</td>
</tr>
<tr>
<td></td>
<td>[35 80 44 85 46 85 46 87 45 89]</td>
</tr>
<tr>
<td></td>
<td>C-Sector Inflation*</td>
</tr>
<tr>
<td></td>
<td>[21 21 21 22 22]</td>
</tr>
<tr>
<td></td>
<td>DSGE Model with Financial Fricitions (medians)</td>
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<tr>
<td></td>
<td>Output</td>
</tr>
<tr>
<td></td>
<td>[33 44 44 46 51]</td>
</tr>
<tr>
<td></td>
<td>Consumption</td>
</tr>
<tr>
<td></td>
<td>[32 38 32 36 49]</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
</tr>
<tr>
<td></td>
<td>[35 41 39 36 35]</td>
</tr>
<tr>
<td></td>
<td>Hours</td>
</tr>
<tr>
<td></td>
<td>[33 44 46 44 42]</td>
</tr>
<tr>
<td></td>
<td>C-Sector Inflation</td>
</tr>
<tr>
<td></td>
<td>[0 1 3 5 8]</td>
</tr>
<tr>
<td></td>
<td>GZ spread</td>
</tr>
<tr>
<td></td>
<td>[48 29 31 33 35]</td>
</tr>
<tr>
<td></td>
<td>Bank Equity</td>
</tr>
<tr>
<td></td>
<td>[24 23 24 24 25]</td>
</tr>
<tr>
<td></td>
<td>Corporate Equity</td>
</tr>
<tr>
<td></td>
<td>[30 26 17 16 16]</td>
</tr>
<tr>
<td></td>
<td>C-Sector Price of Capital</td>
</tr>
<tr>
<td></td>
<td>[19 24 35 40 45]</td>
</tr>
<tr>
<td></td>
<td>I-Sector Price of Capital</td>
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<td>[74 74 74 73 71]</td>
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<tr>
<td></td>
<td>Average Price of Capital</td>
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<tr>
<td></td>
<td>[47 49 55 57 58]</td>
</tr>
<tr>
<td></td>
<td>DSGE Model without Financial Fricitions (medians)</td>
</tr>
<tr>
<td></td>
<td>Output</td>
</tr>
<tr>
<td></td>
<td>[9 8 9 12 19]</td>
</tr>
<tr>
<td></td>
<td>Consumption</td>
</tr>
<tr>
<td></td>
<td>[25 22 9 8 12]</td>
</tr>
<tr>
<td></td>
<td>Investment</td>
</tr>
<tr>
<td></td>
<td>[4 5 12 15 20]</td>
</tr>
<tr>
<td></td>
<td>Hours</td>
</tr>
<tr>
<td></td>
<td>[7 10 14 17 22]</td>
</tr>
<tr>
<td></td>
<td>C-Sector Inflation</td>
</tr>
<tr>
<td></td>
<td>[0 1 3 4 6]</td>
</tr>
<tr>
<td></td>
<td>C-Sector Price of Capital</td>
</tr>
<tr>
<td></td>
<td>[11 15 18 18 16]</td>
</tr>
<tr>
<td></td>
<td>I-Sector Price of Capital</td>
</tr>
<tr>
<td></td>
<td>[19 22 31 33 32]</td>
</tr>
<tr>
<td></td>
<td>Average Price of Capital</td>
</tr>
<tr>
<td></td>
<td>[15 19 25 26 24]</td>
</tr>
</tbody>
</table>

The FEV of variables denoted with a * are obtained from a seven variable VAR baseline specification with TFP, consumption, output, hours, GZ spread, S&P500 and inflation (consistent with VAR in Figure 1). The FEV of variables denoted with a † are obtained from the baseline VAR specification in which the GZ spread is replaced with the EBP. The FEV of variables denoted with a ‡ are obtained from the baseline VAR specification where investment replace consumption.
for 9.3%, 9.7%, 16.0%, 11.4% of the variance in output, consumption, investment, and hours respectively, and 66.6% of variance in the GZ spread.\textsuperscript{35} This is consistent with the notion that risk shocks, independently from real disturbances, affecting the supply of credit can have significant real effects.

Table 5: Variance decomposition — business cycle frequencies (6-32 quarters)

<table>
<thead>
<tr>
<th></th>
<th>TFP news baseline model</th>
<th>TFP news simple model</th>
<th>Financial shocks simple model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>52.3</td>
<td>69.7</td>
<td>9.3</td>
</tr>
<tr>
<td>Consumption</td>
<td>50.8</td>
<td>65.9</td>
<td>9.7</td>
</tr>
<tr>
<td>Total Investment</td>
<td>42.6</td>
<td>74.6</td>
<td>16.0</td>
</tr>
<tr>
<td>Total Hours</td>
<td>50.1</td>
<td>80.2</td>
<td>11.4</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>37.3</td>
<td>13.7</td>
<td>66.6</td>
</tr>
</tbody>
</table>

Decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption and investment. The spectral density is computed from the model’s state space representation with 500 bins for frequencies covering the range of periodicities. We report median shares.

5 Conclusion

This paper examines the empirical significance and dynamic effects of TFP news shocks in the context of financial frictions using complementary VAR and DSGE methods. The VAR model identifies two robust stylized facts. First, a shock to future TFP is associated with a significant decline of credit spread indicators, with highly predictive content as recently emphasized by (Gilchrist and Zakrajsek (2012)) along with a broad-based expansion in activity. The credit spread indicators include the GZ spread and the excess bond premium. The decline in credit spread indicators is associated with an improvement in the balance sheet conditions of financial intermediaries, suggesting that credit supply conditions are critical for the propagation of news shocks. Second, we independently identify a single shock that seeks to explain as much as possible of the un-forecastable movements in the excess bond premium. This single shock explains approximately 75% in the forecast error variance of

\textsuperscript{35}It is interesting to note that quantitatively the role assigned by the model to financial shocks is broadly consistent with the VAR decomposition results for activity aggregates and EBP component of the GZ spread reported in Gilchrist and Zakrajsek (2012).
the latter. Importantly, the dynamic macro effects generated by this shock are qualitatively and quantitatively very similar to the macro effects generated by the TFP news shock. This finding provides strong support for the notion that movements in credit spread indicators are tightly linked with news shocks.

We employ a DSGE model with financial frictions and suggest it is a useful structural framework to understand the propagation of news shocks emphasizing credit supply frictions. The model analysis shows that the critical mechanism for the strong macro effects of news shocks relies on the linkages between leveraged equity, bond prices, and excess premiums which vary inversely with the balance sheet condition of intermediaries, consistent with the VAR evidence. Moreover, the estimated model generates dynamic responses and quantitative estimates of TFP news shocks very similar to those obtained from the VAR model. The consistent assessment of news shocks across methods provides support for the traditional ‘news view’ of business cycles.

Our analysis suggests several avenues for future research that go beyond the scope of this paper. Our model features an exogenous TFP process where agents receive signals about future TFP developments. Whereas this is a parsimonious and flexible way to introduce ‘news’ shocks in a medium scale DSGE it nevertheless is silent about the drivers of TFP dynamics. We believe that endogenous medium term developments in TFP may interact with short term financing frictions in ways that have not been emphasized in the literature, and such interactions may be important to understand why some technologies are successfully adopted while others never make it to the technology frontier. One possible avenue to unify the traditional notion of TFP news with endogenous TFP is to introduce imperfect learning (noisy signals) about the profitability of new innovations. In such an environment noisy signals will give rise to forecast errors about future profitability and eventually productivity (as a fraction of innovations are adopted) and has the ability to generate cycles due to expectation shifts (Pigou cycles) as emphasized in the traditional news literature within an endogenous TFP framework. Moreover, introducing constrained banks to fund innovation activity as in Queralto (2019) has the potential to amplify forecast errors due to noise.
These features combined have two potentially interesting implications. First, the emergence of wasteful financing booms which are eventually reversed when the impact of noise dies out, boom-bust patterns in the spirit of Pigou cycles. Second, how and to what extent high frequency noise interacts with medium term TFP dynamics. Of course, the challenge remains of how such a model will better account for the S-shaped delayed pattern for TFP documented in this paper. We leave a detailed exploration of such considerations for future work.

References


ON-LINE APPENDIX
News Shocks under Financial Frictions*

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March 2021

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1 Supporting details and results

1.1 Robustness to max FEV credit spread shock indicator

Figure 1 displays the variance shares explained by the max FEV EBP shock discussed in the main body, section 2.

![Graphs showing variance decomposition for different variables](attachment:image.png)

Figure 1: **Variance Decomposition** FEV of variable ‘x’ of the max FEV EBP shock (median solid line). The shaded gray areas are the 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. Vertical axes show percentages.

Figure 2 displays IRFs to a TFP news shock (as shown in Figure 1 in the main body) and the median responses to a single shock that maximizes the FEV of the GZ spread over forecast horizons six to thirty-two quarters (red dashed line). As shown in the main body for the EBP, also when we use the GZ spread as target variable to identify the shock that maximizes variation in the spread, the responses to this shock are qualitatively and quantitatively very similar to the responses to a TFP news shock.

1.2 Robustness to VAR methodology

The results in the main body of the paper are generated using the Francis et al. (2014) identification approach (referred to as Max share method). This section reports VAR findings using two alternative approaches. First, the identification scheme in Barsky and Sims (2011)
that recovers the news shock by maximizing the variance of TFP over the horizons zero to 40 quarters, and the restriction that the news shock does not move TFP on impact. Second, the identification scheme in Kurmann and Sims (2016), that recovers the news shock by maximizing the FEV of TFP at a very long horizon (80 quarters) without however imposing the zero impact restriction on TFP conditional on the news shock.\textsuperscript{1, 2}

Figures 3 and 4 show the responses obtained from the two alternative identification methods in relation to the responses shown in the main body. The IRFs are qualitatively and quantitatively very similar to each other. Qualitatively, all methods suggest that TFP rises significantly above zero only with a significant delay, except that the Kurmann and Sims (2016) method allows TFP to jump on impact (though the response is not significant different from zero). Importantly, the results suggest that the identified news shocks from

\textsuperscript{1}These authors argue that allowing TFP to jump freely on impact, conditional on a news shock, produces robust inference to cyclical measurement error in the construction of TFP.

\textsuperscript{2}A third, alternative identification proposed in the literature is the Forni et al. (2014) long-run identification scheme. This method identifies the news shock by imposing the zero impact restriction on TFP, and seeks to maximise the impact of the news shock on TFP in the long run. As such it is very similar in spirit to the Max Share method we employ as a baseline identification. Responses are qualitatively and quantitatively very similar between these two identification schemes. We don not show these results for space considerations, but IRFs are available upon request.
the three alternative methods are qualitatively and in the majority of cases quantitatively very similar to each other.

Figure 3: TFP news shock. Impulse responses to a TFP news shock from a seven-variable VAR. The black solid line shows the median using the baseline news shock identification and the shaded gray areas are the corresponding 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The dashed blue lines show the median and posterior bands when using the Barsky and Sims (2011) identification. The units of the vertical axes are percentage deviations.

1.3 Robustness of VAR results to alternative samples

In addition to the results reported in the main body of the paper for the sample 1984Q1-2017Q1, we also report results for a sample without the Great Recession period (1984Q1-2007Q3). Independently, we identify responses to a TFP news shock and, using the agnostic approach in Uhlig (2003), we identify the single shock that maximizes the forecast error variance of the EBP at business cycle frequencies. Figure 5 shows responses to these shocks based on seven-variable VARs estimated with 3 lags (to account for the relatively short sample length). The results for the shorter sample without the Great Recession are consistent with the ones shown in the main body. Most notably, the EBP declines significantly on impact and both the max FEV EBP shock and the TFP news shock trigger very similar dynamic responses.
Figure 4: **TFP news shock.** Impulse responses to a TFP news shock from a seven-variable VAR. The black solid line shows the median using the baseline news shock identification and the shaded gray areas are the corresponding 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The dashed blue lines show the median and posterior bands when using the Kurmann and Sims (2016) identification. The units of the vertical axes are percentage deviations.

Figure 5: **TFP news shock and max-EBP shock.** Sample without Great Recession. Median IRFs to a TFP news shock (solid black line) and a max FEV EBP shock (dashed red line) from seven-variable VARs. The shaded gray areas are the 16% and 84% posterior bands of the TFP news shock generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.
1.4 Robustness to VAR results: TFP news and financial shocks

In this section, we identify TFP news and financial shocks within the same VAR framework. We identify the TFP news shock as described in section 2.2 and then identify the financial shock as the innovation to the excess bond premium (EBP), similar to the approach in Gilchrist and Zakrajsek (2012). The IRFs to the identified financial shock displayed in Figure 6 are qualitatively very similar to the ones generated from the max FEV EBP shock and the TFP news shock (Figure 3). One key difference that distinguishes between a financial shock and a news TFP shock is the behavior of inflation. The former is an inflationary shock, i.e. inflation rises with activity, while the latter is a dis-inflationary shock, i.e. inflation declines with activity.

Finally, we have undertaken an additional robustness exercise with respect to the identification of the news TFP shock. Specifically, the TFP news shock is identified as the shock that maximizes the variance of TFP at the 40 quarter horizon with a zero impact restriction but crucially where the latter is now a linear combination of the reduced form innovations from the remaining variables in the VAR, excluding the EBP. In other words, this identification does not assume a-priori any correlation between movements in EBP and future TFP caused by a TFP shock. Figure 7 below displays the IRFs from a TFP news shock identified as described above. Importantly, the IRFs from this alternative identification are qualitatively consistent with the ones based on the baseline identification used in the main body of the paper.

Figure 6: Financial shock — reduced from innovation to EBP. Impulse responses to a financial shock from a seven-variable VAR. The shaded gray areas are the 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.
Figure 7: TFP news shock — alternative identification. Impulse responses to a TFP news shock from a seven-variable VAR. The shaded gray areas are the 16% and 84% posterior bands generated from the posterior distribution of VAR parameters. The units of the vertical axes are percentage deviations.

1.5 Robustness of DSGE model results

We scrutinise our baseline DSGE model results in four dimensions. First, we extend our baseline DSGE model by incorporating a wedge between the model implied sectoral spreads and the corresponding corporate spread concepts in the data. The wedge follows the process

\[ \text{wedge}_t = \rho_{\text{wedge}} \text{wedge}_{t-1} + \varepsilon_{\text{wedge},t}, \]

where \( \rho_{\text{wedge}} \in (0, 1) \) and \( \varepsilon_{\text{wedge},t} \) is i.i.d. \( N(0, \sigma_{\text{wedge}}^2) \). The wedge is introduced as an reduced form way to account for variation in the spread that could reflect factors we do not model, such as agents’ default risk (although our VAR findings do not suggest this is a major consideration) or other non-fundamental factors in the pricing of corporate bond as recently argued by Gilchrist and Zakrajsek (2012). We report the variance decomposition at business cycle frequencies for our baseline model and the extended model with measurement error in the corporate spread equations in Table 1. Results are consistent across the two model specifications in the way that they point towards a quantitatively important role of TFP news shocks.

Second, we estimate the baseline model using a sample that excludes the Great Recession (1984Q1-2007Q3), addressing concerns about misspecification of the monetary policy rule when the policy rate approaches the zero lower bound, as well as concerns that high volatility
in corporate bond spreads and disruptions in financial markets may, at least partly, drive the important role of TFP news shocks. It is evident from the variance decomposition provided in Table 2 that the DSGE model’s prediction on the quantitative importance of TFP news shocks as drivers of aggregate fluctuations is robust to excluding the Great Recession from the sample.

Third, we estimate a one-sector model without financial frictions similar to the ones described in Fujiwara et al. (2011), Khan and Tsoukalas (2012), and Justiniano et al. (2010). Table 3 shows variance decomposition results for our baseline model and the one-sector model without financial frictions. Consistent with the comparison of the baseline model with a two-sector model without financial sector in the main body, the absence of the financial sector also limits the importance of TFP news shocks in the one sector model, but a much more prominent role is assigned to the MEI shock. These results are consistent with the findings reported in the studies mentioned above.

---

3In comparison to our baseline setup, this model version turns off the financial channel, i.e. the balance sheet identity (15), the leverage constraint (16), the evolution of equity capital (17), and the financial constraint (9) that describe the financial sector as well as equations (7), (10) and (11) that allow capital services producers to raise funds from households. The one-sector model can be written as a special case of the two-sector model. It imposes a perfectly competitive investment sector and perfect capital mobility.
<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>Baseline with measurement errors in spread eqs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all TFP</td>
<td>all TFP</td>
</tr>
<tr>
<td></td>
<td>unanticipated</td>
<td>news</td>
</tr>
<tr>
<td>Output</td>
<td>19.8</td>
<td>52.3</td>
</tr>
<tr>
<td>Consumption</td>
<td>15.0</td>
<td>50.8</td>
</tr>
<tr>
<td>Investment</td>
<td>13.8</td>
<td>42.6</td>
</tr>
<tr>
<td>Total Hours</td>
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<td>50.1</td>
</tr>
<tr>
<td>Real Wage</td>
<td>10.2</td>
<td>49.0</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>3.0</td>
<td>36.0</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
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<td>4.0</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>4.5</td>
<td>37.3</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>3.9</td>
<td>23.4</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>3.3</td>
<td>10.7</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>13.3</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE models and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities.
Table 2: Variance Decomposition at Business Cycle Frequencies

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th></th>
<th>Baseline model without Great Recession</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all TFP</td>
<td>all TFP</td>
<td>all Bank</td>
<td>all other</td>
</tr>
<tr>
<td>unanticipated</td>
<td>news</td>
<td>equity</td>
<td>shocks</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>19.8</td>
<td>52.3</td>
<td>7.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Consumption</td>
<td>15.0</td>
<td>50.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Investment</td>
<td>13.8</td>
<td>42.6</td>
<td>6.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Total Hours</td>
<td>10.0</td>
<td>50.1</td>
<td>4.9</td>
<td>0.1</td>
</tr>
<tr>
<td>Real Wage</td>
<td>10.2</td>
<td>49.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>3.0</td>
<td>36.0</td>
<td>4.5</td>
<td>0.1</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.6</td>
<td>4.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>4.5</td>
<td>37.3</td>
<td>12.0</td>
<td>7.8</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>3.9</td>
<td>23.4</td>
<td>0.3</td>
<td>69.9</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>3.3</td>
<td>10.7</td>
<td>8.0</td>
<td>11.9</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>13.3</td>
<td>21.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE models and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities.
Table 3: Variance decomposition – business cycle frequencies (6-32 quarters)

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>One sector model without financial frictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all TFP unanticipated</td>
<td>all TFP news</td>
</tr>
<tr>
<td>Output</td>
<td>19.8</td>
<td>52.3</td>
</tr>
<tr>
<td>Consumption</td>
<td>15.0</td>
<td>50.8</td>
</tr>
<tr>
<td>Investment</td>
<td>13.8</td>
<td>42.6</td>
</tr>
<tr>
<td>Total Hours</td>
<td>10.0</td>
<td>50.1</td>
</tr>
<tr>
<td>Real Wage</td>
<td>10.2</td>
<td>49.0</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>3.0</td>
<td>36.0</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.6</td>
<td>4.0</td>
</tr>
<tr>
<td>GZ Spread</td>
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<td>37.3</td>
</tr>
<tr>
<td>Bank Equity</td>
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<td>23.4</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>3.3</td>
<td>10.7</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>13.3</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

1.6 Risk appetite shocks

Full decomposition of model described in section 4.3. The streamlined version of the model discussed in section 4.3 is obtained from the baseline version. All structural equations are identical and as described in the detailed model Appendix 3. The difference, compared to the baseline model, is the removal of equations that describe the exogenous processes for the investment sector mark-up, equation (3.79), preference, equation (3.83), MEI, equation (3.86), stationary TFP in the C sector, equation (3.87), stationary TFP in the I sector, equation (3.88), as these shocks are not considered in the estimation. We report the variance decomposition corresponding to the streamlined version with “risk appetite” shocks in Table 4. As discussed in section 4.3 this model version allows for a significant role of financial shocks in terms of real activity variables and the GZ spread. Moreover, financial shocks account for 58.9 percent of the variance in bank equity. TFP news shocks’ quantitative importance is very similar to the baseline model, in fact slightly increased for activity variables.
Table 4: Variance decomposition: model with risk appetite shocks

<table>
<thead>
<tr>
<th>z</th>
<th>v</th>
<th>η_{mp}</th>
<th>λ_ζ^{C}</th>
<th>λ_w</th>
<th>ς_C</th>
<th>Ξ</th>
<th>Ξ^4</th>
<th>Ξ^8</th>
<th>Ξ^12</th>
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<th>z^{12}</th>
<th>All TFP news cols. 11-13</th>
<th>All Financial cols. 7-10</th>
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</thead>
<tbody>
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<td>0.3</td>
<td>0.5</td>
<td>1.8</td>
<td>0.9</td>
<td>2.2</td>
<td>3.9</td>
<td>0.4</td>
<td>2.9</td>
<td>43.1</td>
<td>3.5</td>
<td>23.2</td>
<td>69.7</td>
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<td>Consumption</td>
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<td>3.2</td>
<td>0.5</td>
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<td>0.4</td>
<td>2.0</td>
<td>3.9</td>
<td>0.4</td>
<td>3.3</td>
<td>38.3</td>
<td>1.6</td>
<td>25.9</td>
<td>65.9</td>
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<tr>
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<td>0.9</td>
<td>1.0</td>
<td>1.2</td>
<td>3.6</td>
<td>6.6</td>
<td>0.7</td>
<td>5.2</td>
<td>36.2</td>
<td>3.9</td>
<td>34.5</td>
<td>74.6</td>
</tr>
<tr>
<td>Total Hours</td>
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<td>0.3</td>
<td>0.4</td>
<td>2.6</td>
<td>1.1</td>
<td>2.7</td>
<td>4.7</td>
<td>0.5</td>
<td>3.6</td>
<td>29.4</td>
<td>4.8</td>
<td>46.0</td>
<td>80.2</td>
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<tr>
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<td>9.2</td>
<td>5.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>34.7</td>
<td>2.1</td>
<td>28.2</td>
<td>64.9</td>
</tr>
<tr>
<td>Nominal Int. Rate</td>
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<td>2.4</td>
<td>4.3</td>
<td>19.9</td>
<td>1.8</td>
<td>0.6</td>
<td>0.9</td>
<td>1.5</td>
<td>0.1</td>
<td>0.9</td>
<td>16.3</td>
<td>2.1</td>
<td>47.2</td>
<td>65.6</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>3.8</td>
<td>0.8</td>
<td>0.6</td>
<td>25.3</td>
<td>5.7</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.0</td>
<td>30.7</td>
<td>2.3</td>
<td>32.3</td>
<td>65.2</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>0.0</td>
<td>3.1</td>
<td>0.2</td>
<td>0.9</td>
<td>0.1</td>
<td>15.3</td>
<td>1.6</td>
<td>12.5</td>
<td>4.1</td>
<td>48.5</td>
<td>4.5</td>
<td>0.6</td>
<td>8.6</td>
<td>13.7</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.2</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>39.5</td>
<td>15.6</td>
<td>20.2</td>
<td>2.0</td>
<td>21.1</td>
<td>0.1</td>
<td>0.0</td>
<td>1.1</td>
<td>1.3</td>
</tr>
<tr>
<td>Rel. Price of Inv.</td>
<td>0.9</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>5.4</td>
<td>16.6</td>
<td>30.0</td>
<td>3.0</td>
<td>23.8</td>
<td>7.7</td>
<td>0.6</td>
<td>11.0</td>
<td>19.4</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>12.6</td>
<td>44.8</td>
<td>0.0</td>
<td>4.6</td>
<td>0.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>19.7</td>
<td>1.4</td>
<td>16.0</td>
<td>37.0</td>
</tr>
</tbody>
</table>

z = TFP growth shock in consumption sector, v = TFP growth shock in investment sector, η_{mp} = monetary policy shock, λ_ζ^{C} = C-sector price markup shock, λ_w = wage markup shock, ς_C = consumption sector bank equity shock, Ξ = risk appetite shock, Ξ_x = x quarters ahead risk appetite news shock, z_x = x quarters ahead consumption sector TFP growth news shock. Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.

**Baseline model with risk appetite shocks.** We also report results from an extended baseline model with risk appetite shocks. This extended model considers all shocks featured in the baseline model and incorporates the risk appetite shocks as described in the streamlined version above. Table 5 below reports the variance decomposition. The key finding from this exercise is that TFP news shocks’ quantitative importance for real activity variables is broadly similar to the baseline model. The variance shares of output and consumption are somewhat smaller, compared to the baseline in the main text, but still substantial at 41.5% and 32.8% respectively. The variance shares for investment and hours worked estimated at 40.1% and 51.1% respectively are very similar to the shares reported for the baseline model. Hence the role of TFP news shocks is very robust. In the baseline model, risk appetite shocks remain important for the variance in the GZ spread and bank equity, with FEV shares estimated at 30.8% of the former and 48.5% of the latter. However, the quantitative importance of risk appetite shocks for real activity variables is at best very limited when the full menu of shocks is present in the estimation. To gain some insight into this finding we isolate MEI shocks, that soak up a non-negligible share of variation in real activity variables. As emphasized by Justiniano et al. (2011), MEI shocks can be thought of as ad-hoc proxies for...
financial market frictions and are thus, similar in flavour to risk appetite shocks. Thus, they compete directly with risk appetite shocks for accounting in the FEV of the observables. In the extended model, MEI shocks account for 8.8%, 8.9%, 6.1% of the FEV in output, investment, and hours worked. These shares are comparable in magnitude to the FEV explained by the risk appetite shocks in the streamlined model as reported in Table 5 in the main text, where MEI shocks are not present in the menu of shocks. Both type of shocks generate similar dynamics in terms of co-movements of real activity variables. However, MEI shocks are investment supply shifters, whereas risk appetite shocks are investment demand shifters and imply different covariances of the relative price of investment and other observables. Although many data moments inform the estimation and consequently determine the relative significance of shocks, we note in particular the strong negative correlation between inflation and the (change in) the relative price of investment and the negative (near zero) correlations between inflation and growth of output (investment) in the data. Risk appetite shocks predict counterfactual correlations compared to MEI shocks and for this reason appear to be displaced when MEI shocks compete with them in the estimation.

Table 5: Variance decomposition: baseline model with risk appetite shocks

<table>
<thead>
<tr>
<th></th>
<th>( z )</th>
<th>all ( z ) news</th>
<th>all ( v ) news</th>
<th>( q )</th>
<th>all ( \Xi ) news</th>
<th>all other shocks</th>
<th>all TFP surprise cols. 1–3</th>
<th>all TFP cols. 2–4</th>
<th>MEI and preference cols. 5–6</th>
<th>all risk appetite cols. 7–8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>21.1</td>
<td>37.3</td>
<td>4.9</td>
<td>4.2</td>
<td>8.8</td>
<td>3.6</td>
<td>1.5</td>
<td>0.4</td>
<td>18.2</td>
<td>26.0</td>
</tr>
<tr>
<td>Consumption</td>
<td>14.0</td>
<td>27.6</td>
<td>3.3</td>
<td>5.2</td>
<td>0.0</td>
<td>48.4</td>
<td>0.0</td>
<td>0.0</td>
<td>1.5</td>
<td>17.3</td>
</tr>
<tr>
<td>Investment</td>
<td>6.9</td>
<td>33.9</td>
<td>5.9</td>
<td>6.2</td>
<td>8.9</td>
<td>3.6</td>
<td>1.5</td>
<td>0.4</td>
<td>32.7</td>
<td>12.8</td>
</tr>
<tr>
<td>Total Hours</td>
<td>6.5</td>
<td>47.9</td>
<td>3.7</td>
<td>3.2</td>
<td>6.1</td>
<td>23.1</td>
<td>1.1</td>
<td>0.3</td>
<td>28.9</td>
<td>10.2</td>
</tr>
<tr>
<td>Real Wage</td>
<td>7.9</td>
<td>28.9</td>
<td>3.3</td>
<td>9.6</td>
<td>0.0</td>
<td>6.9</td>
<td>0.0</td>
<td>0.0</td>
<td>43.3</td>
<td>11.2</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>2.4</td>
<td>26.1</td>
<td>0.2</td>
<td>0.7</td>
<td>2.8</td>
<td>5.8</td>
<td>0.5</td>
<td>0.2</td>
<td>61.3</td>
<td>2.7</td>
</tr>
<tr>
<td>C-Sector Inflation</td>
<td>0.0</td>
<td>2.7</td>
<td>0.0</td>
<td>0.4</td>
<td>0.0</td>
<td>9.0</td>
<td>0.0</td>
<td>0.0</td>
<td>87.8</td>
<td>0.0</td>
</tr>
<tr>
<td>GZ Spread</td>
<td>7.0</td>
<td>25.9</td>
<td>5.2</td>
<td>5.2</td>
<td>9.4</td>
<td>0.1</td>
<td>1.7</td>
<td>29.2</td>
<td>16.3</td>
<td>12.3</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>9.0</td>
<td>36.7</td>
<td>0.9</td>
<td>1.5</td>
<td>0.4</td>
<td>0.2</td>
<td>40.9</td>
<td>7.6</td>
<td>2.7</td>
<td>9.9</td>
</tr>
<tr>
<td>Rel. Price of Investment</td>
<td>0.3</td>
<td>5.6</td>
<td>2.3</td>
<td>3.6</td>
<td>10.7</td>
<td>2.1</td>
<td>16.3</td>
<td>5.5</td>
<td>53.5</td>
<td>2.6</td>
</tr>
<tr>
<td>Corporate Equity</td>
<td>2.6</td>
<td>10.8</td>
<td>7.9</td>
<td>15.7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>63.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

\( z = \text{TFP growth shock in consumption sector}, \ v = \text{TFP growth shock in investment sector}, \ \Xi = \text{risk appetite shock}, \ q = \text{marginal efficiency of investment (MEI) shock}, \ \beta = \text{preference shock}. \text{The model includes 4, 8 and 12 quarter ahead news shocks for } z, v \text{ and } \Xi. \text{Business cycle frequencies considered in the decomposition correspond to periodic components with cycles between 6 and 32 quarters. The decomposition is performed using the spectrum of the DSGE model and an inverse first difference filter to reconstruct the levels for output, consumption, total investment, the real wage, the relative price of investment, bank equity and corporate equity. The spectral density is computed from the state space representation of the model with 500 bins for frequencies covering the range of periodicities. We report median shares.}
1.7 DSGE based forecasting results

We use our DSGE model as the data generating process and perform the forecasting regressions as in Gilchrist and Zakrajsek (2012) to assess the predictive ability of credit spreads for economic activity.

In particular, we first draw parameters from the posterior distributions, simulate the model and reconstruct the time series in levels. We then use these time series to estimate the following forecasting specification,

\[ \nabla^h Y_{t+h} = \alpha + \sum_{i=1}^{p} \beta_i \nabla Y_{t-i} + \gamma_1 RFF_t + \gamma_2 \text{Spread}_t + \epsilon_{t+h}, \tag{1.1} \]

where \( \nabla^h Y_{t+h} \equiv \frac{c}{1+h} \ln \left( \frac{Y_{t+h}}{Y_{t-1}} \right) \) denotes output growth. The forecasting horizon is denoted by \( h \) and \( c = 400 \) is a scaling constant calibrated consistent with the quarterly frequency of our data. \( RFF_t \) denotes the real interest rate defined as the difference of the nominal interest rate and expected (consumer’s) inflation for the next quarter. \( \text{Spread}_t \) denotes the credit spread, and \( \epsilon_{t+h} \) is the forecast error. The lag length \( p \) is determined by the Akaike Information Criterion (AIC). We estimate the equation using ordinary least squares so that our procedure and the forecasting specification resemble exactly the setup in Gilchrist and Zakrajsek (2012). The only exception is that we omit the term spread as a right-hand variable since the model does not include bonds with different maturity structure that would allow us to generate time series for this variable.

We draw 200 times from the posterior distributions and estimate equation (1.1).\(^4\) Table 6 summarizes the results of this exercise where we focus on one and four quarter forecasting horizons. Consistent with the findings in Gilchrist and Zakrajsek (2012) the spread is a statistically highly significant predictor of economic activity at either the one or four quarter horizon, while the real interest rate is at best marginally significant. At the one-quarter horizon, the median for the real interest rate is insignificant for both model specifications.

\(^4\)The lag length \( p \) is determined based on the AIC each time for a set of time series based on a particular draw. The maximum number of lags considered is eight which is never chosen in any specification.
At the four-quarter horizon, the median is significant at the 10% level, but only as long as the spread is not included as explanatory variable. Comparing results based on models with and without the spread, shows that including the spread in the forecasting equation leads to an increase in the adjusted R-squared.

<table>
<thead>
<tr>
<th>Financial Indicator</th>
<th>Forecast horizon: 1 quarter</th>
<th></th>
<th></th>
<th>Forecast horizon: 4 quarters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>percentiles</td>
<td>percentiles</td>
<td></td>
<td>percentiles</td>
<td>percentiles</td>
<td></td>
</tr>
<tr>
<td>Real Interest Rate</td>
<td>0.003 0.028 0.057</td>
<td>-0.187 -0.141 -0.086</td>
<td></td>
<td>0.076 0.120 0.150</td>
<td>-0.138 -0.102 -0.055</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>-0.316 -0.291 -0.267</td>
<td>-0.401 -0.380 -0.364</td>
<td></td>
<td>-0.029 [0.000] [0.000]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.224 0.248 0.268</td>
<td>0.296 0.310 0.329</td>
<td></td>
<td>0.277 0.323 0.362</td>
<td>0.479 0.493 0.509</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Dependent variable is $\nabla^b Y_{t+h}$, where $Y_t$ denotes real GDP in quarter $t$ and $h$ is the forecast horizon. In addition to the specified financial indicator in quarter $t$, each specification also includes a constant and $p$ lags of $\nabla^b Y_{t+1}$ (not reported), where $p$ is determined by the AIC. Entries in the OLS coefficients associated with each financial indicator. Entries in brackets correspond to $p$-values. Each estimate is based on data simulated from the DSGE model with corresponding trends added. Time series length is 133 quarters (after 100 quarters are discarded) which corresponds to the length of our baseline horizon (1984Q1-2017Q1). We generate 200 sets of time series by drawing from the posterior distributions of the DSGE model parameters.
1.8 Specification for the Minnesota prior in the VAR

The prior for the VAR coefficients $A$ is of the form

$$\text{vec}(A) \sim N(\beta, V),$$

where $\beta$ is one for variables which are in log-levels, and zero for the corporate bond spread as well as inflation. The prior variance $V$ is diagonal with elements,

$$V_{i,jj} = \begin{cases} 
\frac{a_1}{p^2} & \text{for coefficients on own lags} \\
\frac{a_2 \sigma_{ii}}{p^2 \sigma_{jj}} & \text{for coefficients on lags of variable } j \neq i \\
a_3 \sigma_{ii} & \text{for intercepts}
\end{cases} \tag{1.2}$$

where, $p$ denotes the number of lags. Here $\sigma_{ii}$ is the residual variance from the unrestricted $p$-lag univariate autoregression for variable $i$. The degree of shrinkage depends on the hyperparameters $a_1, a_2, a_3$. We set $a_3 = 1$ and we select $a_1, a_2$ by searching on a grid and selecting the prior that maximizes the in-sample fit of the VAR, as measured by the Bayesian Information Criterion.$^5$

1.9 Calibration and estimation

**Calibration.** Table 7 describes the calibrated parameters referred to in section 3.2. We set the quarterly depreciation rate to be equal across sectors, $\delta_C = \delta_I = 0.025$. From the steady state restriction $\beta = \pi_C/R$, we set $\beta = 0.997$. The shares of capital in the production functions, $a_C$ and $a_I$, are assumed equal across sectors and fixed at 0.3. These are standard values used in the extant literature. The steady state value for the ratios of nominal investment to consumption is calibrated to be consistent with the average value in the data.

The steady state sectoral inflation rates are set to the sample averages and the sectoral steady state mark-ups are assumed to be equal to 15%. We also calibrate the steady state (deterministic) growth of TFP in the consumption/investment sectors in line with the sample average growth rates of output in the two sectors. This yields $g_a = 0.15\%$ and $g_v = 0.48\%$ per

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$^5$The grid of values we use is: $a_1 = (1e-5:1e-4:9e-5, 1e-4:1e-4:9e-4, 0.001:0.001:0.009, 0.01:0.01:0.5)$, $a_2 = (0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5)$. We consider all possible pairs of $a_1$ and $a_2$ in the above grids.
### Table 7: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_C$</td>
<td>0.025</td>
<td>Consumption sector capital depreciation</td>
</tr>
<tr>
<td>$\delta_I$</td>
<td>0.025</td>
<td>Investment sector capital depreciation</td>
</tr>
<tr>
<td>$a_c$</td>
<td>0.3</td>
<td>Consumption sector share of capital</td>
</tr>
<tr>
<td>$a_I$</td>
<td>0.3</td>
<td>Investment sector share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.997</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\pi_C - 1$</td>
<td>0.66</td>
<td>Steady state consumption sector net inflation rate (percent quarterly)</td>
</tr>
<tr>
<td>$\pi_I - 1$</td>
<td>0.14</td>
<td>Steady state investment sector net inflation rate (percent quarterly)</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>0.15</td>
<td>Steady state price markup</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>0.15</td>
<td>Steady state wage markup</td>
</tr>
<tr>
<td>$g_a$</td>
<td>0.15</td>
<td>Steady state C-sector TFP growth (percent quarterly)</td>
</tr>
<tr>
<td>$g_v$</td>
<td>0.48</td>
<td>Steady state I-sector TFP growth (percent quarterly)</td>
</tr>
<tr>
<td>$p_{i \tau}$</td>
<td>0.445</td>
<td>Steady state investment / consumption</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>0.965</td>
<td>Fraction of bankers that survive</td>
</tr>
<tr>
<td>$R^B - R$</td>
<td>0.5</td>
<td>Steady state spread (percent quarterly)</td>
</tr>
<tr>
<td>$\frac{Q^S}{Q^{S \pi}}$</td>
<td>0.25</td>
<td>Corporate bonds over equity market capitalization</td>
</tr>
</tbody>
</table>

**Notes.** $\beta$, $\pi_C$, $\pi_I$, $g_a$, $g_v$, $p_{i \tau}$, $R^B - R$ and $\frac{Q^S}{Q^{S \pi}}$ are based on sample averages.

There are three parameters specific to financial intermediation. The parameter $\theta_B$, which determines the banker’s average life span does not have a direct empirical counterpart and is fixed at 0.965, similar to the value used by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). This value implies an average survival time of bankers of slightly over six years. The parameters $\varpi$ and the steady state leverage ratio are implied by steady state values and the estimate for $\lambda_B$. Our value for $\varpi = 0.0051$ is very close to the calibration in Gertler and Kiyotaki (2010) and the steady state leverage ratio implied by these values (3.3005) is within the range of values reported in the literature and the average leverage ratio we compute from the data. Also the parameters for governing fixed equity adjustment costs, $\gamma^h = 0.0286$ and $\gamma = 0.0299$, are pinned down by steady state ratios as shown in Appendix 3.6.
2 Sample, Data Sources and Time Series Construction

There are several considerations for focusing attention on a Great Moderation period. Adrian and Shin (2010) and Jermann and Quadrini (2012) argue that the importance of the financial sector for the determination of credit and asset prices, which is the main focus of our study, has risen significantly during this period. Further, Jermann and Quadrini (2009) discuss a variety of financial innovations that were taking place or intensified in the 1980s, including banking liberalization, and flexibility in debt issuance through the introduction of the Asset Backed Securities market. The corporate bond market—relative to equity markets—has grown tremendously as a source of finance, suggesting that developments in the corporate bond market may more accurately reflect future economic conditions. According to the Securities Industry and Financial Markets Association (SIFMA) over the period 1990 to 2013 the volume of US corporate bonds outstanding more than quantipled from $1.35 trillion to $7.46 trillion. The same body reports that in 2010, total corporate debt was 5.1 times common stock issuance. Philippon (2009) argues that corporate bond spreads may contain news about future corporate fundamentals and provides evidence that information extracted from corporate bond markets, in contrast to the stock market, is informative for U.S. business fixed investment.

Table 8 provides an overview of the data used to construct the observables. All the data transformations we have made in order to construct the dataset used for the estimation of the model are described in detail below. As described in the main body, a subset of variables are used for estimating the various VAR specifications and they enter in levels. The majority of the raw data described below were retrieved from the Federal Reserve of St.Luis FRED database. The exceptions are, the Market equity for banks which is from CRSP, TFP data series which is from Fernald (2014) at the Federal reserve bank of San Francisco, and the GZ spread and excess bond premium which are from the Federal reserve board.6

**Sectoral definition.** To allocate a sector to the consumption or investment category,

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6The TFP data can be accessed at www.frbsf.org/economic-research/economists/jfernald/quarterly_tfp.xls.
Table 8: Time Series used to construct the observables and steady state relationships

<table>
<thead>
<tr>
<th>Time Series Description</th>
<th>Units</th>
<th>Code</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product</td>
<td>CP, SA, billion $</td>
<td>GDP</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Gross Private Domestic Investment</td>
<td>CP, SA, billion $</td>
<td>GPDI</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Real Gross Private Domestic Investment</td>
<td>CVM, SA, billion $</td>
<td>GPDIC1</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Personal Consumption Exp: Durable Goods</td>
<td>CP, SA, billion $</td>
<td>FCDG</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Real Personal Consumption Exp: Durable Goods</td>
<td>CVM, SA, billion $</td>
<td>PCDGCC96</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Personal Consumption Expenditures: Services</td>
<td>CP, SA, billion $</td>
<td>PCESV</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Real Personal Consumption Expenditures: Services</td>
<td>CVM, SA, billion $</td>
<td>PCESVC96</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Personal Consumption Exp: Nondurable Goods</td>
<td>CP, SA, billion $</td>
<td>FCND</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Real Personal Consumption Exp: Nondurable Goods</td>
<td>CVM, SA, billion $</td>
<td>PCNDGC96</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Civilian Noninstitutional Population</td>
<td>NSA, 1000s</td>
<td>CNP160V</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Non-farm Business Sector: Compensation Per Hour</td>
<td>SA, Index 2005=100</td>
<td>COMPNSPB</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Non-farm Business Sector: Hours of All Persons</td>
<td>SA, Index 2005=100</td>
<td>HOANBS</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Effective Federal Funds Rate</td>
<td>NSA, percent</td>
<td>FEDFUNDS</td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>Implicit GDP deflator</td>
<td>SA, percent</td>
<td></td>
<td>St. Louis FED FRED</td>
</tr>
<tr>
<td>S&amp;P 500 Index</td>
<td></td>
<td></td>
<td>Robert Shiller</td>
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<tr>
<td>BAA corporate spread</td>
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<td>St. Louis FED FRED</td>
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<td>GZ Spread</td>
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<td>Excess bond premium</td>
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<td>Market Equity</td>
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<td>Corporate equity (non financial corporate sector)</td>
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<td>SLOOS</td>
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<td>TFP utilization adjusted</td>
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CP = current prices, CVM = chained volume measures, SA = seasonally adjusted, NSA = not seasonally adjusted, FRB = Federal Reserve Board.

we used the 2005 Input-Output tables. The Input-Output tables track the flows of goods and services across industries and record the final use of each industry’s output into three broad categories: consumption, investment and intermediate uses (as well as net exports and government). First, we determine how much of a 2-digit industry’s final output goes to consumption as opposed to investment or intermediate uses.

Then we adopt the following criterion: if the majority of an industry’s final output is allocated to final consumption demand it is classified as a consumption sector; otherwise, if the majority of an industry’s output is allocated to investment or intermediate demand, it is classified as an investment sector. Using this criterion, mining, utilities, transportation and warehousing, information, manufacturing, construction and wholesale trade industries are classified as the investment sector and retail trade, real estate, rental and leasing, professional and business services, educational services, health care and social assistance, arts, entertainment, recreation, accommodation and food services and other services except government are classified as the consumption sector.\(^7\)

\(^7\)The investment sectors’ NAICS codes are: 21 22 23 31 32 33 42 48 49 51 (except 491). The
Real and nominal variables. We describe the exact source of each data series below as provided by FRED.


Consumption sector NAICS codes are: 6 7 11 44 45 53 54 55 56 81. This information is provided by the Bureau of Economic analysis (Use Tables/Before Redefinitions/Producer Value (http://www.bea.gov/industry/io_annual.htm)). We have checked whether there is any migration of 2-digit industries across sectors for our sample. The only industry which changes classification (from consumption to investment) during the sample is “information” which for the majority of the sample can be classified as investment and we classify it as such.


Effective Federal Funds Rate: Board of Governors of the Federal Reserve System (US), Effective Federal Funds Rate [FEDFUNDS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/FEDFUNDS, Sep 15, 2017.


The raw data are transformed as follows for the analysis. Consumption (in current
prices) is defined as the sum of personal consumption expenditures on services and personal consumption expenditures on non-durable goods. The times series for real consumption is constructed as follows. First, we compute the shares of services and non-durable goods in total (current price) consumption. Then, total real consumption growth is obtained as the chained weighted (using the nominal shares above) growth rate of real services and growth rate of real non-durable goods. Using the growth rate of real consumption we construct a series for real consumption using 2005 as the base year. The consumption deflator is calculated as the ratio of nominal over real consumption. In the DSGE model inflation of consumer prices is the growth rate of the consumption deflator. In the VAR model we use the log change in the GDP deflator as our inflation measure, however results are nearly identical when we use the consumption deflator or CPI inflation. Analogously, we construct a time series for the investment deflator using series for (current price) personal consumption expenditures on durable goods and gross private domestic investment and chain weight to arrive at the real aggregate. The relative price of investment is the ratio of the investment deflator and the consumption deflator. Real output is GDP expressed in consumption units by dividing current price GDP with the consumption deflator.

The hourly wage is defined as total compensation per hour. Dividing this series by the consumption deflator yields the real wage rate. Hours worked is given by hours of all persons in the non-farm business sector. All series described above as well as the equity capital series (described below) are expressed in per capita terms using the series of non-institutional population, ages 16 and over. The nominal interest rate is the effective federal funds rate. We use the monthly average per quarter of this series and divide it by four to account for the quarterly frequency of the model. The time series for hours is in logs. Moreover, all series used in estimation (including the financial time series described below) are expressed in deviations from their sample average.

Financial variables.

The GZ spread and excess bond premium. The GZ spread and excess bond premium series is directly obtained from the FED reserve board (‘Updating the recession risk
and the excess bond premium'); The Excel file with the GZ spread can be accessed at: https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp.csv. The methodology is described in Gilchrist and Zakrajsek (2012).

The **BAA spread** series is downloaded directly from FRED; the monthly rates are converted to quarterly using the three month average for the quarter; Federal Reserve Bank of St. Louis, Moody’s Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity [BAA10YM], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/BAA10YM, Sep 16, 2017.

The **S&P 500 index** is obtained from Robert Shiller’s website (http://www.econ.yale.edu/shiller/data.htm) and has been converted to a real per capita index by dividing with the consumption deflator and non-institutional population, ages 16 and over.

**Market equity.** The market value of commercial bank’s equity is constructed using monthly data from the Centre for Research in Security Prices (CRSP) and have been accessed from the Wharton research data services (WRDS): https://wrds Wharton.upenn.edu/. From the raw data we retain companies with the following SIC codes to cover the commercial banking sector: 6021 (National Commercial Banks), 6022 (State Commercial Banks), 6029 (Commercial Banks, not elsewhere classified), 6081 (Branches and Agencies of Foreign Banks), 6153 (Short-Term Business Credit Institutions, except Agricultural), 6159 (Miscellaneous Business Credit Institutions) and 6111 (Federal and Federally-Sponsored Credit Agencies). Market value is calculated as the product of Price (PRC) and Shares Outstanding (SHROUT). We transform the data to quarterly frequency by considering the market value on the last trading day per quarter. The final series for total equity is generated by taking the log after dividing by Civilian Noninstitutional Population and the consumption deflator.

**Senior officer opinion survey of bank lending practices (SLOOS).** The SLOOS is downloaded directly from FRED (Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms). Board of Governors of the Federal Reserve System (US), Net Percentage of Domestic Banks Tightening Standards for Commercial and Industrial Loans to Large and Middle-Market Firms
The survey panel contains domestic banks headquartered in all 12 Federal Reserve Districts, with a minimum of 2 and a maximum of 12 domestic banks in the panel from each district. In general, up to 60 domestically chartered U.S. commercial banks participated in each survey from 1990 through mid-2012; beginning with the July 2012 survey, the size of the domestic panel was increased to include as many as 80 institutions. As described in the Federal Register Notice authorizing the SLOOS, the panel of domestic respondents as of September 30, 2011 contained 55 banks, 34 of which had assets of $20 billion or more. The combined assets of the respondent banks totaled $7.5 trillion and accounted for 60 percent of the $10.9 trillion in total assets at domestically chartered institutions. The respondent banks also held between 40 percent and 80 percent of total commercial bank loans outstanding in each major loan category regularly queried in the survey, with most categories falling in the upper end of that range. The particular survey question we consider is the net percentage of domestic respondents reporting tightening lending standards for commercial and industry loans for large and medium-sized firms.

3 Model Details and Derivations

We provide the model details and derivations required for solution and estimation of the model. We begin with the pricing and wage decisions of firms and households, the financial sector followed by the normalization of the model to render it stationary, the description of the steady state and the log-linearized model equations.

3.1 Intermediate and Final Goods Producers

Intermediate producers pricing decision. A constant fraction $\xi_{p,x}$ of intermediate firms in sector $x = C, I$ cannot choose their price optimally in period $t$ but reset their price — as
in Calvo (1983) — according to the indexation rule,

\[ P_{C,t}(i) = P_{C,t-1}(i) \pi_{C,t-1}^{pC} \pi_{C,t}^{1-pC}, \]

\[ P_{I,t}(i) = P_{I,t-1}(i) \pi_{I,t-1}^{pI} \pi_{I,t}^{1-pI} \left[ \left( \frac{A_t}{A_{t-1}} \right)^{-1} \left( \frac{V_t}{V_{t-1}} \right) \right]^{-\frac{1}{1-pI}}, \]

where \( \pi_{C,t} \equiv \frac{P_{C,t}}{P_{C,t-1}} \) and \( \pi_{I,t} \equiv \frac{P_{I,t}}{P_{I,t-1}} \left( \frac{A_t}{A_{t-1}} \right)^{-1} \left( \frac{V_t}{V_{t-1}} \right) \) is gross inflation in the two sectors and \( \pi_C, \pi_I \) denote steady state values. The factor that appears in the investment sector expression adjusts for investment specific progress.

The remaining fraction of firms, \( (1 - \xi_{p,x}) \), in sector \( x = C, I \) can adjust the price in period \( t \). These firms choose their price optimally by maximizing the present discounted value of future profits.

The resulting aggregate price index in the consumption sector is,

\[ P_{C,t} = \left[ (1 - \xi_{p,C}) \tilde{P}_{C,t}^{pC} + \xi_{p,C} \left( \frac{\pi_{C,t-1}}{\pi_C} \right)^{1-pC} P_{C,t-1} \right]^{\lambda_{p,t}^{C}}. \]

The aggregate price index in the investment sector is,

\[ P_{I,t} = \left[ (1 - \xi_{p,I}) \tilde{P}_{I,t}^{pI} + \xi_{p,I} \left( \frac{\pi_{I,t-1}}{\pi_I} \right)^{1-pI} \left[ \left( \frac{A_t}{A_{t-1}} \right)^{-1} \left( \frac{V_t}{V_{t-1}} \right) \right]^{1-pI} \right]^{\lambda_{p,t}^{I}}. \]

**Final goods producers.** Profit maximization and the zero profit condition for final good firms imply that sectoral prices of the final goods, \( P_{C,t} \) and \( P_{I,t} \), are CES aggregates of the prices of intermediate goods in the respective sector, \( P_{C,t}(i) \) and \( P_{I,t}(i) \),

\[ P_{C,t} = \int_0^1 P_{C,t}(i) \frac{1}{\lambda_{p,t}^{pC}} \, di \]

\[ P_{I,t} = \int_0^1 P_{I,t}(i) \frac{1}{\lambda_{p,t}^{pI}} \, di \]

The elasticity \( \lambda_{p,t}^{x} \) is the time varying price markup over marginal cost for intermediate firms. It is assumed to follow the exogenous stochastic process,

\[ \log(1 + \lambda_{p,t}^{x}) = (1 - \rho_{\lambda_{p,t}^{x}}) \log(1 + \lambda_{p,t}^{x}) + \rho_{\lambda_{p,t}^{x}} \log(1 + \lambda_{p,t-1}^{x}) + \varepsilon_{p,t}^{x}, \]

where \( \rho_{\lambda_{p,t}^{x}} \in (0, 1) \) and \( \varepsilon_{p,t}^{x} \) is i.i.d. \( N(0, \sigma_{\lambda_{p,t}^{x}}^2) \), with \( x = C, I \).
3.1.1 Household’s wage setting

Each household $j \in [0, 1]$ supplies specialized labor, $L_t(j)$, monopolistically as in Erceg et al. (2000). A large number of competitive “employment agencies” aggregate this specialized labor into a homogenous labor input which is sold to intermediate goods producers in a competitive market. Aggregation is done according to the following function,

$$L_t = \left[ \int_0^1 L_t(j) \frac{1}{1+\lambda_{w,t}} dj \right]^{1+\lambda_{w,t}}.$$

The desired markup of wages over the household’s marginal rate of substitution (or wage mark-up), $\lambda_{w,t}$, follows the exogenous stochastic process,

$$\log(1 + \lambda_{w,t}) = (1 - \rho_w) \log(1 + \lambda_w) + \rho_w \log(1 + \lambda_{w,t-1}) + \varepsilon_{w,t},$$

where $\rho_w \in (0, 1)$ and $\varepsilon_{w,t}$ is i.i.d. $N(0, \sigma_{\lambda_{w}}^2)$.

Profit maximization by the perfectly competitive employment agencies implies the labor demand function,

$$L_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t,$$  \hspace{1cm} (3.1)

where $W_t(j)$ is the wage received from employment agencies by the supplier of labor of type $j$, while the wage paid by intermediate firms for the homogenous labor input is,

$$W_t = \left[ \int_0^1 W_t(j) \frac{1}{1+\lambda_{w,t}} dj \right]^{\lambda_{w,t}}.$$

Following Erceg et al. (2000), in each period, a fraction $\xi_w$ of the households cannot freely adjust its wage but follows the indexation rule,

$$W_{t+1}(j) = W_t(j) \left( \frac{\pi_{c,t} e^{zt+\frac{aw}{1-a_n} \gamma_n}}{\pi_{c} e^{\beta_n+\frac{aw}{1-a_n} \gamma_n}} \right)^{\xi_w} \left( \frac{\pi_{c} e^{\beta_n+\frac{aw}{1-a_n} \gamma_n}}{\pi_{c,t} e^{zt+\frac{aw}{1-a_n} \gamma_n}} \right)^{1-\xi_w}.$$

The remaining fraction of households, $(1 - \xi_w)$, chooses an optimal wage, $W_t(j)$, by maximizing,

$$E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ -b_{t+s} \varphi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} + \Lambda_{t+s} W_t(j) L_{t+s}(j) \right] \right\},$$

...
subject to the labor demand function (3.1). The aggregate wage evolves according to,

\[ W_t = \left(1 - \xi_w \right) \left( \hat{W}_t \right)^{1/\lambda_w} + \xi_w \left[ \left( \pi_c e^{\delta_m + \frac{\mu_m}{1-\mu_m}} \right)^{1-\xi_w} \left( \pi_{c,t-1} e^{\xi_w + \frac{\mu_m}{1-\mu_m}} \right)^{1/\lambda_w} \right]^{1/\lambda_w}, \]

where \( \hat{W}_t \) is the optimally chosen wage.

### 3.2 Physical capital producers

Capital producers in sector \( x = C, I \) use a fraction of investment goods from final goods producers and undepreciated capital stock from capital services producers (as described above) to produce new capital goods, subject to investment adjustment costs as proposed by Christiano et al. (2005). These new capital goods are then sold in perfectly competitive capital goods markets to capital services producers. The technology available for physical capital production is given as,

\[ O'_{x,t} = O_{x,t} + \mu_t \left( 1 - S \left( \frac{I_{x,t}}{I_{x,t-1}} \right) \right) I_{x,t}, \]

where \( O_{x,t} \) denotes the amount of used capital at the end of period \( t \), \( O'_{x,t} \) the new capital available for use at the beginning of period \( t + 1 \). The investment adjustment cost function \( S(\cdot) \) satisfies the following: \( S(1) = S'(1) = 0 \) and \( S''(1) = \kappa > 0 \), where \( ^n \text{d} \) denote differentiation. The optimization problem of capital producers in sector \( x = C, I \) is given as,

\[
\max_{I_{x,t}, O_{x,t}} \sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ Q_{x,t} \left[ O_{x,t} + \mu_t \left( 1 - S \left( \frac{I_{x,t}}{I_{x,t-1}} \right) \right) I_{x,t} \right] - Q_{x,t} O_{x,t} - \frac{P_{l,t}}{P_{C,t}} I_{x,t} \right\},
\]

where \( Q_{x,t} \) denotes the price of capital (i.e. the value of installed capital in consumption units). The first order condition for investment goods is,

\[
\frac{P_{l,t}}{P_{C,t}} = Q_{x,t} \mu_t \left[ 1 - S \left( \frac{I_{x,t}}{I_{x,t-1}} \right) - S' \left( \frac{I_{x,t}}{I_{x,t-1}} \right) \frac{I_{x,t}}{I_{x,t-1}} \right] + \beta E_t Q_{x,t} + 1 + \frac{\Lambda_{t+1}}{\Lambda_t} S' \left( \frac{I_{x,t+1}}{I_{x,t}} \right) \left( \frac{I_{x,t+1}}{I_{x,t}} \right)^2.
\]

From the capital producer’s problem it is evident that any value of \( O_{x,t} \) is profit maximizing. Let \( \delta_x \in (0, 1) \) denote the depreciation rate of capital and \( \bar{K}_{x,t-1} \) the capital stock available at the beginning of period \( t \) in sector \( x = C, I \). Then setting \( O_{x,t} = (1 - \delta) \xi^K_{x,t} \bar{K}_{x,t-1} \) implies
the available (sector-specific) capital stock in sector \(x\), evolves according to,

\[
\tilde{K}_{x,t} = \left(1 - \delta_x\right) \xi_{x,t-1} \tilde{K}_{x,t-1} + \mu_t \left(1 - S \left(\frac{I_{x,t}}{I_{x,t-1}}\right)\right) I_{x,t}, \quad x = C, I; \tag{3.2}
\]
as described in the main text.

3.3 Financial Intermediaries

This section describes in detail how the setup of Gertler and Karadi (2011) is adapted for the two sector model and describes in detail how the equations for financial intermediaries in the main text are derived.

The balance sheet for the consumption or investment sector branch can be expressed as,

\[
P_{C,t} Q_{x,t} S_{x,t} = P_{C,t} N_{x,t} + B_{x,t}, \quad x = C, I,
\]
where \(S_{x,t}\) denotes the quantity of financial claims held by the intermediary branch and \(Q_{x,t}\) denotes the sector-specific price of a claim. The variable \(N_{x,t}\) represents the bank’s wealth (or equity) at the end of period \(t\) and \(B_{x,t}\) are the deposits the intermediary branch obtains from households. The sector-specific assets held by the financial intermediary pay the stochastic return \(R_{x,t+1}^B\) in the next period. Intermediaries pay at \(t+1\) the non-contingent real gross return \(R_t\) to households for their deposits made at time \(t\). Then, the intermediary branch equity evolves over time as,

\[
N_{x,t+1} P_{C,t+1} = R_{x,t+1}^B \pi_{C,t+1} P_{C,t} Q_{x,t} S_{x,t} - R_t B_{x,t}
\]

\[
N_{x,t+1} \frac{P_{C,t+1}}{P_{C,t}} = R_{x,t+1}^B \pi_{C,t+1} Q_{x,t} S_{x,t} - R_t (Q_{x,t} S_{x,t} - N_{x,t})
\]

\[
N_{x,t+1} = \left[\left(R_{x,t+1}^B \pi_{C,t+1} - R_t\right) Q_{x,t} S_{x,t} + R_t N_{x,t}\right] \frac{1}{\pi_{C,t+1}}.
\]

The premium, \(R_{x,t+1}^B \pi_{C,t+1} - R_t\), as well as the quantity of assets, \(Q_{x,t} S_{x,t}\), determines the growth in bank’s equity above the riskless return. The bank will not fund any assets with a negative discounted premium. It follows that for the bank to operate in period \(i\) the following inequality must hold,

\[
E_t \beta^i \Lambda_{t+1+i}^B \left(R_{x,t+1+i}^B \pi_{C,t+1+i} - R_{t+i}\right) \geq 0, \quad i \geq 0,
\]

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where $\beta^i \Lambda_{t+1+i}$ is the bank’s stochastic discount factor, with,

$$\Lambda_{t+1}^B \equiv \frac{\Lambda_{t+1}}{\Lambda_t},$$

where $\Lambda_t$ is the Lagrange multiplier on the household’s budget equation. Under perfect capital markets, arbitrage guarantees that the risk premium collapses to zero and the relation always holds with equality. However, under imperfect capital markets, credit constraints rooted in the bank’s inability to obtain enough funds may lead to positive risk premia. As long as the above inequality holds, banks will keep building assets by borrowing additional funds from households. Accordingly, the intermediary branch objective is to maximize expected terminal wealth,

$$V_{x,t} = \max E_t \sum_{i=0} (1 - \theta_B) \theta_B^i \beta^i \Lambda_{t+1+i} N_{x,t+1+i},$$

$$= \max E_t \sum_{i=0} (1 - \theta_B) \theta_B^i \beta^i \Lambda_{t+1+i} [(R^B_{x,t+1+i} \pi_{C,t+1+i} - R_{t+i}) Q_{x,t+i} S_{x,t+i} + R_{t+i} N_{x,t+i}],$$

where $\theta_B \in (0, 1)$ is the fraction of bankers at $t$ that survive until period $t + 1$.

Following the setup in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) the banks are limited from infinitely borrowing additional funds from households by a moral hazard/costly enforcement problem. On the one hand, the agent who works in the bank can choose, at the beginning of each period, to divert the fraction $\lambda_B$ of available funds and transfer it back to the household. On the other hand, depositors can force the bank into bankruptcy and recover a fraction $1 - \lambda_B$ of assets. Note that the fraction, $\lambda_B$, which intermediaries can divert is the same across sectors to guarantee that the household is indifferent between lending funds between different branches.

Given this tradeoff, depositors will only lend funds to the intermediary when the latter’s maximized expected terminal wealth is larger or equal to the gain from diverting the fraction $\lambda_B$ of available funds. This incentive constraint can be formalized as,

$$V_{x,t} \geq \lambda_B Q_{x,t} S_{x,t}, \quad 0 < \lambda_B < 1.$$  \hspace{1cm} (3.4)

Using equation (3.3), the expression for $V_{x,t}$ can be written as the following first-order
difference equation,

\[ V_{x,t} = \nu_{x,t} Q_{x,t} S_{x,t} + \eta_{x,t} N_{x,t}, \]

with,

\[ \nu_{x,t} = E_t \left\{ (1 - \theta_B) \Lambda_{t+1}^B (R_{x,t+1}^B \pi_{C,t+1} - R_t) + \theta_B \beta Z_{1,t+1}^x \right\}, \]

\[ \eta_{x,t} = E_t \left\{ (1 - \theta_B) \Lambda_{t+1}^B R_t + \theta_B \beta Z_{2,t+1}^x \right\}, \]

and,

\[ Z_{1,t+1+i}^x \equiv \frac{Q_{x,t+1+i} S_{x,t+1+i}}{Q_{x,t+i} S_{x,t+i}}, \quad Z_{2,t+1+i}^x \equiv \frac{N_{x,t+1+i}}{N_{x,t+i}}. \]

The variable \( \nu_{x,t} \) can be interpreted as the expected discounted marginal gain of expanding assets \( Q_{x,t} S_{x,t} \) by one unit while holding wealth \( N_{x,t} \) constant. The interpretation of \( \eta_{x,t} \) is analogous: it is the expected discounted value of having an additional unit of wealth, \( N_{x,t} \), holding the quantity of financial claims, \( S_{x,t} \), constant. The gross growth rate in assets is denoted by \( Z_{1,t+1}^x \) and the gross growth rate of net worth is denoted by \( Z_{2,t+1}^x \).

Then, using the expression for \( V_{x,t} \), we can express the intermediary’s incentive constraint (3.4) as,

\[ \nu_{x,t} Q_{x,t} S_{x,t} + \eta_{x,t} N_{x,t} \geq \lambda_B Q_{x,t} S_{x,t}. \]

As indicated above, under perfect capital markets banks will expand borrowing until the risk premium collapses to zero which implies that in this case \( \nu_{x,t} \) equals zero as well. Imperfect capital markets however, limit the possibilities for this kind of arbitrage because the intermediaries are constrained by their equity capital. If the incentive constraint binds it follows that,

\[ Q_{x,t} S_{x,t} = \frac{\eta_{x,t}}{\lambda_B - \nu_{x,t}} N_{x,t} \]

\[ = \varrho_{x,t} N_{x,t}. \quad (3.5) \]

In this case, the quantity of assets which the intermediary can acquire depends on the equity capital, \( N_{x,t} \), as well as the intermediary’s leverage ratio, \( \varrho_{x,t} \), limiting the bank’s ability to
acquire assets. This leverage ratio is the ratio of the bank’s intermediated assets to equity.
The bank’s leverage ratio is limited to the point where its maximized expected terminal wealth equals the gains from diverting the fraction \( \lambda_B \) from available funds. However, the constraint (3.5) binds only if \( 0 < \nu_{x,t} < \lambda_B \) (given \( N_{x,t} > 0 \)). This inequality is always satisfied with our estimates.

Using the leverage ratio (3.5) we can express the evolution of the intermediary’s wealth as,

\[
N_{x,t+1} = \left[ (R_{x,t+1}^B \pi_{C,t+1} - R_t) \psi_{x,t} + R_t \right] \frac{N_{x,t}}{\pi_{C,t+1}}.
\]

From this equation it also follows that,

\[
Z_{2,t+1}^x = \frac{N_{x,t+1}}{N_{x,t}} = \left[ (R_{x,t+1}^B \pi_{C,t+1} - R_t) \psi_{x,t} + R_t \right] \frac{1}{\pi_{C,t+1}},
\]

and,

\[
Z_{1,t+1}^x = \frac{Q_{x,t+1}^S S_{x,t+1}}{Q_{x,t} S_{x,t}} = \frac{\psi_{x,t+1} N_{x,t+1}}{\psi_{x,t} N_{x,t}} = \frac{\psi_{x,t+1}}{\psi_{x,t}} Z_{2,t+1}^x.
\]

Financial intermediaries which are forced into bankruptcy are replaced by new entrants. Therefore, total wealth of financial intermediaries is the sum of the net worth of existing, \( N_{x,t}^e \), and new ones, \( N_{x,t}^n \),

\[
N_{x,t} = N_{x,t}^e + N_{x,t}^n.
\]

The fraction \( \theta_B \) of bankers at \( t - 1 \) which survive until \( t \) is equal across branches. Then, the law of motion for existing bankers is given by,

\[
N_{x,t}^e = \theta_B \left[ (R_{x,t}^B \pi_{C,t} - R_{t-1}) \psi_{x,t-1} + R_{t-1} \right] \frac{N_{x,t-1}}{\pi_{C,t}}, \quad 0 < \theta_B < 1.
\] (3.6)

where a main source of variation is the ex-post excess return on assets, \( R_{x,t}^B \pi_{C,t} - R_{t-1} \).

New banks receive startup funds from their respective household, equal to a small fraction of the value of assets held by the existing bankers in their final operating period. Given that the exit probability is i.i.d., the value of assets held by the existing bankers in their final
operating period is given by \((1 - \theta_B)Q_{x,t}S_{x,t}\). The transfer to new intermediaries is a fraction, \(\varpi\), of this value, leading to the following formulation for new banker’s wealth,

\[
N^n_{x,t} = \varpi Q_{x,t}S_{x,t}, \quad 0 < \varpi < 1.
\]  

(3.7)

Existing banker’s net worth (3.6) and entering banker’s net worth (3.7) lead to the law of motion for total net worth,

\[
N_{x,t} = (\theta_B[R^B_{x,t}\pi_{C,t} - R_{t-1}]\varphi_{x,t-1} + R_{t-1}\frac{N_{x,t-1}}{\pi_{C,t}} + \varpi Q_{x,t}S_{x,t})\varsigma_{x,t}.
\]

where \(\varsigma_{x,t}\) is s shock to bank’s equity capital. The excess return, \(x = C, I\) can be defined as,

\[
R^S_{x,t} = R^B_{x,t+1}\pi_{C,t+1} - R_t.
\]

Since \(R_t\), \(\lambda_B\), \(\varpi\) and \(\theta_B\) are equal across sectors, the institutional setup of the two representative banks in the two sectors is symmetric. Both branches hold deposits from households and buy assets from firms in the sector they provide specialized lending. Their performance differs because the demand for capital differs across sectors resulting in sector-specific prices of capital, \(Q_{x,t}\), and nominal rental rates for capital, \(R^K_{x,t}\). Note that the institutional setup of banks does not depend on firm-specific factors. Gertler and Karadi (2011) show that this implies a setup with a continuum of banks is equivalent to a formulation with a representative bank. Owing to the symmetry of the banks this also holds for our formulation of financial intermediaries in the two-sector setup.

### 3.4 Resource Constraints

The resource constraint in the consumption sector is,

\[
C_t + (a(u_{C,t})\xi^K_{C,t}K_{C,t-1} + a(u_{I,t})\xi^K_{I,t}K_{I,t-1})\frac{A_tV_{t}^{\frac{1}{1-a}}}{V_{t}^{1-\frac{1}{1-a}}} = a_t A_t L_{C,t}^{1-a_c} K_{C,t}^{a_c} - A_t V_{t}^{\frac{1}{1-a}} F_C.
\]

The resource constraint in the investment sector is,

\[
I_{t,t} + I_{C,t} = v_t V_t L_{I,t}^{1-a_i} K_{I,t}^{a_i} - V_{t}^{1-\frac{1}{1-a_i}} F_I.
\]
Hours worked are aggregated as,
\[ L_t = L_{I,t} + L_{C,t}. \]

Bank equity is aggregated as,
\[ N_t = N_{I,t} + N_{C,t}. \]

### 3.5 Stationary Economy

The model includes two non-stationary TFP shocks, \( A_t \) and \( V_t \). This section shows how we normalize the model to render it stationary. Lower case variables denote normalized stationary variables.

The model variables can be stationarized as follows:

\[
\begin{align*}
    k_{x,t} &= \frac{K_{x,t}}{V_t^{\frac{1}{1-a}}}, & \dot{k}_{x,t} &= \frac{\dot{K}_{x,t}}{V_t^{\frac{1}{1-a}}}, & k_t &= \frac{K_t}{V_t^{\frac{1}{1-a}}}, \tag{3.8} \\
    i_{x,t} &= \frac{I_{x,t}}{V_t^{\frac{1}{1-a}}}, & \dot{i}_t &= \frac{\dot{I}_t}{V_t^{\frac{1}{1-a}}}, & c_t &= \frac{C_t}{A_tV_t^{\frac{1}{1-a}}}, \tag{3.9} \\
    r^K_{C,t} &= \frac{P^K_{C,t}}{P_{C,t}}A_t^{-1}V_t^{\frac{1-a}{1-a}}, & \dot{r}^K_{I,t} &= \frac{\dot{P}^K_{I,t}}{P_{C,t}}A_t^{-1}V_t^{\frac{1-a}{1-a}}, & w_t &= \frac{W_t}{P_{C,t}A_tV_t^{\frac{1-a}{1-a}}}. \tag{3.10}
\end{align*}
\]

From

\[
\begin{align*}
    \frac{P_{I,t}}{P_{C,t}} &= \frac{mc_{C,t}}{mc_{I,t}} \frac{1-a}{1-a} \frac{A_t}{V_I} \left( \frac{K_{I,t}}{L_{I,t}} \right)^{-a} \left( \frac{K_{C,t}}{L_{C,t}} \right)^{a} \\
    &= \frac{mc_{C,t}}{mc_{I,t}} \frac{1-a}{1-a} \frac{A_t}{V_I} \left( \frac{k_{I,t}}{L_{I,t}} \right)^{-a} \left( \frac{k_{C,t}}{L_{C,t}} \right)^{a},
\end{align*}
\]

follows that,

\[ p_{i,t} = \frac{P_{I,t}}{P_{C,t}} A_t^{-1}V_t^{\frac{1-a}{1-a}}. \tag{3.11} \]

and the multipliers are normalized as,

\[ \lambda_t = \Lambda_t A_t V_t^{\frac{1-a}{1-a}}, \quad \phi_{x,t} = \Phi_{x,t} V_t^{\frac{1-a}{1-a}}. \tag{3.12} \]

where \( \Phi_{x,t} \) denotes the multiplier on the respective capital accumulation equation. Using
the growth of investment, it follows that the prices of capital can be normalized as,

\[ q^{T}_{x,t} = Q^{T}_{x,t} A^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{c}}} , \quad q^{h}_{x,t} = Q^{h}_{x,t} A^{-1} V_{t}^{\frac{1-a_{i}}{1-a_{i}}}, \quad q_{x,t} = Q_{x,t} A^{-1} V_{t}^{\frac{1-a_{c}}{1-a_{c}}} . \]

with the price of capital in sector \( x \), defined as,

\[ q^{T}_{x,t} = \phi^{x,t} / \lambda_{t}, \quad x = C, I. \]

Then, it follows from entering bankers wealth equation (3.7) that,

\[ n^{n}_{x,t} = N^{n}_{x,t} A^{-1} V_{t}^{\frac{1-a}{1-a}}, \]

Total wealth, wealth of existing and entering bankers has to grow at the same rate,

\[ n^{e}_{x,t} = N^{e}_{x,t} A^{-1} V_{t}^{\frac{1-a}{1-a}} , \quad n_{x,t} = N_{x,t} A^{-1} V_{t}^{\frac{1-a}{1-a}} . \]

### 3.5.1 Intermediate goods producers

Firm’s production function in the consumption sector:

\[ c_{t} = a_{lt} L_{C,t}^{1-a_{c}} k_{C,t}^{a_{c}} - F_{C}. \]  \hspace{1cm} (3.13)

Firm’s production function in the investment sector:

\[ i_{t} = v_{lt} L_{I,t}^{1-a_{i}} k_{I,t}^{a_{i}} - F_{I}. \]  \hspace{1cm} (3.14)

Marginal costs in the consumption sector:

\[ mc_{C,t} = (1 - a_{c})^{a_{c}-1} a_{c}^{-a_{c}} (r^{K}_{C,t})^{a_{c}} w_{t}^{1-a_{c}} a_{lt}^{-1}. \]  \hspace{1cm} (3.15)

Marginal costs in the investment sector:

\[ mc_{I,t} = (1 - a_{i})^{a_{i}-1} a_{i}^{-a_{i}} (r^{K}_{I,t})^{a_{i}} v_{t}^{1-a_{i}} p_{i,t}^{-1} \text{ with } p_{i,t} = \frac{P_{I,t}}{P_{C,t}}. \]  \hspace{1cm} (3.16)

Capital labour ratios in the two sectors:

\[ \frac{k_{C,t}}{L_{C,t}} = \frac{w_{t}}{r^{K}_{C,t}} \frac{a_{c}}{1-a_{c}} , \quad \frac{k_{I,t}}{L_{I,t}} = \frac{w_{t}}{r^{K}_{I,t}} \frac{a_{i}}{1-a_{i}}. \]  \hspace{1cm} (3.17)
3.5.2 Firms’ pricing decisions

Price setting equation for firms that change their price in sector \( x = C, I \):

\[
0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_{p,x}^{s} \beta^{s} \lambda_{t+s} \tilde{x}_{t+s} \left[ \tilde{p}_{t,t} \tilde{\Pi}_{t,t+s} - (1 + \lambda_{p,t+s}^{x}) \mu c_{x,t+s} \right] \right\},
\]

with

\[
\tilde{\Pi}_{t,t+s} = \prod_{k=1}^{s} \left[ \left( \frac{\pi_{x,t+k-1}}{\pi_{x}} \right)^{p_{x}} \left( \frac{\pi_{x,t+k}}{\pi_{x}} \right)^{-1} \right]
\]

and \( \tilde{x}_{t+s} = \left( \frac{\tilde{P}_{x,t}}{P_{x,t}} \right)^{1+\lambda_{p,t+s}} \frac{1}{\lambda_{p,t+s}} x_{t+s} \).

Aggregate price index in the consumption sector:

\[
1 = \left( 1 - \xi_{x,p} \right) \left( \tilde{p}_{x,t} \right)^{\frac{1}{\beta_{p,t}}} + \xi_{x,p} \left[ \left( \frac{\pi_{x,t-1}}{\pi_{x}} \right)^{p_{x}} \left( \frac{\pi_{x,t}}{\pi_{x}} \right)^{-1} \right] \frac{1}{\beta_{p,t}}.
\]

It further holds that

\[
\frac{\pi_{l,t}}{\pi_{c,t}} = \frac{p_{i,t}}{p_{i,t-1}}.
\]

3.5.3 Household’s optimality conditions and wage setting

Marginal utility of income:

\[
\lambda_{t} = \frac{b_{t}}{c_{t} - h c_{t-1} \left( \frac{A_{t-1}}{A_{t}} \right) \left( \frac{V_{t-1}}{V_{t}} \right)^{\frac{a_{c}}{1-a_{c}}}} - \beta h \frac{b_{t+1}}{c_{t+1} \left( \frac{A_{t+1}}{A_{t}} \right) \left( \frac{V_{t+1}}{V_{t}} \right)^{\frac{a_{c}}{1-a_{c}}}} - h c_{t}.
\]

Euler equation:

\[
\lambda_{t} = \beta E_{t} \lambda_{t+1} \left( \frac{A_{t}}{A_{t+1}} \right) \left( \frac{V_{t}}{V_{t+1}} \right)^{\frac{a_{c}}{1-a_{c}}} R_{t} \frac{1}{\pi_{c,t+1}}.
\]

Labor supply

\[
\lambda_{t} w_{t} = b_{t} \varphi (L_{C,t} + L_{I,t})^{\nu}.
\]

Purchase of financial claims

\[
1 = E_{t} \beta \frac{\lambda_{t+1}}{\lambda_{t}} \frac{1}{v_{t+1}^{\frac{a_{c}}{1-a_{c}}}} \frac{P_{x,t}^{b}}{\pi_{c,t+1}}
\]
3.5.4 Capital services

Optimal capital utilization:

\[ r^K_{C,t} = a_C'(u_{C,t}), \quad r^K_{I,t} = a_I'(u_{I,t}). \]

Definition of capital services:

\[ k_{C,t} = u_C \xi^K_{C,t} (V_{t-1} / V_t)^{\frac{1}{1-\alpha}}, \quad k_{I,t} = u_I \xi^K_{I,t} (V_{t-1} / V_t)^{\frac{1}{1-\alpha}}. \] (3.22)

Optimal choice of available capital in sector \( x = C, I \):

\[ \phi_{x,t} = \beta E_t \xi^K_{x,t+1} \left\{ \lambda_{t+1} \left( V_{t+1} / V_t \right)^{\frac{1}{1-\alpha}} (r^K_{x,t+1} u_{x,t+1} - a(u_{x,t+1})) + (1 - \delta) E_t \phi_{x,t+1} \left( V_{t+1} / V_t \right)^{\frac{1}{1-\alpha}} \right\}, \] (3.23)

Optimal financing from households

\[ 0 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ r^K_{x,t+1} u_{x,t+1} q^h_{x,t} - a_x(u_{x,t+1}) q^h_{x,t} \right] - \gamma^h v_{t+1}^{\frac{1}{1-\alpha}} \]
\[ - \Gamma^h s^h_x \left[ \left( s^h_{x,t} v_{t+1}^{\frac{1}{1-\alpha}} - e \left( V_{t+1}^{\frac{1}{1-\alpha}} \right) g^h \right) \right], \quad \left( s^h_{x,t} v_{t+1}^{\frac{1}{1-\alpha}} - e \left( V_{t+1}^{\frac{1}{1-\alpha}} \right) g^h \right) \] (3.24)

Optimal financing from financial intermediaries

\[ 0 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ r^K_{x,t+1} u_{x,t+1} q^x_{x,t} - a_x(u_{x,t+1}) q^x_{x,t} \right] - \gamma v_{t+1}^{\frac{1}{1-\alpha}} \]
\[ - \Gamma^x s_x \left[ \left( s_x v_{t+1}^{\frac{1}{1-\alpha}} - e \left( V_{t+1}^{\frac{1}{1-\alpha}} \right) g^x \right) \right], \quad \left( s_x v_{t+1}^{\frac{1}{1-\alpha}} - e \left( V_{t+1}^{\frac{1}{1-\alpha}} \right) g^x \right) \] (3.25)

Household’s return on claims

\[ R^h_{x,t} = \left[ r^K_{x,t+1} u_{x,t+1} + q^h_{x,t+1} (1 - \delta_x) - a_x(u_{x,t+1}) \right] \left( z_{t+1} v_{t+1}^{\frac{1}{1-\alpha}} / q^h_{x,t} \right). \] (3.26)

Total value of acquired capital

\[ q^T_{x,t} k_{x,t} = q^h_{x,t} s^h_{x,t} + q_{x,t} s_{x,t}. \] (3.27)
3.5.5 Physical capital producers

Optimal choice of investment in sector \( x = C, I \):

\[
\lambda_t p_{x,t} = \phi_{x,t} \mu_t \left[ 1 - S \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{\frac{1}{\gamma_x}} \right) - S' \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{\frac{1}{\gamma_x}} \right) \right] + \beta E_t \phi_{x,t+1} \mu_{t+1} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{1}{\gamma_x}} \left( \frac{i_{x,t+1}}{i_{x,t}} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{1}{\gamma_x}} \right)^2. \tag{3.28}
\]

Accumulation of capital in sector \( x = C, I \):

\[
\bar{k}_{x,t} = (1 - \delta_x) \xi_x t \bar{k}_{x,t-1} \left( \frac{V_{t-1}}{V_t} \right)^{\frac{1}{\gamma_x}} + \mu_t \left( 1 - S \left( \frac{i_{x,t}}{i_{x,t-1}} \left( \frac{V_t}{V_{t-1}} \right)^{\frac{1}{\gamma_x}} \right) \right) \bar{i}_{x,t}. \tag{3.29}
\]

3.5.6 Household’s wage setting

Household’s wage setting:

\[
E_t \sum_{s=0}^{\infty} \beta^s \xi_w \lambda_{t+s} \tilde{L}_{t+s} \left[ \bar{w}_t \tilde{\Pi}_{t+s}^w - (1 + \lambda_{w,t+s}) b_{t+s} \bar{v}_{t+s} \lambda_{t+s} \right] = 0, \tag{3.30}
\]

with

\[
\tilde{\Pi}_{t+s}^w = \prod_{k=1}^{s} \left[ \left( \frac{\pi_{C,t+k-1} e^{a_t + k - 1 - \frac{n}{1-\gamma_v} v_t + k - 1}}{\pi_{c,e}^a + \frac{n}{1-\gamma_v} g_v} \right)^{\xi_w} \left( \frac{\pi_{C,t+k} e^{a_t + k - 1 - \frac{n}{1-\gamma_v} v_t + k - 1}}{\pi_{C,e}^a + \frac{n}{1-\gamma_v} g_v} \right)^{-1} \right]
\]

and

\[
\tilde{L}_{t+s} = \left( \frac{\bar{w}_t \tilde{\Pi}_{t+s}^w}{w_{t+s}} \right)^{\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s}.
\]

Wages evolve according to

\[
w_t = \left\{ \left( 1 - \xi_w \right) \bar{w}_t \frac{1}{w_{t+s}} + \xi_w \left[ \left( \frac{\pi_{c,t-1} e^{a_t - 1 - \frac{n}{1-\gamma_v} v_t - 1}}{\pi_{c,e}^a + \frac{n}{1-\gamma_v} g_v} \right)^{\xi_w} \left( \frac{\pi_{c,t} e^{a_t + \frac{n}{1-\gamma_v} v_t}}{\pi_{c,e}^a + \frac{n}{1-\gamma_v} g_v} \right)^{-1} \right] \right\}^{\frac{1}{\xi_w}} \lambda_{w,t}. \]

3.5.7 Financial Intermediation

The stationary stochastic discount factor can be expressed as,

\[
\lambda_{t+1}^B = \frac{\lambda_{t+1}^1}{\lambda_t}.
\]
Then, one can derive expressions for \( \nu_{x,t} \) and \( \eta_{x,t} \),

\[
\nu_{x,t} = E_t \{ (1 - \theta_B) \lambda^{B}_{t+1} \frac{A_t}{A_{t+1}} \left( \frac{V_t}{V_{t+1}} \right)^{\frac{u}{\pi}} (R^B_{x,t+1} \pi_{C,t+1} - R_t) + \theta_B \beta \nu_{x,t+1} \},
\]

\[
\eta_{x,t} = E_t \{ (1 - \theta_B) \lambda^{B}_{t+1} \frac{A_t}{A_{t+1}} \left( \frac{V_t}{V_{t+1}} \right)^{\frac{u}{\pi}} R_t + \theta_B \beta \eta_{x,t+1} \},
\]

with

\[
\bar{z}_{1,t+1+i} = \frac{q_{x,t+1} s_{x,t+1+i}}{q_{x,t} s_{x,t+i}} \frac{A_t}{A_{t+1}} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}}, \quad \bar{z}_{2,t+1+i} = \frac{n_{x,t+1+i} A_t}{n_{x,t+i} A_{t+1}} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}}.
\]

It follows that if the bank’s incentive constraint binds it can be expressed as,

\[
\nu_{x,t} q_{x,t} s_{x,t} + \eta_{x,t} n_{x,t} = \lambda_B q_{x,t} s_{x,t}
\]

\[
\iff q_{x,t} s_{x,t} = \varrho_{x,t} n_{x,t},
\]

with the leverage ratio given as,

\[
\varrho_{x,t} = \frac{\eta_{x,t}}{\lambda_B - \nu_{x,t}}.
\]

It further follows that:

\[
\bar{z}_{2,t+1} = \frac{n_{x,t+1} A_{t+1}}{n_{x,t} A_t} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}} = \left[ (R^B_{x,t+1} \pi_{C,t+1} - R_t) \varrho_{x,t} + R_t \right] \frac{1}{\pi_{C,t+1}},
\]

and

\[
\bar{z}_{1,t+1} = \frac{q_{x,t+1} s_{x,t+1+i}}{q_{x,t} s_{x,t+i}} \frac{A_t}{A_{t+1}} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}} = \frac{\varrho_{x,t+1} n_{x,t+1+i} A_{t+1}}{\varrho_{x,t} n_{x,t+i} A_t} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}} = \frac{\varrho_{x,t+1}}{\varrho_{x,t}} \bar{z}_{2,t+1}.
\]

The normalized equation for bank’s wealth accumulation is,

\[
n_{x,t} = (\theta_B [(R^B_{x,t} \pi_{C,t} - R_{t-1}) \varrho_{x,t-1} + R_{t-1}] A_{t-1} \left( \frac{V_{t-1}}{V_t} \right)^{\frac{u}{\pi}} n_{x,t-1} \pi_{C,t+1} + \varpi q_{x,t} s_{x,t}) \bar{s}_{x,t}.
\]

The leverage equation:

\[
q_{x,t} s_{x,t} = \varrho_{x,t} n_{x,t}.
\]

Bank’s stochastic return on assets can be described in normalized variables as:

\[
R^B_{x,t+1} = \frac{r^K_{x,t+1} u_{x,t+1} + q_{x,t+1} (1 - \delta_x) \sigma(u_{x,t+1})}{q_{x,t}} \left( \frac{V_{t+1}}{V_t} \right)^{\frac{u}{\pi}} \frac{A_t}{A_{t+1}} \frac{V_{t+1}}{V_t} \frac{1 - \alpha}{1 - \eta},
\]

knowing from the main model that

\[
r^K_{x,t} = \frac{R^K_{x,t} P_{x,t}^{-\alpha}}{A_t^{-\alpha} V_t^{\frac{1 - \alpha}{1 - \eta}}}.
\]
3.5.8 Monetary policy and market clearing

Monetary policy rule:

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\pi_{C,t}}{\pi_C} \right)^{\phi_s} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_{\Delta Y}} \right]^{1-\rho_R} \eta_{mp,t}, \]

Resource constraint in the consumption sector:

\[ c_t + (a(u_{C,t})_t^K v_{C,t,k_{C,t}}-1 + a(u_{I,t})_t^K v_{I,t,k_{I,t}}-1) \left( \frac{V_{t-1}}{V_t} \right)^{\gamma} = a_t L_t^{1-a_t} k_{C,t}^{a_t} - F_C. \]

Resource constraint in the investment sector:

\[ i_t = v_t L_t^{1-a_t} k_{I,t}^{a_t} - F_I. \]

Definition of GDP:

\[ y_t = c_t + p_t i_t + \left( 1 - \frac{1}{y_t} \right) y_t. \] (3.31)

Moreover

\[ L_t = L_{I,t} + L_{C,t}, \quad i_t = i_{C,t} + i_{I,t}, \quad n_t = n_{C,t} + n_{I,t}. \]

3.6 Steady State

This section describes the model’s steady state.

From the optimal choice of available capital (3.23) and the optimal choice of investment (3.28) in both sectors:

\[ r^C_C = \left( \frac{e^{1-\gamma g v}}{\beta} - (1 - \delta_C) \right) p_t, \] (3.32)

\[ r^I_I = \left( \frac{e^{1-\gamma g v}}{\beta} - (1 - \delta_I) \right) p_t. \] (3.33)

From firm’s price setting in both sectors (3.18),

\[ m c_C = \frac{1}{1 + \lambda_p^C}, \quad m c_I = \frac{1}{1 + \lambda_p^I}. \] (3.34)
Using equations (3.34) and imposing knowledge of the steady state expression for \( r_K^C \) and \( r_K^I \), one can derive expressions for the steady state wage from the equations that define marginal costs in the two sectors ((3.15) and (3.16)).

**Consumption sector:**

\[
w = \left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}}. \tag{3.35}
\]

**Investment sector:**

\[
w = \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}. \tag{3.36}
\]

Since labour can move across sectors the steady state wage has to be the same in the consumption and investment sector. The equality is verified by \( p_i \). An expression for \( p_i \) can be found by setting (3.35) equal to (3.36):

\[
\left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}} = \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}
\]

\[
\iff \left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}} = \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}
\]

\[
\iff p_i = \left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}} \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}
\]

\[
\iff p_i = \left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}} \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}
\]

\[
\iff p_i = \frac{\left( \frac{1}{1 + \lambda^C} (1 - a_c)^{1-a_c} a_c e^{r_K^C} \right)^{\frac{1}{1-a_c}} \left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}}{\left( \frac{1}{1 + \lambda^I} (1 - a_i)^{1-a_i} a_i e^{r_K^I} \right)^{\frac{1}{1-a_i}}}
\]

Knowing \( w, r_K^C, r_K^I \), the expressions given in (3.17) can be used to find the steady state capital-to-labour ratios in the two sectors:

\[
k_C = \frac{w}{L_C} \frac{a_c}{r_K^C} \frac{1}{1 - a_c}, \tag{3.38}
\]

\[
k_I = \frac{w}{L_I} \frac{a_i}{r_K^I} \frac{1}{1 - a_c}. \tag{3.39}
\]

The zero profit condition for intermediate goods producers in the consumption sector,
\( c - r^K_C k_C - w L_C = 0 \), and (3.13) imply:

\[
L_C^{1-a_c} k_C^{a_c} - F_C - r^K_C k_C - w L_C = 0 \\
\Leftrightarrow \frac{F_C}{L_C} = \left( \frac{k_C}{L_C} \right)^{a_c} - r^K_C k_C - w.
\]

Analogously the zero profit condition for intermediate goods producers in the investment sector, \( i - r^I_I k_I - w L_I = 0 \), and (3.14) imply:

\[
\frac{F_I}{L_I} = \left( \frac{k_I}{L_I} \right)^{a_i} - r^I_I k_I - w.
\]

These expressions pin down the steady state consumption-to-labour and investment-to-labour ratios which follow from the intermediate firms’ production functions ((3.13) and (3.14)):

\[
\frac{c}{L_C} = \left( \frac{k_C}{L_C} \right)^{a_c} - \frac{F_C}{L_C}, \quad \frac{i}{L_I} = \left( \frac{k_I}{L_I} \right)^{a_i} - \frac{F_I}{L_I}.
\]

\[
1 + \lambda_p^C = \frac{c + F_C}{c} \Leftrightarrow \lambda_p^C c = F_C, \quad \text{and} \quad 1 + \lambda_p^I = \frac{i + F_I}{i} \Leftrightarrow \lambda_p^I i = F_I.
\]

This and the steady state consumption-to-labour ratio can be used to derive an expression for steady state consumption:

\[
c = \left( \frac{k_C}{L_C} \right)^{a_c} L_C - F_C \\
\Leftrightarrow c = \left( \frac{k_C}{L_C} \right)^{a_c} L_C - \lambda_p^C c \\
\Leftrightarrow c = \frac{1}{1 + \lambda_p^C} \left( \frac{k_C}{L_C} \right)^{a_c} L_C.
\]

Analogously one can derive an expression for steady state investment:

\[
i = \frac{1}{1 + \lambda_p^I} \left( \frac{k_I}{L_I} \right)^{a_i} L_I.
\]
Combining these two expressions leads to,

\[ \pi_c^i = \frac{1}{1+\lambda_p} \left( \frac{k_I}{L_I} \right)^{a_i} L_I p_i \]

\[ \L_I \L_C = \pi_c^i \frac{1}{1+\lambda_p} \left( \frac{k_C}{L_C} \right)^{a_c} \L_C p_i^{-1}. \]

Total labour \( L \) is set to unity in the steady state. However, since \( a_i \) and \( a_c \) are not necessarily calibrated to be equal one needs to fix another quantity in addition to \( L = 1 \). We fix the steady state investment-to-consumption ratio, \( \pi_c^i \), which equals 0.426 in the data. This allows us to derive steady state expressions for labour in the two sectors. Steady state labour in the investment sector is given by

\[ L_I = 1 - \L_C, \]

and the two equations above imply that steady state labour in the consumption sector can be expressed as,

\[ L_C = \left( 1 + \pi_c^i \frac{1}{1+\lambda_p} \left( \frac{k_c}{L_C} \right)^{a_c} \L_C p_i^{-1} \right)^{-1}. \]

The steady state values for labour in the two sectors imply:

\[ k_c = \frac{k_C}{L_C} L_C, \quad k_I = \frac{k_I}{L_I} L_I, \quad c = \frac{c}{L_C} \L_C, \quad i = \frac{i}{L_I} \L_I, \quad F_C = \frac{F_C}{L_C} L_C, \quad F_I = \frac{F_I}{L_I} \L_I. \]

It follows from (3.22) that,

\[ k_c = \bar{k}_C e^{-\frac{1}{\Gamma_c} g_v}, \quad \text{and} \quad k_I = \bar{k}_I e^{-\frac{1}{\Gamma_I} g_v}. \]

The accumulation equation of available capital (3.29) can be used to solve for investment in the two sectors:

\[ i_c = k_C \left( e^{\frac{1}{\Gamma_c} g_v} - (1 - \delta_C) \right), \]

\[ i_I = k_I \left( e^{\frac{1}{\Gamma_I} g_v} - (1 - \delta_I) \right). \]

From the definition of GDP (3.31):

\[ y = c + p_i i + \left( 1 - \frac{1}{g} \right) y. \]
From the marginal utility of income (3.20):
\[
\lambda = \frac{1}{c - hce^{-ga - \frac{\sigma_e}{1-\sigma_i}g_a} - \frac{\beta h}{ce^{ga + \frac{\sigma_e}{1-\sigma_i}g_a} - hc}}.
\]

From the household’s wage setting (3.30)
\[
\sum_{s=0}^{\infty} \beta^s \xi_s^a \lambda L \left[ w - (1 + \lambda_w) \varphi \frac{L^\nu}{\lambda} \right] = 0,
\]
follows the expression for \( L \):
\[
w - (1 - \lambda_w) \varphi \frac{L^\nu}{\lambda} = 0 \quad \Rightarrow \quad L = \left[ \frac{w\lambda}{(1 + \lambda_w)\varphi} \right]^{\frac{1}{\nu}}.
\]
This expression can be solved for \( \varphi \) to be consistent with \( L = 1 \):
\[
1 = \left[ \frac{w\lambda}{(1 + \lambda_w)\varphi} \right]^{\frac{1}{\nu}} \quad \Leftrightarrow \varphi = \frac{\lambda w}{1 + \lambda_w}.
\]
It further holds from equation (3.19) that,
\[
\frac{\pi I}{\pi C} = e^{gs + \frac{\sigma_a^e}{1-\sigma_i}g_v}.
\]
A system of 10 equations (3.32, 3.33, 3.35, 3.37, 3.39, 3.40, 3.41, 3.42, 3.43) can be solved for the 10 steady state variables \( k_C, k_I, w, i_C, i_I, r^K_C, r^K_I, L_C, L_I \) and \( p_i \). The steady state values for the remaining variables follow from the expressions above.

Given these steady state variables, the remaining steady state values are mainly related to financial intermediaries and household’s provision of funds for capital acquisition. These can be derived as follows.

The nominal interest rate is given from the Euler equation as,
\[
R = \frac{1}{\beta} e^{gs + \frac{\sigma_e}{1-\sigma_i}g_v} \pi^-C.
\]
The bank’s stationary stochastic discount factor can be expressed in the steady state as
\[
\lambda^B = 1.
\]
The steady state price of capital is given by
\[ q_{x,t} = p_{x,t}. \]
The steady state leverage equation is given by
\[ \frac{q_{x,s_x}}{n_x} = q_x. \]
Given the non-linearity in the leverage ratio, we solve numerically for the steady state expressions for \( \eta \) and \( \nu \) using,
\[ \nu_x = (1 - \theta_B) \lambda^B e^{-g_x - \frac{a_x}{1 - \pi_x}} (R^B\pi_C - R) + \theta_B \beta z_1 \nu_x, \]
\[ \eta_x = (1 - \theta_B) \lambda^B e^{-g_x - \frac{a_x}{1 - \pi_x}} R + \theta_B \beta z_2 \eta_x, \]
with
\[ z_2^x = \left[ (R^B\pi_C - R) q_x + R \right] \frac{1}{\pi_C}, \quad \text{and} \quad z_1^x = z_2^x, \]
and the steady state leverage ratio,
\[ q_x = \frac{\eta_x}{\lambda_B - \nu_x}. \]
Recall that the parameter \( \lambda_B \) is estimated. This information, the calibrated value for \( \theta_B \) and the weighted quarterly average of the corporate spreads \( (R^B - R = 0.5\%) \) allows us then to pin down \( \varpi \) using the bank’s wealth accumulation equation,
\[ \varpi = \left[ 1 - \theta_B \left( (R^B\pi_C - R) q_x + R \right) e^{-g_x - \frac{a_x}{1 - \pi_x}} \right] \left( \frac{q_x s_x}{n_x} \right)^{-1}. \]
The steady state equations for bank’s stochastic return can be solved to pin down
\[ q_x = \frac{q^K_x e^{g_x}}{R^B e^{-g_x} + \left( \frac{a_x}{1 - \pi_x} \right) g_x - (1 - \delta_x)}. \]
Household’s purchase of financial claims, equation (3.21), implies in steady state
\[ R^h_x = \frac{\pi e^{-g_x + \left( \frac{a_x}{1 - \pi_x} \right) g_x}}{\beta}. \]
Household’s sectoral returns on financial claims, equation (3.26) can be used to solve for \( q^h_C \) and \( q^h_I \)
\[ R^h_x = \left[ q^K_x + q^h_x (1 - \delta_x) \right] \frac{e^{g_x}}{\left( \frac{1 - a_x}{1 - \pi_x} \right) g_x q^h_x}. \]
Since the price of capital \( q^T_C = q^T_I = 1 \) and in the data the corporate bonds over equity
market capitalization, \( \frac{q_x s_x}{q_x s_x^*} = 0.25 \), the additive identity for capital claims from equity and
debt markets, equation (3.27), pins down \( s_x^h \) as follows

\[
s_x^h = \frac{q^h_x k_x}{1 + \frac{q_x s_x}{q_x s_x^*}} q^h_x.
\]

This allows in turn to use equation (3.27) to pin down \( s_x \). The conditions for bank’s and
firm’s optimal financing, equations (3.24) and (3.25), pin down the fixed cost to portfolio
adjustment

\[
\gamma^h = \beta r K \frac{q^h}{q^T} e^{-\left(\frac{1}{1-\alpha}\right) g_v},
\]

\[
\gamma = \beta r K \frac{q}{q^T} e^{-\left(\frac{1}{1-\alpha}\right) g_v}.
\]

### 3.7 Log-linearized Economy

This section collects the log-linearized model equations. The log-linear deviations of all
variables are defined as

\[
\hat{\varsigma}_t \equiv \log \varsigma_t - \log \varsigma,
\]

except for

\[
\hat{z}_t \equiv z_t - g_a,
\]

\[
\hat{v}_t \equiv v_t - g_v,
\]

\[
\hat{\lambda}^C_{p,t} \equiv \log(1 + \lambda^C_{p,t}) - \log(1 + \lambda^C_p),
\]

\[
\hat{\lambda}^I_{p,t} \equiv \log(1 + \lambda^I_{p,t}) - \log(1 + \lambda^I_p),
\]

\[
\hat{\lambda}_{w,t} \equiv \log(1 + \lambda_{w,t}) - \log(1 + \lambda_w).
\]
3.7.1 Firm’s production function and cost minimization

Production function for the intermediate good producing firm \( i \) in the consumption sector:

\[
\hat{c}_t = \frac{c + F_t}{c} [\hat{a}_{lt} + a_c \hat{k}_{C,t} + (1 - a_c) \hat{L}_{C,t}].
\]

Production function for the intermediate good producing firm \( i \) in the investment sector:

\[
\hat{i}_t = \frac{i + F_t}{i} [\hat{v}_{lt} + a_i \hat{k}_{I,t} + (1 - a_i) \hat{L}_{I,t}].
\]

Capital-to-labour ratios for the two sectors:

\[
\hat{r}_K^{C,t} - \hat{w}_t = \hat{L}_{C,t} - \hat{k}_{C,t}, \quad \hat{r}_K^{I,t} - \hat{w}_t = \hat{L}_{I,t} - \hat{k}_{I,t}. \tag{3.44}
\]

Marginal cost in both sectors:

\[
\hat{m}c_{C,t} = a_c \hat{r}_K^{C,t} + (1 - a_c) \hat{w}_t - \hat{a}_{lt}, \quad \hat{m}c_{I,t} = a_i \hat{r}_K^{I,t} + (1 - a_i) \hat{w}_t - \hat{v}_{lt} - \hat{p}_{i,t}. \tag{3.45}
\]

3.7.2 Firm’s prices

Price setting equation for firms that change their price in sector \( x = C, I \):

\[
0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_{p,x}^s \beta^s \left[ \hat{p}_{x,t} \hat{\Pi}_{t,t+s} - \hat{\lambda}^x_{p,t+s} - \hat{m}c_{x,t+s} \right] \right\},
\]

with

\[
\hat{\Pi}_{t,t+s} = \sum_{k=1}^{s} \left[ t_{p,x} \hat{\pi}_{t+k-1} - \hat{\pi}_{t+k} \right].
\]

Solving for the summation

\[
\frac{1}{1 - \xi_{p,x} \beta} \hat{p}_{x,t} = E_t \left\{ \sum_{s=0}^{\infty} \xi_{p,x}^s \beta^s \left[ - \hat{\Pi}_{t,t+s} + \hat{\lambda}^x_{p,t+s} + \hat{m}c_{x,t+s} \right] \right\}
\]

\[
= - \hat{\Pi}_{t,t} + \hat{\lambda}^x_{p,t} + \hat{m}c_{x,t} - \frac{\xi_{p,x} \beta}{1 - \xi_{p,x} \beta} \hat{\Pi}_{t,t+1}
\]

\[
+ \xi_{p,x} \beta E_t \left\{ \sum_{s=1}^{\infty} \xi_{p,x}^{s-1} \beta^{s-1} \left[ - \hat{\Pi}_{t+1,t+s} + \hat{\lambda}^x_{p,t+s} + \hat{m}c_{x,t+s} \right] \right\}
\]

\[
= \hat{\lambda}^x_{p,t} + \hat{m}c_{x,t} + \frac{\xi_{p,x} \beta}{1 - \xi_{p,x} \beta} \hat{\Pi}_{t,t+1},
\]

where we used \( \hat{\Pi}_{t,t} = 0 \).
Prices evolve as

\[ 0 = (1 - \xi_{p,x})\hat{p}_{x,t} + \xi_{p,x}(t_{p,x}\hat{\pi}_{t-1} - \hat{\pi}), \]

from which we obtain the Phillips curve in sector \( x = C, I \):

\[
\hat{\pi}_{x,t} = \frac{\beta}{1 + t_{p,c}}E_t\hat{\pi}_{x,t+1} + \frac{t_{p,c}}{1 + t_{p,c}}\hat{\pi}_{x,t-1} + \kappa_x\hat{\pi}_{x,t} + \kappa_x\hat{\lambda}_{p,t},
\]

with \( \kappa_x = \frac{(1 - \xi_{p,x})(1 - \xi_{p,x})}{\xi_{p,x}(1 + t_{p,c})} \).

From equation (3.19) it follows that

\[
\hat{\pi}_{I,t} - \hat{\pi}_{C,t} = \hat{p}_{I,t} - \hat{p}_{I,t-1}.
\]

### 3.7.3 Households

**Marginal utility:**

\[
\hat{\lambda}_t = \frac{e^G}{e^G - h\beta} \left[ \hat{b}_t + \left( \hat{z}_t + \frac{a_c}{1 - a_i} \hat{v}_t \right) - \left( \frac{e^G}{e^G - h} \left( \hat{c}_t + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{v}_t \right) - \frac{h}{e^G - h} \hat{c}_{t-1} \right) \right] \\
- \frac{h\beta}{e^G - h\beta} E_t \left[ \hat{b}_{t+1} - \left( \frac{e^G}{e^G - h} \left( \hat{c}_{t+1} + \hat{z}_{t+1} + \frac{a_c}{1 - a_i} \hat{v}_{t+1} \right) - \frac{h}{e^G - h} \hat{c}_t \right) \right]
\]

\[\Leftrightarrow \hat{\lambda}_t = \alpha_1 E_t \hat{c}_{t+1} - \alpha_2 \hat{c}_t + \alpha_3 \hat{c}_{t-1} + \alpha_4 \hat{z}_t + \alpha_5 \hat{b}_t + \alpha_6 \hat{v}_t, \tag{3.47}\]

with

\[
\alpha_1 = \frac{h\beta e^G}{(e^G - h\beta)(e^G - h)}, \quad \alpha_2 = \frac{e^{2G} + h^2\beta}{(e^G - h\beta)(e^G - h)}, \quad \alpha_3 = \frac{he^G}{(e^G - h\beta)(e^G - h)}, \\
\alpha_4 = \frac{h\beta e^G \rho_z - he^G}{(e^G - h\beta)(e^G - h)}, \quad \alpha_5 = \frac{e^G - h\beta \rho_b}{e^G - h\beta}, \quad \alpha_6 = \frac{(h\beta e^G \rho_v - he^G) \alpha_6}{1 - a_i}, \\
e^G = e^{\rho_i + \frac{\alpha_6}{1 - a_i}} g_o.
\]

This assumes the shock processes for \( \hat{z}_t \) and \( \hat{b}_t \).
Euler equation:
\[ \hat{\lambda}_t = \hat{R}_t + E_t(\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{v}_{t+1} \frac{a_c}{1 - a_i} - \hat{\pi}_{C,t+1}). \] (3.48)

Purchase of financial claims
\[ E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \hat{z}_{t+1} = \left( \frac{a_c}{1 - a_i} \right) \hat{v}_{t+1} + \hat{R}^h_{x,t} - \hat{\pi}_{C,t+1} = 0. \] (3.49)

### 3.7.4 Investment and Capital

Capital utilization in both sectors:
\[ \hat{r}^K_{C,t} = \chi_{C,t} \hat{u}_{C,t}, \quad \hat{r}^K_{I,t} = \chi_{I,t} \hat{u}_{I,t}, \quad \text{where} \quad \chi_x = \frac{a''_x(1)}{a'_x(1)}. \] (3.50)

Choice of investment for the consumption sector:
\[ \hat{q}_{C,t} = e^{2(1 - a_i \sigma_v)} \kappa \left( \hat{i}_{C,t} - \hat{i}_{C,t-1} + \frac{1}{1 - a_i} \hat{v}_t \right) - \beta e^{2(1 - a_i \sigma_v)} \kappa E_t \left( \hat{i}_{C,t+1} - \hat{i}_{C,t} + \frac{1}{1 - a_i} \hat{v}_{t+1} \right) \]
\[ + \hat{p}_{i,t} - \hat{\mu}, \] (3.51)

with \( \hat{q}_{C,t} = \hat{\phi}_{C,t} - \hat{\lambda}_t. \)

Choice of investment for the investment sector:
\[ \hat{q}_{I,t} = e^{2(1 - a_i \sigma_v)} \kappa \left( \hat{i}_{I,t} - \hat{i}_{I,t-1} + \frac{1}{1 - a_i} \hat{v}_t \right) - \beta e^{2(1 - a_i \sigma_v)} \kappa E_t \left( \hat{i}_{I,t+1} - \hat{i}_{I,t} + \frac{1}{1 - a_i} \hat{v}_{t+1} \right) \]
\[ + \hat{p}_{I,t} - \hat{\mu}, \] (3.52)

with \( \hat{q}_{I,t} = \hat{\phi}_{I,t} - \hat{\lambda}_t. \)

Capital services input in both sectors:
\[ \hat{k}_{C,t} = \hat{u}_{C,t} + \xi^K_{C,t} + \hat{k}_{C,t-1} - \frac{1}{1 - a_i} \hat{v}_t, \quad \hat{k}_{I,t} = \hat{u}_{I,t} + \xi^K_{I,t} + \hat{k}_{I,t-1} - \frac{1}{1 - a_i} \hat{v}_t. \] (3.53)

Capital accumulation in the consumption and investment sector:
\[ \hat{k}_{C,t} = (1 - \delta_C)e^{-\frac{1}{1-a_i} \sigma_v} \left( \hat{k}_{C,t-1} + \xi^K_{C,t} - \frac{1}{1 - a_i} \hat{v}_t \right) + \left( 1 - (1 - \delta_C)e^{-\frac{1}{1-a_i} \sigma_v} \right) \hat{i}_{C,t}. \] (3.54)
\[ \hat{k}_{I,t} = (1 - \delta_I)e^{-\frac{1}{1-a_i} \sigma_v} \left( \hat{k}_{I,t-1} + \xi^K_{I,t} - \frac{1}{1 - a_i} \hat{v}_t \right) + \left( 1 - (1 - \delta_I)e^{-\frac{1}{1-a_i} \sigma_v} \right) \hat{i}_{I,t}. \] (3.55)
Optimal financing from households

\[
0 = \beta q_h T x \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{q}_{x,t} - \hat{q}_{x,t}^T + \hat{u}_{x,t+1} \right) - \beta q_h a'(u_x) \hat{u}_{x,t+1} \\
- \gamma e \left( \frac{1}{1-a_r} \right) g_x \left[ \left( \frac{1}{1-a_r} \right) \hat{v}_{t+1} \right] - \kappa e \left( \frac{1}{1-a_r} \right) g_x \left[ \hat{v}_{x,t} + \left( \frac{1}{1-a_r} \right) \hat{v}_t \right].
\] (3.56)

Optimal financing from financial intermediaries

\[
0 = \beta q_x T x \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{q}_{x,t} - \hat{q}_{x,t}^T + \hat{u}_{x,t+1} \right) - \beta q_x a'(u_x) \hat{u}_{x,t+1} \\
- \gamma e \left( \frac{1}{1-a_r} \right) g_x \left[ \left( \frac{1}{1-a_r} \right) \hat{v}_{t+1} \right] - \kappa e \left( \frac{1}{1-a_r} \right) g_x \left[ \hat{v}_{x,t} + \left( \frac{1}{1-a_r} \right) \hat{v}_t \right],
\] (3.57)

Household’s return on claims

\[
\hat{R}_h x,t = \frac{1}{r_x^K + q_h^h(1-\delta)} \left[ r_x^K (\hat{q}_x^T + \hat{u}_{x,t+1}) + q_h^h (1-\delta) \hat{q}_x^T + \hat{z}_{t+1} - \hat{q}_{x,t}^T + \hat{z}_{t+1} - \left( \frac{1-a_c}{1-a_i} \right) \hat{v}_{t+1} \right],
\] (3.58)

Total value of acquired capital

\[
\hat{q}_{x,t}^T + \hat{k}_{x,t} = \frac{q_h^h s_h}{q_x^h k_x} \left( \hat{q}_{x,t}^h + \hat{s}_{x,t} \right) + \frac{q_x s_x}{q_x^h k_x} \left( \hat{q}_{x,t} + \hat{s}_{x,t} \right).
\] (3.59)

3.7.5 Wages

The wage setting equation for workers renegotiating their salary:

\[
0 = E_t \left\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \left[ \hat{w}_t + \hat{\Pi}_{w,t+s} - \hat{L}_{w,t+s} - \hat{b}_{t+s} - \nu \hat{L}_{t+s} + \hat{L}_{t+s} \right] \right\},
\]

with

\[
\hat{\Pi}_{w,t+s} = \sum_{k=1}^{s} \left[ \xi_w \left( \hat{\pi}_{c,t+k-1} + \hat{z}_{t+k-1} + \frac{a_c}{1-a_i} \hat{v}_{t+k-1} \right) - \left( \hat{\pi}_{c,t+k} + \hat{z}_{t+k} + \frac{a_c}{1-a_i} \hat{v}_{t+k} \right) \right],
\]

and

\[
\hat{L}_{t+s} = \hat{L}_{t+s} - \left( 1 + \frac{1}{\lambda_w} \right) \left( \hat{w}_t + \hat{\Pi}_{w,t+s} - \hat{w}_{t+s} \right).
\]
Then using the labor demand function,
\begin{align*}
0 &= E_t \left\{ \sum_{s=0}^{\infty} \xi^s \beta^s \left[ \hat{\tilde{w}}_t + \hat{\tilde{\Pi}}^w_{t,t+s} - \hat{\lambda}_{w,t+s} - \hat{b}_{t+s} \right] \\
&\quad - \nu \left( L_{t+s} - \left( 1 + \frac{1}{\lambda_w} \right) \left( \hat{\tilde{w}}_t + \hat{\tilde{\Pi}}^w_{t,t+s} - \hat{\tilde{w}}_{t+s} \right) \right) \right\} \\
\Leftrightarrow 0 &= E_t \left\{ \sum_{s=0}^{\infty} \xi^s \beta^s \left[ \hat{\tilde{w}}_t \left( 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right) \right) + \hat{\tilde{\Pi}}^w_{t,t+s} - \hat{\lambda}_{w,t+s} - \hat{b}_{t+s} \right] \\
&\quad - \nu \left( \tilde{L}_{t+s} - \left( 1 + \frac{1}{\lambda_w} \right) \left( \hat{\tilde{\Pi}}^w_{t,t+s} - \tilde{w}_{t+s} \right) + \hat{\lambda}_{t+s} \right) \right\}.
\end{align*}

Solving for the summation,
\begin{align*}
\frac{\nu_w}{1 - \xi_w \beta} \hat{\tilde{\psi}}_t &= E_t \left\{ \sum_{s=0}^{\infty} \xi^s \beta^s \left[ - \left( 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right) \right) \hat{\tilde{\Pi}}^w_{t,t+s} + \hat{\psi}_{t+s} \right] \right\} \\
&= - \nu_w \hat{\tilde{\Pi}}^w_{t,t+1} + \hat{\psi}_t + E_t \left\{ \sum_{s=0}^{\infty} \xi^s \beta^s \left[ - \nu_w \hat{\tilde{\Pi}}^w_{t,t+s} + \hat{\psi}_{t+s} \right] \right\} \\
&= \hat{\psi}_t - \frac{\xi_w \beta}{1 - \xi_w \beta} \nu_w \hat{\tilde{\Pi}}^w_{t,t+1} + \xi_w \beta E_t \left\{ \sum_{s=0}^{\infty} \xi^s \beta^s \left[ - \nu_w \hat{\tilde{\Pi}}^w_{t,t+1+s} + \hat{\psi}_{t+1+s} \right] \right\} \\
&= \hat{\psi}_t + \frac{\xi_w \beta}{1 - \xi_w \beta} \nu_w \hat{\tilde{\Pi}}^w_{t,t+1}.
\end{align*}

where
\begin{align*}
\hat{\psi}_t &\equiv \hat{\lambda}_{w,t} + \hat{b}_t + \nu \hat{L}_t + \nu \left( 1 + \frac{1}{\lambda_w} \right) \hat{w}_t - \hat{\lambda}_t, \\
\nu_w &\equiv 1 + \nu \left( 1 + \frac{1}{\lambda_w} \right),
\end{align*}
and recall that $\hat{\Pi}_{t,t}^w = 0$.

Wages evolve as,
\begin{align*}
\hat{w}_t &= (1 - \xi_w) \hat{w}_t + \xi_w \left( \hat{w}_{t-1} + \tau_c \hat{v}_t \hat{\pi}_{c,t-1} + \tau_w \left( \hat{\pi}_{c,t-1} + \frac{a_c}{1 - a_i} \hat{v}_{t-1} \right) - \hat{\pi}_{c,t} - \hat{z}_t - \frac{a_c}{1 - a_i} \hat{v}_t \right) \\
\Leftrightarrow \hat{w}_t &= (1 - \xi_w) \hat{w}_t + \xi_w (\hat{w}_{t-1} + \hat{\Pi}_{t,t-1}^w).
\end{align*}

Equation (3.62) can be solved for $\hat{w}_t$. This expression, as well as the formulation for $\hat{\psi}_t$ given in (3.61) can be plugged into equation (3.60). After rearranging this yields the wage Phillips curve,
\[
\begin{align*}
\hat{w}_t &= \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{w}_{t+1} - \kappa_w \hat{g}_{w,t} + \frac{\nu_w}{1 + \beta} \hat{\pi}_{c,t-1} - \frac{1 + \beta \nu_w}{1 + \beta} \hat{\pi}_{c,t} \\
&+ \frac{\beta}{1 + \beta} E_t \hat{\pi}_{c,t+1} + \kappa_w \hat{\lambda}_{w,t} + \frac{\nu_w}{1 + \beta} (\hat{z}_{t-1} + \frac{a_c}{1 - a_i} \hat{v}_{t-1}) \\
&- \frac{1 + \beta \nu_w - \rho_z \beta}{1 + \beta} \hat{z}_t - \frac{1 + \beta \nu_w - \rho_v \beta}{1 + \beta} \frac{a_c}{1 - a_i} \hat{v}_t.
\end{align*}
\]

(3.63)

where

\[
\begin{align*}
\kappa_w &\equiv \frac{(1 - \xi_w \beta)(1 - \xi_w)}{\xi_w (1 + \beta)(1 + \nu (1 + \xi_w))}, \\
\hat{g}_{w,t} &\equiv \hat{w}_t - (\nu \hat{L}_t + \hat{b}_t - \hat{\lambda}_t).
\end{align*}
\]

3.7.6 Financial sector

The part of the economy concerned with the banking sector is described by the following equations:

The stochastic discount factor:

\[
\hat{\lambda}_t^B = \hat{\lambda}_t - \hat{\lambda}_{t-1}.
\]

(3.64)

Definition of \( \nu \) for \( x = C, I \):

\[
\hat{\nu}_x,t = (1 - \theta_B z^x_t) [\hat{\lambda}_t^B + \hat{z}_{t+1} - \frac{a_c}{1 - a_i} \hat{v}_{t+1}] \\
+ \frac{1 - \theta_B \beta z^x_t}{R_x^B \pi_C} [R_x^B \pi_C \hat{R}_x,t+1 + R_x^B \pi_C \hat{\pi}_{C,t+1} + R \hat{R}_t] + \theta_B \beta z^x_t [\hat{z}_{t+1} + \hat{\nu}_{x,t+1}].
\]

(3.65)

Definition of \( \eta \):

\[
\hat{\eta}_x,t = (1 - \theta_B \beta z^x_2) [\hat{\lambda}_t^B + \hat{z}_{t+1} - \frac{a_c}{1 - a_i} \hat{v}_{t+1} + \hat{R}_t] \\
+ \theta_B \beta z^x_2 [\hat{z}_{2,t+1} + \hat{\eta}_{t+1}], \quad x = C, I.
\]

(3.66)

Definition of \( z^1 \):

\[
\hat{z}^x_{1,t} = \hat{\phi}_x,t - \hat{\phi}_{x,t-1} + \hat{z}^x_2, \quad x = C, I.
\]

(3.67)
Definition of $z_2$ for $x = C, I$:

$$
\ddot{z}_{2,t} = \frac{\pi C}{(R^B_x - R)q_x + R} [R^B_x \dot{q}_x [\dot{R}^B_x + \dot{\pi}_{C,t}] + \frac{R}{\pi C} (1 - q_x) \hat{\pi}_{t-1} + (R^B_x \pi C - R) \frac{q_x}{\pi C} \dot{\hat{\pi}}_{x,t-1}] - \dot{\pi}_{C,t}.
$$

(3.68)

The leverage ratio:

$$\dot{\hat{\pi}}_{x,t} = \frac{\nu}{\lambda_B - \nu} \hat{\nu}_{x,t}, \quad x = C, I.
$$

(3.69)

The leverage equation:

$$\dot{\hat{q}}_{x,t} + \dot{\hat{s}}_{x,t} = \dot{\hat{\pi}}_{x,t} + \dot{\hat{n}}_{x,t}.
$$

(3.70)

The bank’s wealth accumulation equation

$$\dot{n}_{x,t} = \theta_B \frac{\theta x}{\pi C} e^{-\frac{g}{\pi_C} \hat{\pi}_t} \left[ R_x^B \pi C [\dot{R}^B_x + \dot{\pi}_{C,t}] + \left( \frac{1}{q_x} - 1 \right) R \hat{\pi}_{t-1} + (R_x^B \pi C - R) \dot{\hat{\pi}}_{x,t-1} \right]
\quad + \theta_B e^{-\frac{g}{\pi_C} \hat{\pi}_t} \left[ (R^B_x \pi C - R) \dot{q}_x + R \left[ -\frac{\alpha_c}{1 - \alpha_i} \hat{\nu}_t + \dot{\hat{n}}_{x,t-1} - \ddot{\pi}_{C,t} \right] + (1 - \theta_B e^{-\frac{g}{\pi_C} \hat{\pi}_t} \left[ (R^B_x \pi C - R) \dot{q}_x + R \right]) \left[ \dot{\hat{q}}_{x,t} + \dot{\hat{s}}_{x,t} \right] + \xi_{x,t}, \quad x = C, I.
$$

(3.71)

The bank’s stochastic return on assets in sector $x = C, I$:

$$\hat{R}^B_{x,t} = \frac{1}{\gamma^x + q_x (1 - \delta_x)} \left[ \gamma^x \left( \gamma^x + \hat{\pi}_{C,t} + \hat{\pi}_{I,t} \right) + q_x (1 - \delta_x) \dot{q}_{x,t} \right] - \dot{\hat{q}}_{x,t-1} + \xi_{x,t} + \dot{\hat{\pi}}_t - \frac{1 - \alpha_c}{1 - \alpha_i} \hat{\nu}_t.
$$

(3.72)

Excess (nominal) return:

$$\hat{R}^S_{x,t} = \frac{R^B_x \pi C}{R_x^B \pi C - R} (\hat{R}^B_{x,t+1} + \ddot{\pi}_{C,t+1}) - \frac{R}{R_x^B \pi C - R} \hat{R}_{t}, \quad x = C, I.
$$

(3.73)

3.7.7 Monetary policy and market clearing

Monetary policy rule:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[ \phi_x \ddot{\pi}_{c,t} + \phi_{\Delta Y} (\dot{y}_t - \dot{y}_{t-1}) \right] + \hat{\eta}_{mp,t}
$$

(3.74)

Resource constraint in the consumption sector:

$$\ddot{c}_t + \left( \frac{K^C c}{C} \ddot{u}_{C,t} + \frac{K^I c}{C} \ddot{u}_{I,t} \right) e^{\frac{1}{\gamma^x} q_x} = \frac{c + F_c}{c} [\ddot{u}_t + a_c \ddot{L}_{C,t} + (1 - a_c) \ddot{L}_{I,t}]
$$

(3.75)
Resource constraint in the investment sector:

\[ \hat{i}_t = \frac{i + F_I}{\hat{c}_t} \left[ \hat{c}_t + a_i \hat{k}_{I,t} + (1 - a_i) \hat{L}_{I,t} \right] \]  

(3.76)

Definition of GDP:

\[ \hat{y}_t = \frac{c}{c + p_i} \hat{c}_t + \frac{p_i}{c + p_i} (\hat{c}_t + \hat{p}_{I,t}) + \hat{g}_t. \]  

(3.77)

Market clearing:

\[ \frac{L_C}{L} \hat{L}_{C,t} + \frac{L_I}{L} \hat{L}_{I,t} = \hat{L}_t, \quad \frac{L_C}{C} \hat{C}_{C,t} + \frac{L_I}{I} \hat{I}_{I,t} = \hat{I}_t, \quad \frac{n_C}{n} \hat{n}_{C,t} + \frac{n_I}{n} \hat{n}_{I,t} = \hat{n}_t. \]  

(3.78)

### 3.7.8 Exogenous processes

The 11 exogenous processes of the model can be written in log-linearized form as follows:

**Price markup in sector** \( x = C, I \):

\[ \hat{\lambda}_{p,t} = \rho \hat{\lambda}_{p,t-1} + \epsilon_{p,t}. \]  

(3.79)

**The TFP growth (consumption sector):**

\[ \hat{z}_t = \rho \hat{z}_{t-1} + \epsilon_t^z. \]  

(3.80)

**The TFP growth (investment sector):**

\[ \hat{v}_t = \rho \hat{v}_{t-1} + \epsilon_t^v. \]  

(3.81)

**Wage markup:**

\[ \hat{\lambda}_{w,t} = \rho \hat{\lambda}_{w,t-1} + \epsilon_{w,t}. \]  

(3.82)

**Preference:**

\[ \hat{b}_t = \rho \hat{b}_{t-1} + \epsilon_t^b. \]  

(3.83)

**Monetary policy:**

\[ \hat{\eta}_{mp,t} = \epsilon_{t}^{mp}. \]  

(3.84)

**Government spending:**

\[ \hat{g}_t = \rho \hat{g}_{t-1} + \epsilon_t^g. \]  

(3.85)
The Marginal Efficiency of Investment (MEI):
\[ \hat{\mu}_t = \rho \hat{\mu}_{t-1} + \varepsilon_t^\mu \] (3.86)

The TFP stationary (consumption sector):
\[ \hat{a}_{lt} = \rho \hat{a}_{l,t-1} + \varepsilon_t^{a_l} \] (3.87)

The TFP stationary (investment sector):
\[ \hat{v}_{lt} = \rho \hat{v}_{l,t-1} + \varepsilon_t^{v_l} \] (3.88)

Bank equity capital:
\[ \hat{s}_{x,t} = \rho \hat{s}_{x,t-1} + \varepsilon_{x,t}^s \] (3.89)

The entire log-linear model is summarized by equations (3.44) - (3.59) and (3.63) - (3.78) as well as the shock processes (3.79) - (3.89).

3.8 Measurement equations

For estimation, model variables are linked with observables using measurement equations. Letting a superscript "d" denote observable series, then the model’s measurement equations are as follows:

Real consumption growth,
\[ \Delta C^d_t = \log \left( \frac{C_t}{C_{t-1}} \right) = \log \left( \frac{c_t}{c_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{\nu}_t, \]

Real investment growth,
\[ \Delta I^d_t = \log \left( \frac{I_t}{I_{t-1}} \right) = \log \left( \frac{i_t}{i_{t-1}} \right) + \frac{1}{1 - a_i} \hat{\nu}_t, \]

Real wage growth,
\[ \Delta W^d_t = \log \left( \frac{W_t}{W_{t-1}} \right) = \log \left( \frac{w_t}{w_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{\nu}_t, \]
Real output growth,
\[ \Delta Y^d_t \equiv \log \left( \frac{Y_t}{Y_{t-1}} \right) = \log \left( \frac{y_t}{y_{t-1}} \right) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{\nu}_t, \]

Consumption sector inflation,
\[ \pi^d_{C,t} \equiv \pi_{C,t} = \hat{\pi}_{C,t} \quad \text{and} \quad \hat{\pi}_{C,t} = \log(\pi_{C,t}) - \log(\pi_C), \]

Relative price of investment
\[ \Delta \left( \frac{P^I_t}{P^C_t} \right)^d \equiv \log \left( \frac{p^I_t}{p^C_{t-1}} \right) + \hat{z}_t + \frac{a_c - 1}{1 - a_i} \hat{\nu}_t, \]

Total hours worked,
\[ L^d_t \equiv \log L_t = \hat{L}_t, \]

Nominal interest rate (federal funds rate),
\[ R^d_t \equiv \log R_t = \log \hat{R}_t, \]

Corporate bond spread,
\[ R^{S,d}_t \equiv \log R^S_t = (e^{\log R^B_{C,t+1} e^{\hat{\pi}_{C,t+1}}} - e^{\log \hat{R}_t}) * w_C + (e^{\log R^B_{I,t+1} e^{\hat{\pi}_{C,t+1}}} - e^{\log \hat{R}_t}) * w_I, \]

where \( w_C \) and \( w_I \) are steady state shares of assets (as a fraction of bank equity) in banks portfolios in the consumption and investment sector respectively.

Real total bank equity capital growth,
\[ \Delta N^d_t \equiv \log \left( \frac{N_t}{N_{t-1}} \right) = e^{g + \frac{a_c}{1 - a_i} \hat{g}} \left( \frac{n_C}{n_C + n_I} (\hat{n}_{C,t} - \hat{n}_{C,t-1}) + \frac{n_I}{n_C + n_I} (\hat{n}_{I,t} - \hat{n}_{I,t-1}) + \hat{z}_t + \frac{a_c}{1 - a_i} \hat{\nu}_t \right). \]

Capital claims from equity markets
\[ \Delta S^{h,d}_t \equiv \log \left( \frac{S^h_t}{S^{h-1}_t} \right) = e^{1 - \frac{a_i}{1 - a_i} \hat{g}} \left( \frac{s^h_C}{s^h_C + s^h_I} (s^h_{C,t} - s^h_{C,t-1} + \hat{q}^h_{C,t} - \hat{q}^h_{C,t-1}) + \left( 1 - \frac{s^h_C}{s^h_C + s^h_I} \right) (s^h_{I,t} - s^h_{I,t-1} + \hat{q}^h_{I,t} - \hat{q}^h_{I,t-1}) + \frac{1}{1 - a_i} \hat{\nu}_t \right). \]
References


