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A generalised model of crop lodging

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Abstract

The lodging of cereal crops due to high wind and rain is of considerable significance in many parts of the world, leading to major economic losses and yield reductions. In earlier papers the authors have developed a model of the lodging of winter wheat that identified the major parameters of the problem and enabled the relationship between root and stem lodging to be examined. It has formed the basis of a methodology used in the UK for guidance to farmers and agronomists on ways of reducing lodging risk. However the authors would be the first to acknowledge that there are limitations to the model that make it difficult to apply for a wide range of crops – particularly in the specification of the wind field and the root / soil interaction, and in allowing for stem lodging elsewhere than at the base of the stem. This paper thus describes the development of a generalised model that overcomes these shortcomings. After a discussion of the lodging phenomenon in general and a description of earlier work, the basis of the new model is set out, based upon a mechanical model of the wind / plant /soil interactions that capture most of the important physical processes. The manner in which this model can be applied to clarify the nature of the lodging process and calculate lodging risk through a simple graphical formulation is discussed. In particular simple formulae are defined for lodging risk that are a function of a small number of dimensionless variables with identified physical meanings. The model is then applied to the lodging of wheat, oat and oilseed rape crops and considers the sensitivity of the risk calculations to uncertainties in the model parameters. In general it is suggested that the risk of lodging can be determined from very simple functions of dimensionless stem and root lodging velocities.

Keywords – wheat, oats, oilseed-rape, wind, rain

Notation

a	stem radius
A_{CF}	plant drag area for an isolated plant or the plant shear area for a plant in a canopy
c	clay content
C	damping coefficient per plant due to stem material properties
d	effective root ball diameter
E	Young's modulus of the stem
f	moisture content at field capacity
f_n	natural frequency of crop oscillation
F	wind induced force on a plant
$\bar{F}$	mean wind force value
F'	amplitude of the fluctuating wind force
g_{MB}	gust factor of broad banded stem moment
g_{MR}	gust factor of resonant stem moment
i	daily rainfall
i_o	reference daily rainfall
i_s	saturation rainfall
I	turbulence intensity
I_A	second moment of area of the stem
K	damping coefficient per plant due to canopy interactions
k	wave number
l	length of stem
L	depth of the anchorage rooting system
m	mean daily rainfall
$\widetilde{m}$	$\frac{\mathrm{m}}{i_0}$
M	moment on stem
$\bar{M}$	mean moment on stem
M'	fluctuating moment on stem
$\widehat{M}$	peak moment on stem
n	number of stems per plant
p	number of plants per unit area
$p(i)$	rainfall probability density function
$p(\bar{U})$	wind probability density function
R	lodging risk
s	soil shear strength
s_D	soil shear strength at the permanent wilting point
s_W	soil shear strength at field capacity
S_M	stem moment spectrum
S_U	wind spectrum
t	stem wall thickness
T	time
$\bar{U}$	mean velocity
$\bar{U}_{LR}$	root lodging velocity
$\bar{U}_{LS}$	stem lodging velocity
$\bar{U}_S$	saturation velocity
$\widetilde{U}_{LR}$	$\frac{\bar{U}_{LR}}{\lambda}$
$\widetilde{U}_{LS}$	$\frac{\bar{U}_{LS}}{\lambda}$
$\widetilde{U}_S$	$\frac{\bar{U}_S}{\lambda}$

v	empirical visual score
V_{gR}	root lodging velocity from Berry *et al.* (2003)
V_{gS}	stem lodging velocity from Berry *et al.* (2003)
w	moisture content at the permanent wilting point
x	distance up the stem from the ground
X	height of the centre of mass of the crop canopy
y	displacement of the stem at height x
Y	displacement of the top of the stem
z	distance in the y direction from a fixed point at the edge of the crop
z_0	surface roughness length

α	dimensionless parameter in equation (6)
γ	constant in equation (25)
ϵ	honami constant
η	wave velocity
θ	damping ratio
λ	wind pdf parameter
μ	mass of the unit area of the canopy
ρ	density of air
ρ_s	density of the soil
ρ_w	density of water
σ	stem yield stress
σ_{MB}	broad banded moment standard deviation
σ_{MR}	resonant moment standard deviation
τ	averaging time
ω	radial frequency

1. Introduction

Lodging, defined as the permanent displacement of plant stems from their vertical position, is a persistent problem for many crop species throughout the world, especially in regions where crops are irrigated or where rainfall and high winds are common. As a result lodging resistance is considered one of the highest priorities for plant breeders worldwide (Reynolds *et al*., 2008). In the UK lodging is a particular problem in wheat, barley, oats and oilseed rape (figure 1). It has been shown that severe lodging years occur, on average, every 3 to 4 years when 16% of the wheat area and 35% of the oilseed rape areas are lodged (Berry *et al*., 1998; 2013). Yield losses from lodging in cereal crops and oilseed rape typically can be 25%, but can be up to 75% if lodging occurs early in the season (Berry and Spink, 2012; Berry *et al.,* 2013). The cost of lodging from yield losses in a severe lodging year has been estimated at £105 million for wheat and £64 million for oilseed rape (Berry 2013). These estimates do not include the additional costs of greater grain drying, loss of bread making quality and longer harvesting time. Lodging in the other major UK cereals, barley and oats, is

generally considered to be more prevalent than in wheat. If similar lodging costs per hectare are assumed for these cereal species as for wheat, then the total cost of lodging in a severe lodging year in the UK is conservatively estimated at £200 million, or approximately £60 million per year on average. Farmers attempt to reduce the risk of lodging by using lodging resistant cultivars and by using chemical plant growth regulators (PGRs). Little progress has been made in breeding cereal varieties with greater lodging resistance since the introduction of semi-dwarfing genes during the 1970s and 80s, and there is evidence in wheat that the lodging resistance of new varieties may in fact be increasing because yield potential is increasing whilst height remains the same (Kendall et al., 2013). PGRs are used on the majority of UK cereal and oilseed rape crops. However, in future the number of products available may become restricted as a result of changing legislation in Europe (revision of 91/414/EEC). Additionally some oat markets restrict the use of PGRs such as chlormequat to minimise the risk of chemical residues in the grain.

The lodging of winter wheat was considered in a series of papers by the authors in the early 2000s (Baker *et al.,* 1998, Berry *et al*., 2000, 2003, Sterling *et al*., 2003), through the development of a model of the wheat lodging process, ultimately derived from that developed by Baker (1995), and this has since underpinned much of the current understanding of wheat lodging control. For example, it was used to show that two types of lodging (stem and root) are both important. Unexpectedly it identified variation in stem and anchorage strength as very important determinants of lodging resistance and this led to work with wheat breeders to investigate genetic markers for these traits. The wheat lodging model also underpins the most comprehensive guide that growers and agronomists use to minimise lodging risk (Berry and Spink, 2005) as well as being used by the agricultural industry to quantify the effect of PGRs on lodging risk. This work involved a series of field tests to observe and measure the lodging phenomenon, using a mobile wind tunnel that was placed over field grown crops (Sterling *et al*., 2003). The results from these tests were used to develop a simple mathematical model of the lodging process that allowed the lodging wind speeds to be determined for crops of different types. This model assumed that the wheat plant can be represented by a two mass model connected by a weightless stem. The upper mass corresponds to the ear, whilst the lower mass corresponds to the root ball. A wind force due to a step change in wind speed is applied to the upper mass to cause displacement of the stem, which is resisted by the anchorage characteristics of the root mass. This allows the bending moment at the base of the stem to be determined, which is then compared with the strength at the base of the stem, or the root failure moment to determine whether or not lodging occurs. This model has been used to provide a method of quantifying how variation in field and plant characteristics affect lodging risk that would allow PGR use to be targeted more precisely.

However, as noted above, winter wheat is not the only cereal crop to undergo lodging in adverse conditions and significant lodging has been observed in barley, oats and oilseed rape. Now whilst one might expect the wheat lodging model to be generally applicable to other crops, the nature of these crops implies that significant modifications might need to be made to the model to ensure its applicability. Specifically modifications are likely to be required in the following areas.

- Some crops, such as oilseed rape, have a tap root anchorage system (Goodman *et al.,* 2001) which is very different from that assumed in the current lodging model.
- Different crops have different stem structures and plant morphologies between flowering and seed filling that need to be allowed for.
- Oats and oilseed rape have interlocking plant canopies, and, at least in part of the growth period, need to be considered as a complete canopy rather than as individual plants.
- For some crops, oilseed rape and oats in particular, it is clear that stem bending/buckling at any point along the stem can occur, rather than just at the base of the stem.

In this paper we develop the wheat lodging model such that it is applicable to a range of other cereal crops and oilseed rape, and in so doing we derive a much simpler, more realistic and much more straightforward approach than previously developed. In section 2 we set out the basic equations of the revised model that takes into account the effects mentioned above, and also introduce the criteria for stem and root lodging. In section 3 we then consider the method of solution for these equations, which results in a series of very simple closed form expressions for the calculation of lodging risk. In section 4 we use the model to investigate aspects of the lodging phenomenon for wheat, oat and oilseed rape crops. Section 5 then sets out some conclusions and makes suggestions for further work.

a) Stem and root lodging of oilseed rape and oats

b) Transition of oilseed rape crop from independent plants to an interlocked canopy

Figure 1 Oat and rape crops

2. The basic equations of the model for interlocking plants

2.1 Differences from the wheat lodging model

The calculation broadly follows the earlier method outlined in Baker (1995) for wheat lodging, in writing down Newton's Law for the canopy top, applying a fluctuating wind load, calculating the fluctuating bending moment along the stem, and, through models for stem strength and root strength, calculating the critical wind speed and rainfall conditions for stem and root lodging. There are however a number of significant differences.

- The same equations are taken to apply for an interlocking canopy (i.e. after the seed bearing morphological structures, the oat panicles and oilseed rape pods, have formed) as well as for non-interlocking single plants (i.e. before the seed bearing structures have formed) with changed values of a number of parameters to reflect the fact that the dynamics of the whole canopy need to be considered, rather than just individual plants. The developmental stage when the crops become an interlocked canopy is defined here as panicle fully emerged growth stage (GS) 59 (Tottman, 1987) for oats, and all pods developed GS5,9 (Sylvester-Bradley and Makepeace, 1984) for oilseed rape. See figure 1b for an example of non-interlocking and interlocked plants.

- The earlier method used a step input of wind speed to represent the wind gust. However the current model attempts to model the stochastic nature of the wind through a consideration of the wind spectrum, in a way that is similar to that used by those designing dynamic structures (Maguire and Wyatt, 1999). At this stage the model makes no allowance for deterministic honami waves as they pass above the canopy (Finnigan, 2000). These waves are coherent wave like movements of the crop and the air above the canopy, which seem to be more regular than normal atmospheric gusts, and produce spatially and temporally coherent oscillating flows over the top of the canopy. However an indication as to how the effects of such waves might be incorporated into the model, when more experimental information is available, is given in section 4.

- The modelling allows explicit equations to be derived for the mean and unsteady wind induced moments along the length of plant stems, rather than just at the base of the stem as was the case in the earlier model.

- Stem lodging is calculated, as in previous models, by comparing the wind induced bending moment of a stem or plant with the stem resisting moment. Oat plants are usually composed of several stems (shoots), each of which originate at ground level and can stem lodge independently. Oilseed rape plants comprise a single main stem with additional stems which usually originate part way up the main stem. Root lodging follows the same methodology as in the earlier model and is assumed to occur on a per plant basis. However different models of the root strength are used for different rooting morphologies.

- The wheat lodging model used measured time histories of wind speed and rainfall data from a specific site. The current paper describes a different approach based on probability distributions of meteorological parameters. This method has the potential to allow for the effects of climate variability to be investigated, since current methodologies for predicting future climate are based on probability distributions of climate variables (UKCIP 2009).

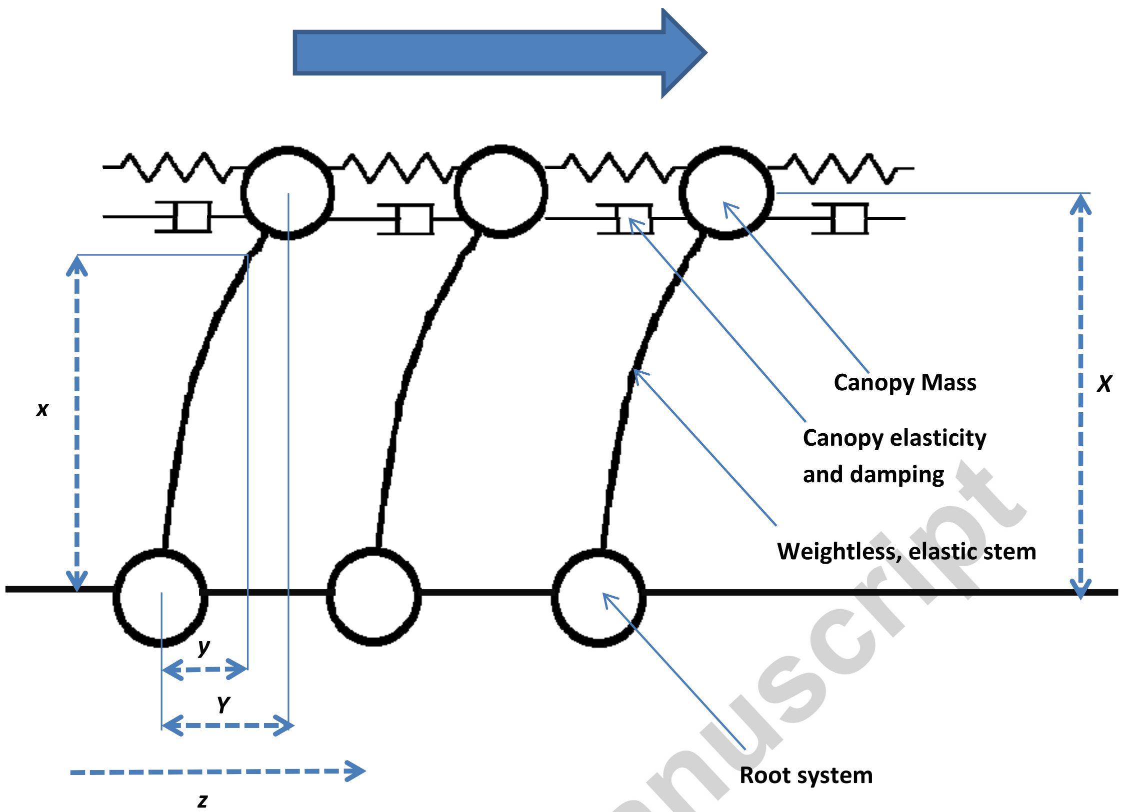


Figure 2 Co-ordinate system

2.2 Forces and moments in the crop

For an isolated plant or a plant in an interlocking crop canopy the basic bending moment equation is given by

$$npEI_A\frac{d^2y}{dx^2} = \mu g(Y - y) + \left(F - K\frac{dy}{dz} - C\frac{dy}{dT}\right)(X - x) - \mu\frac{d^2Y}{dT^2}(X - x) \quad (1)$$

(Figure 2). Here x is distance up the stem from the ground, y is the displacement of the stem at height x, z is the distance in the y direction from a fixed point at the edge of the crop, and T is time. Y is the displacement of the top of the stem, and X is the height of the centre of mass of the crop canopy. n is the number of stems per plant, p is the number of plants per unit area, E is the Young's modulus of the stem, I_A is the second moment of area of the stem, μ is the mass of the unit area of the canopy, and F is the wind induced force on a plant. K and C are the damping coefficients per plant due to canopy interactions and stem material properties respectively. For an isolated plant i.e. not in an interlocking canopy, K=0. The term on the left hand side is the bending moment at a distance x from the ground (the flexural rigidity multiplied by the second derivative of displacement); the first term on the right hand side is the moment at a point in the stem caused by

the displacement of the mass of the crop canopy; the second term on the right hand side represents the moment due to the wind force and the damping of the motion due to canopy interactions and stem material properties; the third term on the right hand side is the moment caused by the inertia of the canopy. By taking *x = y = 0* (i.e. considering crop base conditions only for an isolated plant at an idealised location), equation (1) reduces to the basic equation for the wheat lodging model given in Baker (1995). Essentially this methodology assumes that for the non-interlocking canopy the wind loads plants individually, through penetration of gusts into the upper part of the canopy which load the upper area of each plant. For an interlocking canopy, the wind loads the canopy as a whole through the application of a shear stress on the top of the canopy.

The solution to this equation is somewhat lengthy and requires a number of simplifying assumptions to be made. The methodology is outlined in the Appendix, and we simply present the results of the analysis here for parameters of most importance – the displacements along and at the top of the stem and the bending moment along the stem.

$$\tilde{y} = \frac{\tilde{F}}{\left(\left(\omega_n^2\left(\frac{X}{g}\right)-\omega^2\left(\frac{X}{g}\right)\right)^2+\left(\frac{X}{g}\right)(\theta\omega)^2\right)^{0.5}}\left(\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)\left(\cos\left(\alpha\frac{x}{X}\right)-\cot\left(\alpha\right)\sin\left(\alpha\frac{x}{X}\right)\right)+\omega^2\left(\frac{X}{g}\right)\left(1-\frac{x}{X}\right)+1\right) \quad (2)$$

$$\tilde{Y} = \frac{\tilde{F}}{\left(\left(\omega_n^2\left(\frac{X}{g}\right)-\omega^2\left(\frac{X}{g}\right)\right)^2+\left(\frac{X}{g}\right)(\theta\omega)^2\right)^{0.5}} \quad (3)$$

$$M = -nEI_A\frac{d^2(y)}{d(x)^2} \quad (4)$$

$$\frac{M\prime}{XF\prime} = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\left(\left(\omega_n^2\left(\frac{X}{g}\right)-\omega^2\left(\frac{X}{g}\right)\right)^2+\left(\frac{X}{g}\right)(\theta\omega)^2\right)^{0.5}}\left(\cos\left(\alpha\frac{x}{X}\right)-\cot\alpha\sin\left(\alpha\frac{x}{X}\right)\right) \quad (5)$$

The dimensionless parameter α is

$$\alpha^2 = \left(\frac{\mu g X^2}{npEI}\right) \quad (6)$$

α is related to the natural frequency by the following expressions

$$\omega_n^2\left(\frac{X}{g}\right) = \frac{\alpha}{1-\alpha\cot\left(\alpha\right)} \quad (7)$$

and can thus be obtained by measurements of the natural frequency of the crop or canopy, and does not need to be derived from its component variables which are, in the main, difficult to measure. The damping is given by a generalised damping coefficient θ, that is an unspecified function of *K* and *C* in equation (1) can be obtained from measurements of crop or canopy oscillations, and the component parameters do not need to be specified.

Note however that the wind force is a combination of a mean and a fluctuating force. The mean moment can be obtained by simply letting the frequency of the fluctuation ω approach zero.

$$\frac{\bar{M}}{X\bar{F}} = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\omega_n^2\left(\frac{X}{g}\right)}\left(\cos\left(\alpha\frac{x}{l}\right) - \cot\alpha\sin\left(\alpha\frac{x}{l}\right)\right) \tag{8}$$

Now consider the wind force per plant. The mean component is given by the normal drag coefficient formulation.

$$\bar{F} = 0.5\rho A_{CF}\bar{U}^2 \tag{9}$$

where ρ is the density of air, A_{CF} is the plant drag area for an isolated plant or the plant shear area for a plant in a canopy, i.e. the multiple of an area and a drag / shear force in conventional terminology and $\bar{U}$ is the mean velocity. The amplitude of the fluctuating component is given by

$$F' = \rho A_{CF}\,\bar{U}U' \tag{10}$$

where U' is the fluctuating velocity, which is assumed to be much smaller than the mean velocity.

2.3 Bending moment calculation

From the above equations one can derive the following expressions for the mean and the amplitude of the fluctuating bending moments along the stem.

$$\bar{M} = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\omega_n^2\left(\frac{X}{g}\right)}(0.5\rho A_{CF}X)\left(\cos\left(\alpha\frac{x}{l}\right) - \cot\alpha\sin\left(\alpha\frac{x}{l}\right)\right)\bar{U}^2 \tag{11}$$

$$M' = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\left(\left(\omega_n^2\left(\frac{X}{g}\right)-\omega^2\left(\frac{X}{g}\right)\right)^2+\left(\frac{X}{g}\right)(\theta\omega)^2\right)^{0.5}}(\rho A_{CF}X)\left(\cos\left(\alpha\frac{x}{l}\right) - \cot\alpha\sin\left(\alpha\frac{x}{l}\right)\right)\bar{U}U' \tag{12}$$

By forming the autocorrelation of the fluctuating moments and velocities in equation (12) and integrating, the moment spectrum can be related to the wind spectrum as follows.

$$S_M = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)^2}{\left(\left(\omega_n^2\left(\frac{X}{g}\right)-\omega^2\left(\frac{X}{g}\right)\right)^2+\left(\frac{X}{g}\right)(\theta\omega)^2\right)}(\rho A_{CF}X\bar{U})^2\left(\cos\left(\alpha\frac{x}{l}\right) - \cot\alpha\sin\left(\alpha\frac{x}{l}\right)\right)^2 S_U \tag{13}$$

The right hand side of equation (13) can be taken as having two components – a broad banded component where the fluctuating moment simply follows the fluctuating wind (with an implied aerodynamic admittance of unity), and a resonant component. The broad banded variance is then given by the integration of equation (13) without consideration of the resonant component.

$$\sigma_{MB}{}^2 = \frac{\left((1+\omega_n^2\left(\frac{X}{g}\right)\right)^2}{\omega_n^2\left(\frac{X}{g}\right)^2}(\rho A_{CF} X\bar{U})^2(\cos(\alpha\bar{x}) - \cot\alpha \sin(\alpha\bar{x}))^2 \sigma_U{}^2 = 4\bar{M}^2 I^2 \quad (14)$$

The resonant component can be approximated by the following formulation (Maguire and Wyatt, 1999)

$$\sigma_{MR}{}^2 = \bar{M}^2 I^2 \left(\frac{\pi}{\theta}\right)\left(\frac{n_n S(n_n)}{\sigma_U{}^2}\right) \quad (15)$$

The term in the last brackets is the normalised wind spectral density at the natural frequency of the crop. Now Finnigan (2000) shows that for typical crop canopies, this frequency corresponds with a peak plateau in the normalised spectral density with a numerical value of 0.25. Thus

$$\sigma_{MR}{}^2 = \bar{M}^2 I^2 \left(\frac{\pi}{4\theta}\right) \quad (16)$$

Having found the mean and variance of the fluctuating moment, there is now a need to evaluate the appropriate peak moments of relevance to the lodging problem. It is generally assumed that this is of the form

$$\hat{M} = \bar{M} + ((g_{MB}\sigma_{MB})^2 + (g_{MR}\sigma_{MR})^2)^{0.5} \quad (17)$$

where g_{MB} and g_{MR} are the broad banded and resonant gust factors. The former is a function of averaging time and is effectively the same as the velocity gust factor. It is given by

$$g_{MB} = 0.42 I \ln\left(\frac{3600}{\tau}\right) \quad (18)$$

Cook (1985) where *I* is the turbulence intensity and τ is the averaging time. The resonant gust factor is given by

$$g_{MR} = (2\ln(3600 f_n))^{0.5} + \frac{0.577}{(2\ln(3600 f_n))^{0.5}} \quad (19)$$

Here $f_n = \omega_n/2\pi$ is the natural frequency of crop oscillation. For the range of plant natural frequencies that are of relevance (0.5 to 2Hz) equation (19) gives $g_{MR} = 4.15$ to a good approximation. Now from observations it is known that root lodging takes place over a period of a minute or more. We thus assume in what follows that the resonant component of the fluctuation is of no relevance here, and we take the appropriate value of g_{MB} to be defined for τ = 60 seconds, which from equation (18) is 1.72. Thus, from equations (14) and (17), the peak moment of relevance to root lodging is given by

$$\hat{M} = \bar{M}(1 + 3.44I) \quad (20)$$

Now stem lodging takes place over a much shorter period of time – over one oscillation of the stem, so the full form of equation (17) is appropriate here, with τ taken as one second. Thus from equations (14), (17) and (19), $g_{MB} = 3.43$. This gives

$$\hat{M} = \bar{M}\left(1 + 6.86I\left(1 + 0.0.087\left(\frac{\pi}{4\theta}\right)\right)^{0.5}\right) \tag{21}$$

2.4 Stem lodging criteria

Remembering that the moment is defined as the wind induced moment per plant, the criterion for stem lodging is taken as being when the maximum moment at a point along the stem exceeds the stem bending strength at that point.

$$\hat{M} > (\sigma\pi a^3/4)\left(1 - ((a-t)/a)^4\right)n \tag{22}$$

where σ is the stem yield stress, a is the stem radius and t is the stem wall thickness (all of which vary along the length of the stem) i.e. the fluctuating moment per plant at some point along the stem is greater than the stem resisting moment. The latter is given by classical structural theory that relates the stem breaking stress to the stem radius and wall thickness. From equations (11) and (21) this leads to the following expression, for the occurrence of stem lodging.

$$\frac{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\omega_n^2\left(\frac{X}{g}\right)}\frac{(0.5\rho U^2 A_{CF}X)(\cos(\alpha x/l) - \cot\alpha \sin(\alpha x/l))\left(1+6.86I\left(1+0.0.087\left(\frac{\pi}{4\theta}\right)\right)^{0.5}\right)}{(\sigma\pi a^3/4)\left(1-((a-t)/a)^4\right)n} > 1 \tag{23}$$

We thus define the mean wind velocity for stem lodging as

$$\bar{U}_{LS} = \left(\frac{\omega_n^2\left(\frac{X}{g}\right)n(\sigma\pi a^3/4)\left(1-((a-t)/a)^4\right)}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF}X)(\cos(\alpha x/l) - \cot\alpha \sin(\alpha x/l))\left(1+6.86I\left(1+0.366\left(\frac{\pi}{4\theta}\right)\right)^{0.5}\right)}\right)^{0.5} \tag{24}$$

$\bar{U}_{LS}$ takes on different values along the stem. It can be defined at the base, with $x/l = 0$, with the value of the stem strength at the stem base, or at a value of x/l consistent with the height of any point along the stem, with the value of stem strength being immediately above the point of interest.

2.5 Root lodging criteria

For root lodging we assume the value of peak moment given by equation (20). This leads to the expression.

$$\bar{M}(1 + 3.44I) > \gamma s d^3 \tag{25}$$

Here d should be interpreted as the effective root ball diameter, which will be the actual root ball diameter for a root ball system, but should be taken as $(d^2L)^{1/3}$ for a tap root system, where

L is the length of the rigid part of the tap root that resists overturning. This latter formulation allows for the increased strength in bending of a tap root system, but keeps the essential form of the earlier relationship. s is the soil shear strength and γ is a constant, which will differ for different types of plant and root system. This leads to

$$\frac{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\omega_n^2\left(\frac{X}{g}\right)}\frac{(0.5\rho\bar{U}^2 A_{CF}X)(1+3.44I)}{\gamma d^3} > s \tag{26}$$

s is in turn given by the expressions (Baker 1995, Baker et al 1998)

$$s = s_D - \frac{i}{\left(\frac{\rho_s}{\rho_w}\right)(f-w)L}(s_D - s_w) \tag{27}$$

$$s_w = 1.484\ x\ 10^6 e^{-5f/c}(2.2 - 0.24v)(4.82c - 0.3) \tag{28}$$

$$s_D = 1.125\ x\ 10^6 e^{-5w/c}(2.2 - 0.24v)(4.82c - 0.3) \tag{29}$$

where ρ_s is the density of the soil, ρ_w is the density of water, f is the moisture content at field capacity, w is the moisture content at the permanent wilting point, v is the empirical visual score, and c is the clay content, L is the rooting depth and i is the daily rainfall.

The daily rainfall is chosen as a surrogate to represent the long term rainfall history of the site in question. s_w is thus the soil shear strength at field capacity and s_D is the soil shear strength at the permanent wilting point. s is taken to have a minimum value of s_w. Equation (27) effectively implies that the soil is at field capacity when the rainfall is such as to saturate the ground down to the rooting depth. The above equations lead to the following expression for the rainfall at which lodging will occur.

$$i > \left(1 - \frac{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)}{\omega_n^2\left(\frac{X}{g}\right)}\frac{(0.5\rho\bar{U}^2 A_{CF}X)}{\gamma s_D d^3}\right)\frac{\left(\frac{\rho_s}{\rho_w}\right)(f-w)l}{\left(1-1.32e^{-5\left(\frac{f-w}{c}\right)}\right)} \tag{30}$$

We define a reference rainfall intensity, i_o, at zero wind speed and a velocity at which root lodging occurs as

$$i_o = \frac{\left(\frac{\rho_s}{\rho_w}\right)(f-w)l}{\left(1-1.32e^{-5\left(\frac{f-w}{c}\right)}\right)} \tag{31}$$

$$\bar{U}_{LR} = \left(\frac{\omega_n^2\left(\frac{X}{g}\right)\gamma s_D d^3}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF}X)(1+3.44I)}\right)^{0.5} \tag{32}$$

This leads to the simple expression for the daily rainfall at which lodging occurs as a function of mean wind speed.

$$i > \left(1 - \frac{U^2}{\bar{U}_{LR}^2}\right) i_0 \tag{33}$$

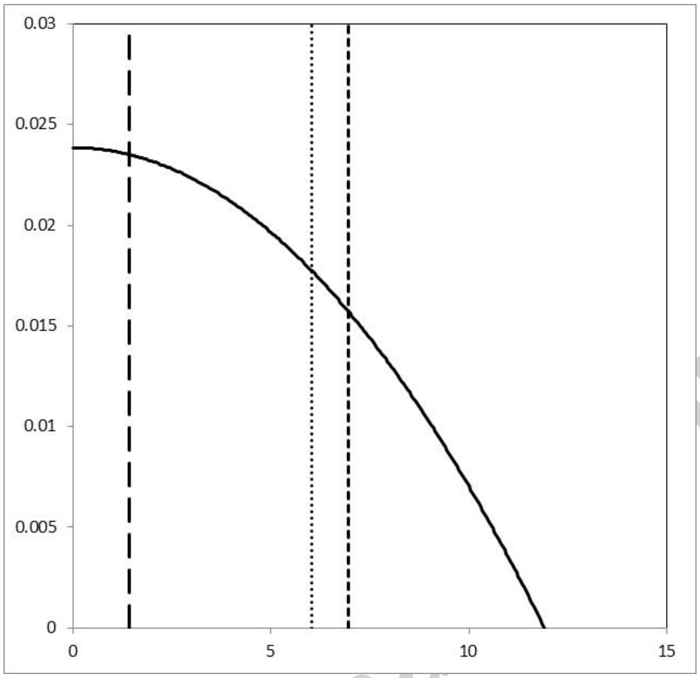


Figure 3 Lodging areas in the mean velocity / daily rainfall plane

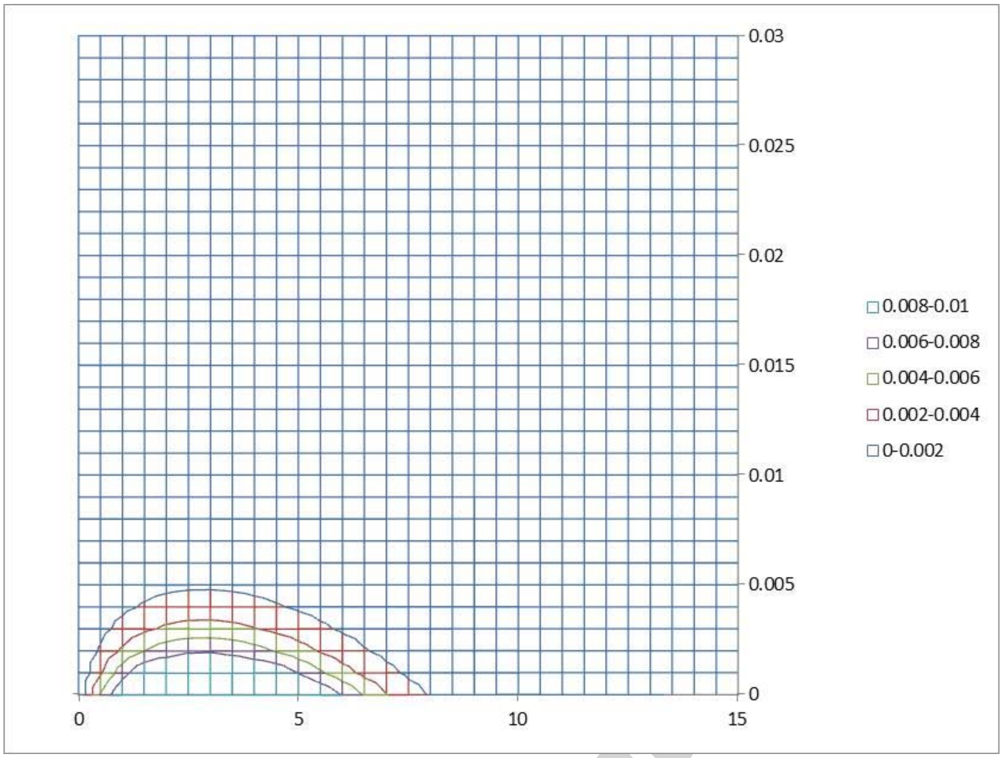


Figure 4 Mean wind / daily rainfall PDF

3. Method of solution

The above equations appear quite complex, but in reality permit a very simple and explicit method of solution. We begin by considering regions of non-lodging and lodging in the $\overline{U}$ versus i plane. This is shown schematically in figure 3. This curve was calculated using a standard set of crop parameters representative of a non-interlocking oat crop – see section 4 below – but are illustrative of a wide range of plant characteristics. Lodging and non-lodging regions are identified. At the extreme left of the figure, for velocities less than $\overline{U}_S$ we have a region where lodging does not occur for any rainfall, and at rainfalls higher than i_S the ground is at field capacity, where $s = s_w$. From equations (27) and (31) above i_S is given by

$$i_s = \left(\frac{\rho_s}{\rho_w}\right)(f - w)L = \left(1 - 1.32e^{-5\left(\frac{f-w}{c}\right)}\right) i_0 \quad (34)$$

The corresponding velocity U_S is given by equation (33), which gives the lower boundary of the root lodging region.

$$\frac{\overline{U}_S}{\overline{U}_{LR}} = \left(1 - \frac{i_s}{i_o}\right)^{0.5} \quad (35)$$

As the wind velocity increases, we have a non-lodging region in the wind velocity / rainfall plane bounded by equation (33) i.e.

$$i = \left(1 - \frac{\overline{U}^2}{\overline{U}_{LR}^2}\right) i_0 \quad (36)$$

This is a simple parabolic relationship. To the left of and beneath this line, no lodging occurs, whilst to the right and above there is a region of root lodging. As the velocity increases further there are two possibilities, either that the rainfall / velocity curve given above falls to zero at the point $\overline{U} = \overline{U}_{LR}$ or that stem lodging occurs at a velocity of $\overline{U}_{LS} < \overline{U}_{LR}$. This is the case shown in figure 3. $\overline{U}_{LS}$ can have different values at different points along the stem, and the figure shows values for stem lodging at the base and at the first node. Only the lowest velocity is of course relevant. Thus the various regions of lodging and non-lodging in the $\overline{U}$ versus *i* plane can be clearly appreciated, using the simple closed form expressions given above.

The next step is to calculate a lodging risk associated with the above lodging characteristic. This requires knowledge of site meteorological conditions, and in particular the probability distribution functions of the mean velocity at crop height $\overline{U}$ and the daily rainfall *i*. As mentioned above we use the latter parameter as a surrogate for the antecedent rainfall that will largely determine the soil moisture conditions. Now we will assume that the probability density function (pdf) for $\overline{U}$ is a Rayleigh distribution of the form

$$p(\overline{U}) = \left(\frac{2}{\lambda}\right)\left(\frac{\overline{U}}{\lambda}\right)e^{-\frac{\overline{U}^2}{\lambda}} \tag{37}$$

where λ characterises the wind climate. Note that this parameter is defined at the top of the crop. The more complex Weibull distribution would have more general validity, but it will be seen below that the Rayleigh distribution allows an explicit formula for risk to be derived, and it is a known fact that for many parts of the UK, the Rayleigh distribution is an adequate description of the wind climate (Cook 1985). The pdf of the daily rainfall is taken to be exponential in form

$$p(\,i) = \left(\frac{1}{m}\right)e^{-\left(\frac{i}{m}\right)} \tag{38}$$

where *m* is the mean daily rainfall. Again, this form is chosen as it allows an explicit risk formula to be derived. Note however the methodology described in this paper can be used with other wind or rainfall probability distributions, although the risk cannot then be expressed explicitly and numerical calculation is required. Also inspection of various meteorological data series shows that there is some interaction between the wind speed and the rainfall pdf's, with higher than expected rainfall occurring in high winds. However this interaction is well to the upper right in the wind speed / rainfall plane, i.e. well into the lodging region, and will be neglected in what follows. The total pdf is thus given by

$$p(U,i) = \left(\frac{1}{m}\right)e^{-\left(\frac{i}{m}\right)}\left(\frac{2}{\lambda}\right)\left(\frac{\overline{U}}{\lambda}\right)e^{-\left(\frac{\overline{U}}{\lambda}\right)^2} \tag{39}$$

Figure 4 shows this pdf for the conditions that will be considered in section 4. To get the total risk we thus integrate this expression over the region of the mean wind speed / rainfall plane where lodging occurs

$$R = \int_0^\infty \left(\frac{2}{\lambda}\right)\left(\frac{\bar{U}}{\lambda}\right) e^{-\left(\frac{\bar{U}}{\lambda}\right)^2} \left(\int_{i(\bar{U})}^\infty \left(\frac{1}{m}\right) e^{-\left(\frac{i}{m}\right)} di\right) d\bar{U} \tag{40}$$

Whilst somewhat complex the above equation can be integrated in a relatively straightforward way. For the case shown in figure 3 where both stem and root lodging occur.

$$R = e^{-\tilde{U}_{LS}^2} + e^{-\left(\frac{1}{\tilde{m}}\right)} \left(\frac{\tilde{U}_{LR}^2 \tilde{m}}{\tilde{U}_{LR}^2 \tilde{m} - 1}\right) \left(e^{-(\tilde{U}_S)^2 \left(\frac{\tilde{U}_{LR}^2 \tilde{m} - 1}{\tilde{U}_{LR}^2 \tilde{m}}\right)} - e^{-\tilde{U}_{LS}^2 \left(\frac{\tilde{U}_{LR}^2 \tilde{m} - 1}{\tilde{U}_{LR}^2 \tilde{m}}\right)} \right) \tag{41}$$

where we define the following dimensionless parameters

$$\tilde{m} = \frac{\mathrm{m}}{i_0} \tag{42}$$

$$\tilde{U}_S = \frac{\bar{U}_S}{\lambda} \tag{43}$$

$$\tilde{U}_{LS} = \frac{\bar{U}_{LS}}{\lambda} \tag{44}$$

$$\tilde{U}_{LR} = \frac{\bar{U}_{LR}}{\lambda} \tag{45}$$

For the case where $\bar{U}_{LS} > \bar{U}_{LR}$ i.e. where only root lodging occurs, the integral becomes

$$R = e^{-\tilde{U}_{LR}^2} + e^{-\left(\frac{1}{\tilde{m}}\right)} \left(\frac{\tilde{U}_{LR}^2 \tilde{m}}{\tilde{U}_{LR}^2 \tilde{m} - 1}\right) \left(e^{-(\tilde{U}_S)^2 \left(\frac{\tilde{U}_{LR}^2 \tilde{m} - 1}{\tilde{U}_{LR}^2 \tilde{m}}\right)} - e^{-\tilde{U}_{LR}^2 \left(\frac{\tilde{U}_{LR}^2 \tilde{m} - 1}{\tilde{U}_{LR}^2 \tilde{m}}\right)} \right) \tag{46}$$

Now in the above expression, λ which characterises the wind climate, must be defined at crop height. It is more usually defined at 10m above the ground, the standard meteorological height. The value at this height λ_{10} can be corrected to crop height via the logarithmic law.

$$\frac{\lambda}{\lambda_{10}} = \frac{\log\left(\frac{10 - 0.8X}{z_0}\right)}{\log\left(\frac{2X - 0.8X}{z_0}\right)} \tag{47}$$

where z_0 is the surface roughness length. This effectively assumes that the displacement height, the effective height of the ground surface "seen" by the wind is 0.8*X*, and that we define our canopy top velocity at *2X* canopy heights above the ground. This assumption is not always completely true, but is again adequate for current illustrative purposes (Dong et al, 2001). The surface roughness length is related to the turbulence intensity by the equation

$$I = \frac{1}{\log\left(\frac{2X - 0.8X}{z_0}\right)} \tag{48}$$

4. Investigations using the lodging model

4.1 Review of the model

As set out above, the model attempts to realistically represent the lodging process, capturing the main physical variables and processes. As noted in section 2 it is similar in some ways to that used in the past by the authors, but in particular moves away from the rather artificial sharp edged gust simulation used in earlier work to a more realistic wind simulation such as that used in the analysis of the dynamics of buildings and other structures. In doing so it defines four fundamental physical

variables that effectively determine the risk of lodging. These are summarised as follows, with the variables split into plant, soil and weather groupings.

$$\widetilde{U}_S = \left(\frac{\omega_n^2 \left(\frac{X}{g}\right) d^3}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF} X)} \gamma s_W \frac{1}{(1+3.44I)\lambda^2} \right)^{0.5} = \widetilde{U}_{LR} \frac{s_W}{s_D} \tag{49}$$

$$\widetilde{U}_{LR} = \left(\frac{\omega_n^2 \left(\frac{X}{g}\right) d^3}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF} X)} \gamma s_D \frac{1}{(1+3.44I)\lambda^2} \right)^{0.5} = \widetilde{U}_S \frac{s_D}{s_W} \tag{50}$$

$$\widetilde{U}_{LS} = \left(\frac{\omega_n^2 \left(\frac{X}{g}\right) n(\sigma\pi a^3/4)\left(1-((a-t)/a)^4\right)}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF} X)(\cos(\alpha x/l) - \cot\alpha \sin(\alpha x/l))} \frac{1}{\left(1+6.86I\left(1+0.366\left(\frac{\pi}{4\theta}\right)\right)^{0.5}\right)\lambda^2} \right)^{0.5} \tag{51}$$

$$\widetilde{m} = \frac{1}{L} \frac{\left(1-1.32e^{-5\left(\frac{f-w}{c}\right)}\right)}{\left(\frac{\rho_S}{\rho_W}\right)(f-w)} m = \frac{1}{L} \frac{\left(1-\frac{s_W}{s_D}\right)}{\left(\frac{\rho_S}{\rho_W}\right)(f-w)} m \tag{52}$$

The first two variables effectively define the windspeed / rainfall curve for root lodging and are functions of plant characteristics, soil characteristics and wind characteristics. They are related to each other through the ratio of the dry and wet soil strengths. The third parameter defines the stem lodging behaviour, and is a function of plant and wind characteristics. There is some interaction between these two sets of parameters however, as the plant damping occurs with the wind characteristics in the definition. The fourth parameter defines the effects of rainfall on root lodging, and is a function of plant, soil and rainfall characteristics. It is effectively a non-dimensionalisation with the rainfall required for saturation (the denominator) with a correction for soil strength ratios. These parameters are of use of themselves in giving an indication of the relative importance of different plant, soil and weather parameters in the lodging process.

Now in Berry *et al.* (2003) the authors defined a root lodging and a stem lodging velocity, based on the description of the wind field as a mean value with a sharp edged sudden gust. In the terms of this paper they have the following forms.

$$V_{gR} = \left(\frac{\omega_n^2 \left(\frac{X}{g}\right) d^3}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)(0.5\rho A_{CF} X)} \gamma s \frac{1}{\left(1+\left(\frac{4}{\pi}\right)\sin\left(\frac{\pi}{\pi 4}\right)e^{-\pi\vartheta}\right)} \right)^{0.5} \tag{53}$$

$$V_{gS} = \left(\frac{\omega_n^2 \left(\frac{X}{g}\right) n(\sigma\pi a^3/4)\left(1-((a-t)/a)^4\right)}{\left(1+\omega_n^2\left(\frac{X}{g}\right)\right)} \frac{1}{\left(1+\left(\frac{4}{\pi}\right)\sin\left(\frac{\pi}{\pi 4}\right)e^{-\pi\vartheta}\right)} \right)^{0.5} \tag{54}$$

Clearly, allowing for the fact that λ, which specifies the wind speed characteristics, is not included due to their dimensional form, these equations are similar in form to equations (50) and (51) but there are significant differences. Firstly the wind / damping term is the same in the root and stem lodging equations, unlike the current methodology, where different averaging times are adopted for

each process. Secondly the root lodging equation contains the soil strength s, rather than the dry soil strength s_D, and is thus effectively a function of rainfall. Finally equation (54) does not allow for the prediction of lodging other than at the stem base.

It will be apparent from the previous sections that the methodology that has been developed provides an easily appreciated graphical way in which the lodging problem can be appreciated, and, in the derivation of simple formulae for risk, offer a practical way in which the risk of a particular crop lodging can be calculated. However it should be emphasised that there are a significant number of assumptions within the method that are unproven and need to be established experimentally. The most important such assumptions are as follows.

- The nature of the wind field itself. As has been pointed out above, at high wind levels wave like interactions between the flow field and crop movements occur. These have not been explicitly accounted for because of lack of experimental information. The subject will be addressed further below.
- The tap root model described briefly above, which defines and effective root ball diameter is completely unproven.
- There is a degree of arbitrariness in the model in a number of ways. Firstly the wind speed at "canopy top", is defined as that at two canopy heights on the basis that it is the wind conditions at this height that are swept down into the crop. The assumption of this height leads to the definition of a height reduction factor for the meteorological standard 10m velocity, which in turn affects the plant forces etc. More experimental information is required here to specify this more adequately. Secondly, and related to the first, the drag force area / shear force area per plant is not well specified for most crops, and has a major effect on the plant forces. Thirdly the averaging times used to define the moments for stem and root lodging are at best an educated guess. Finally, the root strength parameter has not been measured for tap roots. Taken together these uncertainties can be used in effect to "fix" the lodging risk for a particular situation. Perhaps a more positive way of interpreting this would be to regard them as a means by which the model can be calibrated against reality, as was done for winter wheat in Berry *et al.* (2003).

These reservations being made, the simplicity of the final results has much to recommend it. In particular the stem and root lodging parameters may prove to be useful in ranking the lodging resistance of different crop types, and both can be calculated from data that is readily available.

4.2 Risk calculations

Risk calculations have been carried out for three distinct crop types – winter wheat, oats and oilseed rape. The assumed parameters, and the assumed uncertainties in these parameters, are given in table 1. Essentially the wheat characteristics are taken from those used by Berry *et al.* (2003), the oat characteristics from the recent Defra Sustainable Arable LINK project LK09124 'QUOATS - Harnessing new technologies for sustainable oat production and utilisation' and the oilseed rape characteristics form a series of projects described in Berry *et al.* (2013). At a number of points there are "author estimates" of certain parameters. The point made above concerning the arbitrariness of some of the modelling assumptions must be stressed, and the absolute values of the risk not relied on. The most significant assumption is that for the root strength parameter for the tap root model for oilseed rape is taken as having a value of 4.0, ten times the experimentally measured value for wheat. In each case a 100 realisations of the risk have been made, with the input parameters assumed to have rectangular probability distributions with the mean ± limit range. It can be seen that for the wheat crop both root and stem lodging is predicted, with $\widetilde{U}_{LS} < \widetilde{U}_{LR}$. The same is true for the oats crop, but this time two values of $\widetilde{U}_{LS}$ are given, with the values for internode 2 being less than that at the internode 1 (base internode) and thus more critical. For the oilseed rape crop only root lodging is predicted with $\widetilde{U}_{LR} < \widetilde{U}_{LS}$. The uncertainties in the risk calculations can be seen to be relatively large (of the order of ±0.1).

One might expect the lodging risk to change through the season, and in particular over the period of crop interlocking. Whilst there is as yet no experimental data available to be able to study this in detail, a preliminary calculation can be made. For the oats crop the average value of risk was 0.22. Going from non-interlocking to interlocking crops one might expect the natural frequency to decrease, and the damping to increase. It was thus assumed that the natural frequency would fall from 1.4Hz to 0.7Hz, and the damping would increase from 0.05 to 0.5. A decrease in the natural frequency alone increases the risk to 0.27, whilst the increase in the damping decreases the risk to 0.19. When considered together the risk increases slightly to 0.24 i.e. the changes are well within the levels of uncertainty outlined above.

	Wheat		Oats		Oilseed rape	
	Assumed	Limits[9]	Assumed	Limits[9]	Assumed	Limits[9]
Growth stage	65 Mid flowering[1]	-	71-83[1] Grain filling	-	4,5 mid flowering[2]	-
Canopy height *X* (m)	0.43[3]	0.05	0.95[4]	0.15	1.52[7]	0.2
Canopy type	Isolated	-	Isolated	-	Isolated	-
Root type	Ball	-	Ball	-	Tap	-
Root constant γ	0.4[3]	0.1	0.4[5]	0.1	4.0[8]	1.0
Plant / canopy natural frequency f_n (Hz)	0.95[3]	0.2	1.4[5]	0.2	0.50[8]	0.1
Plant / canopy damping ratio θ	0.10[3]	0.02	0.10[5]	0.02	0.10[8]	0.02
Plant drag / shear coeff. area (m^2)	0.025[3]	0.003	0.01[5]	0.003	0.037[8]	0.003
Stem diameter *a* – internode 1 (mm)	3.25[3]	0.5	6[4]	1	11.1[8]	2
Stem diameter *a* – internode 2 (mm)	-	-	7[4]	1	-	-
Stem wall thickness *t* – internode 1 (mm)	0.64[3]	0.15	1[4]	0.2	2.2[8]	0.5
Stem wall thickness *t* – internode 2 (mm)	-	-	1[4]	0.2	-	-
Stem yield strength σ – internode 1 (MPa)	30[3]	5	30[4]	5	25[8]	5
Stem yield trength σ – internode 2 (MPa)	-	-	25[4]	5	-	-
Root plate diameter *d* (mm)	38[3]	10	50[4]	10	16.5[8]	3.0
Root plate depth *l* (mm)	40[3]	10	70[4]	10	123[8]	15
Number of stems/plant *n*	3.2[3]	0.5	2.5[4]	0.5	8.2[8]	1
Visual score *v*	5[3]	1	5[3]	1	5[3]	1
Clay content *c*	0.25[3]	0.03	0.25[3]	0.03	0.25[3]	0.03
Water content at wilting point *w*	0.27[3]	0.03	0.27[3]	0.03	0.27[3]	0.03
Water content at field capacity *f*	0.15[3]	0.03	0.15[3]	0.03	0.15[3]	0.03
λ_{10} (m/s)	4.0[6]	0.5	4.0[6]	0.5	4.0[6]	0.5
Mean daily rainfall *m* (mm)	2.0[6]	0.5	2.0[6]	0.5	2.0[6]	0.5
Turbulence intensity *I*	0.5[7]	0.1	0.5[7]	0.1	0.5[67]	0.1
$\overline{U}_S$ m/s	0.73		1.42		0.29	
$\overline{U}_{LS}$ m/s internode 1	2.20		6.02		8.83	
$\overline{U}_{LS}$ m/s internode 2	-		6.95		-	
$\overline{U}_{LR}$ m/s	6.12		11.91		2.43	
Risk	0.58		0.22		0.55	
Risk – 2 standard deviations	0.52		0.16		0.44	
Risk + 2 standard deviations	0.63		0.28		0.67	

[1] Tottman (1987), [2] Sylvester-Bradley and Makepeace (1984), [3] from Berry et al (2003) [4] Values from QUOATS trial for Balado –only info for internodes 1 and 2 – for other internodes, very similar to internode, [5] From UOB wind tunnel tests, [6] From MO data for Watnall http://www.metoffice.gov.uk/climate/uk/mi/ [7] from Finnigan (2000), [8] from Berry et al (2013), [9] Author estimate,

Table 1 Assumed and calculated parameters

4.3 Parametric analysis

In this section we consider a parametric analysis of the risk of lodging using equations (41) and (46). The analysis is carried out using the dimensionless variables of equations (42) to (45) and is thus non-crop specific. Figure 5 shows the variation of the risk of root lodging with $\widetilde{U}_{LR}$ for different values of $\widetilde{m}$, as these two parameters are the only ones of importance for root lodging. It can be seen that there is a smooth reduction in risk as $\widetilde{U}_{LR}$ increases, with the effect of $\widetilde{m}$ being small, and well within the range of uncertainties discussed in the last section. For small $\widetilde{U}_{LR}$ the risk is constant, reflecting the fact that there is a range of low wind speeds for which root lodging cannot occur. In effect the root lodging risk is largely given by the first term in equation (46), which represents the proportion of risk for dimensionless velocities greater than $\widetilde{U}_{LR}$ – see figure 3. This is because the rainfall pdf fall off very rapidly with rainfall intensity, and most of the risk is concentrated in regions of low rainfall intensity. Figure 6 shows the case where both stem and root lodging occur and equation (41) applies. Figure 6a shows the variation of risk with $\widetilde{U}_{LS}$ for different values of $\widetilde{U}_{LR}$ and $\widetilde{m}$ = 0.1, and figure 6b shows the variation of risk with $\widetilde{U}_{LS}$ for different values of $\widetilde{m}$ for $\widetilde{U}_{LR}$ = 1.0. In general the same point applies as above in that the risk is largely determined by the first term in equation (41), although there is a levelling off of risk at low values of $\widetilde{U}_{LS}$ for the reasons described above and at high values of $\widetilde{U}_{LS}$ where $\widetilde{U}_{LS} > \widetilde{U}_{LR}$ and the value of risk is fixed by the value of $\widetilde{U}_{LR}$.

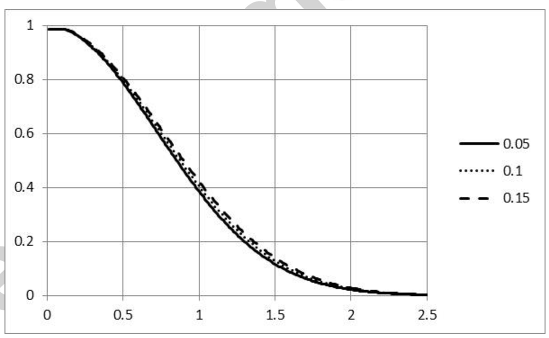
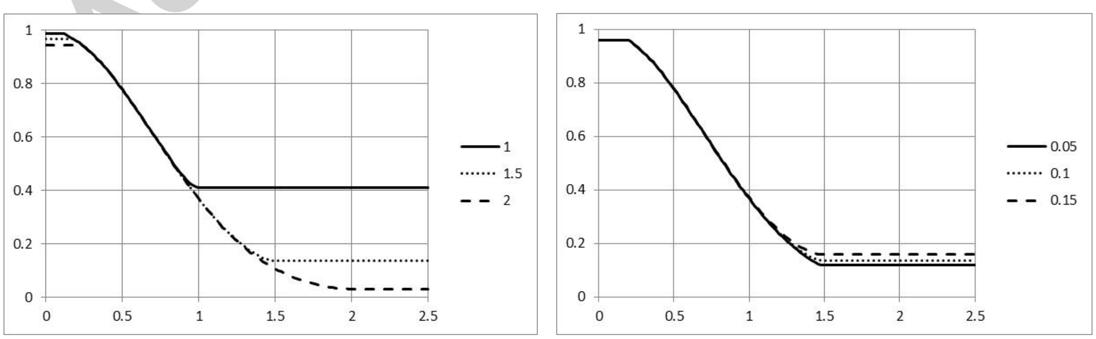


Figure 5 Variation of risk for root lodging



a) $\widetilde{m} = 0.1$ b) $\overline{U}_{LR}$ = 1.5

Figure 6. Variation of risk for stem and root lodging

4.5 Model development

There are two obvious ways in which the model could be developed, although data is required for both. The first is to incorporate a more realistic simulation of honami gusts into the model, and the second is to use the model to investigate the effects of climate change on model risk.

Firstly consider the simulation of honamis. We make the assumption that these gusts occur with a frequency of $\omega = \frac{h\widehat{U}}{X}$ i.e. the honami frequency scales on a peak wind velocity at the top of the canopy and the canopy height. We assume that when honomis occur, the canopy is oscillating at its natural frequency. The constant *h* will be referred to as the honami constant and the work of Finnigan (2000) suggest it is of the order of 1.5 to 2.0. At best this expression can be regarded as very approximate, and more work is needed to substantiate this. Observations suggest that these gusts occur over a period of a few seconds, so we will assume that they can be represented by a sine wave superimposed on a broad band maximum gust that lasts for, say, 10 seconds. For the latter, $g_{MB} = 2.47$, and we assume that the velocity amplitude of the former is given by εI where ε is a constant. Thus from equation (17) we have

$$\widehat{M} = \overline{M}(1 + 4.94I(1 + 0.81\frac{\omega_n\left(\frac{X}{g}\right)^{0.5}}{\theta}\epsilon)) \tag{55}$$

Note that equations (21) and (55) are similar in form, and indeed can be made to give identical values of $\widehat{M}$ if the various constants they contain are suitably chosen. More experimental data on the magnitude and period of occurrence of honami gusts is required to validate the adequacy of this approach.

Now consider the incorporation of the effects of climate change in the model. In the more recent predictions of various panels (e.g. the UK Climate Impacts Programme – UKCIP (2010)) the changes in the climate variables such as rainfall and temperature are given in probabilistic terms i.e. the change in the probability distributions of the variables are given. In principle therefore these predictions are ideally suited to be incorporated into the present model, which takes its climate information from probability distributions of wind speed and rainfall. Now whilst the changes in rainfall probability distributions within, say, UKCIP (2010) can be regarded as reasonably robust, the same is not true concerning the wind speed distributions. The reason for this is that in many climate models wind speeds are amongst the last variables to be calculated in the parameter calculation chain, often from parameters such as evapotranspiration rates, and in general cannot be considered as robust in any way. Thus whilst the current lodging model can in principle take the effects of climate change into account to calculate lodging risk, study of these effects must await more robust wind speed probability distributions from future climate modelling work.

5. Conclusions and suggestions for further work

From the material presented in earlier sections the following conclusions can be drawn.

- The lodging phenomenon for both isolated plants and interlocking canopies can be simply modelled using a second order harmonic equation, provided that the natural frequencies, damping ratios and wind forces are suitably specified in each case. This results in simple expressions for both the mean and the fluctuating moments along the length of the plant stem.
- By applying suitable root and stem lodging criteria, it is possible to define mean wind velocities for the occurrence both root and stem lodging that are simple functions of a wide range of plant, soil and weather parameters.
- The definition of these velocities enables a graphical representation of the lodging process to be developed that clearly indicates regions of lodging and non-lodging in the rainfall / wind velocity plane.
- The stem and root lodging velocities themselves have a practical significance in that the definitions allow a proper appreciation of the plant / soil / weather properties that are relevant to the lodging issue, and indicate where effort in reducing the overall lodging risk should be directed.
- The assumption of a Rayleigh distribution for wind speed probabilities and an exponential distribution for rainfall probabilities allows simple formulae for lodging risk to be derived that are simple functions of dimensionless saturation, stem lodging and root lodging wind velocities, and dimensionless mean rainfall intensity.
- For most practical situations the stem and root lodging risks can be calculated from exponential functions of dimensionless stem and root lodging velocities.
- Again these simple risk equations are of practical importance as they indicate the relationships between the various parameters of this multi-parameter issue, and potentially offer a way for assessing the effect of current and future climate parameters on the risk of lodging occurring, which has some potential economic significance.

Whilst the model presented in this paper thus represents a simple and effective way of studying lodging for a variety of crops, it still requires development in a number of ways.

- The firmer specification of the gust periods of relevance to root and stem lodging.
- The incorporation of honami dynamics into the model.
- The development and testing of a robust tap root model.
- The specification of crop parameters for interlocking canopies of different types.

These developments of course imply the need for field tests on different crops throughout the growing season.

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Appendix. Derivation of displacement and bending moment equations

The equation of motion of a plant canopy is given by equation (1) in section 2.2. This equation is complicated by the damping terms, which makes the solution cumbersome. There are two alternative (and ultimately equivalent) methods of solution. The first would be to retain the damping terms and to solve the equation using complex methods to obtain the bending moment up the stem. At some point in this calculation the rather complicated damping term is replaced by a generalised damping coefficient. The alternative approach is to omit the damping terms, and simply introduce a generalised damping into the solution. The second approach is taken here as this offers a simpler, more easily understandable method.

Thus without the damping terms, equation (1) becomes

$$npEI_A \frac{d^2y}{dx^2} = \mu g(Y - y) + F(X - x) - \mu \frac{d^2Y}{dT^2}(X - x) \quad \text{(A1)}$$

where the symbols have been defined in section 2.2. We now assume that the forcing wind force function and the displacements are given by

$$F = F'e^{j\omega T} \qquad y = y'e^{j\omega T} \qquad Y = Y'e^{j\omega T} \quad \text{(A2)}$$

Substituting these expressions, equation (A1) thus takes on the simple form

$$\frac{d^2(\tilde{y})}{d(\tilde{x})^2} + \alpha^2(\tilde{y}) = \alpha^2\tilde{Y}\left(1 + \tilde{\omega}^2(1 - \tilde{x})\right) + \alpha^2\tilde{F}(1 - \tilde{x}) \quad \text{(A3)}$$

Here we define normalised values for the parameters as follows

$$\tilde{y} = y/X \qquad \tilde{Y} = Y/X \qquad \tilde{x} = x/X \qquad \tilde{\omega} = \omega(X/g)^{0.5} \qquad \tilde{F} = F/\mu g \quad \text{(A4)}$$

and

$$\alpha^2 = \left(\frac{\mu g X^2}{npEI_A}\right) \quad \text{(A5)}$$

The boundary conditions are simply that $\tilde{x} = \tilde{y} = 0$ at the base; that $\tilde{y}=\tilde{Y}$ when $\tilde{x} = 1$ at the top of the stem; and that the bending moment at the base is proportional to the slope at the base i.e. at $\tilde{x} = 0$

$$\frac{d(\tilde{y})}{d(\tilde{x})} = 0 \tag{A6}$$

Equation (A3) has the solution

$$\tilde{y} = Acos(\alpha\tilde{x}\,) + Bsin(\alpha\tilde{x}\,) + \tilde{Y}\left(1 + \tilde{\omega}^2(1 - \tilde{x})\right) + \tilde{F}(1 - \tilde{x}) \tag{A7}$$

Application of the boundary conditions gives

$$A = -\tilde{F} - \tilde{Y}(1 + \tilde{\omega}^2) \tag{A8}$$

and

$$B = -Acot(\alpha) \tag{A9}$$

$$\tilde{Y} = \frac{\tilde{F}}{\left(\frac{\alpha}{1-\alpha\cot(\alpha)} - \tilde{\omega}^2\right)} \tag{A10}$$

Now this is clearly of the standard undamped oscillator form with a natural frequency given by

$$\tilde{\omega}_n{}^2 = \frac{\alpha}{1-\alpha\cot(\alpha)} \tag{A11}$$

Thus we can write

$$\tilde{Y} = \frac{\tilde{F}}{(\tilde{\omega}_n{}^2 - \tilde{\omega}^2)} \tag{A12}$$

Equation (A7) thus becomes

$$\tilde{y} = \frac{\tilde{F}}{(\tilde{\omega}_n{}^2 - \tilde{\omega}^2)}\left((1 + \tilde{\omega}_n{}^2)(\cos(\alpha\tilde{x}) - \cot(\alpha)\sin(\alpha\tilde{x})) + \tilde{\omega}^2(1 - \tilde{x}) + 1\right) \tag{A13}$$

Now the bending moment along the stem is given by

$$M = -\frac{nEI_A}{X}\frac{d^2(\tilde{y})}{d(\tilde{x})^2} \tag{A14}$$

From equations (A13) and (A14) one obtains

$$\frac{M\prime}{XF\prime} = \frac{(1+\tilde{\omega}_n^2)}{(\tilde{\omega}_n^2 - \tilde{\omega}^2)}\left(\cos(\alpha\tilde{x}) - \cot\alpha\sin(\alpha\tilde{x})\right) \tag{A15}$$

As noted above, this equation has the form of an undamped oscillator. To make this more realistic we thus include a generalised damping term to give the following final form.

$$\frac{M\prime}{XF\prime} = \frac{(1+\tilde{\omega}_n^2)}{\left((\tilde{\omega}_n^2 - \tilde{\omega}^2)^2 + (\theta\tilde{\omega})^2\right)^{0.5}}\left(\cos(\alpha\tilde{x}) - \cot\alpha\sin(\alpha\tilde{x})\right) \tag{A16}$$

The damped forms of the displacement equations (A12) and (A13) are given by

$$\tilde{Y} = \frac{\tilde{F}}{\left((\tilde{\omega}_n^2 - \tilde{\omega}^2)^2 + (\theta\tilde{\omega})^2\right)^{0.5}} \qquad \text{(A17)}$$

$$\tilde{y} = \frac{\tilde{F}}{\left((\tilde{\omega}_n^2 - \tilde{\omega}^2)^2 + (\theta\tilde{\omega})^2\right)^{0.5}} \left((1 + \tilde{\omega}_n{}^2)(\cos(\alpha\tilde{x}) + \cot(\alpha)\sin(\alpha\tilde{x})) + \tilde{\omega}^2(1 - \tilde{x}) + 1\right) \qquad \text{(A18)}$$

The equations (A14),(A16), (A17) and (A18) for displacement and bending moment are reproduced as equations (2) to (6) in the main text, but in a dimensional rather than normalised format.

Highlights

- A generalised model is presented to describe lodging for isolated plants and plants in canopies.
- The model results in simple expressions for the bending moment along the length of the plant stem.
- The analysis identifies regions of lodging and non-lodging in the rainfall / wind velocity plane.
- Lodging risk can be calculated for any combination of rain / wind /soil and plant characteristics