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Bank Insolvency Risk and Z-Score Measures: A Refinement

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Abstract

We re-examine the probabilistic foundation of the link between Z-score measures and banks' probability of insolvency, offering an improved measure of that probability without imposing further distributional assumptions. While the traditional measure of the probability of insolvency thus provides a less effective upper bound of the probability of insolvency, it can be meaningfully reinterpreted as a measure capturing the odds of insolvency instead. We similarly obtain refined probabilistic interpretations of the commonly used simple and log-transformed Z-score measures; in particular, the log of the Z-score is shown to be negatively proportional to the log odds of insolvency.

JEL classification: G21

Key words: insolvency risk, Z-score, probability, odds

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1. Introduction

The recent financial crisis has refocused attention on the general importance, impact and measurement of banks' insolvency and liquidity risk. A popular risk measure in the banking and financial stability related literature that reflects a bank's probability of insolvency is the Z-score.[1] Its widespread use[2] is due to its relative simplicity and the fact that it can be calculated using only accounting information; this, in contrast to market-based risk measures, makes it also applicable to the substantial number of unlisted financial institutions.[3] In its general form, allowing for non-normal return distributions, it is generally attributed to Hannan and Hanweck (1988) and Boyd et al. (1993); Boyd and Graham (1986) had previously introduced Z-scores in the special context of normal return distributions.

In this paper, we re-examine the probabilistic foundation of this general approach to proxying a bank's probability of insolvency, demonstrating that it is in fact possible to refine, i.e. improve on, the measure of the probability of insolvency implied by this traditional approach without imposing any further distributional assumptions. We show that while the traditional measure of the probability of insolvency thus provides a less effective upper bound of the probability of insolvency, it can in fact be meaningfully reinterpreted as a measure capturing the odds of insolvency instead.[4] We then further show that this refinement of the probabilistic foundation of Z-score measures implies that the risk measures commonly used in the existing literature, such as the simple Z-score or its log-transformation, are also more closely related to the odds of insolvency than the probability of insolvency itself. As a

[1] This methodology should not be confused with the Altman (1968) Z-score measure used in the corporate finance literature; see Altman (2002, ch. 1) for a discussion.

[2] For some recent papers using this methodology, see e.g. Berger et al. (2014), Delis et al. (2014), Fang et al. (2014), Fu et al. (2014), Hakenes et al. (2014); Beck et al. (2013), Bertay et al. (2013), DeYoung and Torna (2013).

[3] Note, however, that it is possible, if uncommon, to calculate Z-score measures using market information as well.

[4] The odds of an event, i.e. the ratio of the probabilities in favor and against that event, indicate how much more likely it is that the event occurs than that it does not occur. Franklin (2001, ch. 10) argues that ordinary language use of odds predates Pascal and Fermat's discovery of the mathematics of probability in 1654.

consequence, the log of the Z-score in particular emerges from our refinement as an insolvency risk measure that is attractive and unproblematic to use (even as a dependent variable in standard regression analysis), providing more rigorously founded support to its emerging use in the literature.

Section 2 now reviews the traditional probabilistic interpretation of Z-scores, introduces our refinement, and advances related risk measures reflecting the odds of insolvency; Section 3 discusses some implications of our results for applied work; and Section 4 concludes the paper.

2. Z-score measures: a refined probabilistic interpretation

Let us first recapitulate the traditional justification for using Z-scores as a risk measure reflecting a bank's probability of insolvency. In line with most of the existing literature,[5] we define bank insolvency as a state where $(car + roa) \leq 0$, with car the bank's capital-asset ratio and roa its return on assets. Hannan and Hanweck (1988) and Boyd et al. (1993) then pointed out that if roa is a random variable with finite mean μ_{roa} and variance σ^2_{roa}, the Chebyshev inequality allows one to state an upper bound of the probability of insolvency as[6]

$$p(roa \leq -car) \leq Z^{-2} \tag{1}$$

where the Z-score is defined as $Z \equiv \frac{car+\mu_{roa}}{\sigma_{roa}} > 0$; we could refer to the measure Z^{-2} as the traditional insolvency probability bound.[7]

[5]Some authors, e.g. Barry et al. (2011) and Bouvatier et al. (2014), consider an alternative return-on-equity based Z-score measure as first proposed in Goyeau and Tarazi (1992); we derive an analogous refined probabilistic interpretation of such a measure in Appendix B.

[6]As similarly implemented by Roy (1952), this is an application of the (two-sided) Chebyshev inequality (see Ross, 1997, p. 396): it states that for a random variable X with finite mean μ and variance σ^2, it holds for any $k > 0$ that $P\{|X - \mu| \geq k\} \leq \sigma^2/k^2$.

[7]Hannan and Hanweck (1988) used this traditional insolvency probability bound as their proxy of a bank's probability of insolvency (under the additional assumption of symmetry). However, much of the remaining empirical literature has followed Boyd and Graham (1986) and Boyd et al. (1993) by using the simple Z-scores Z as the relevant bank insolvency risk measure instead; as such it is

Equation (1) gives a probabilistic interpretation of Z-scores as a particular non-linear transformation of a bank's probability of insolvency; this could have implications for the correct formulation of empirical models and hypotheses and the meaningful discussion of results. In this context it is therefore important to point out that it is in fact possible to improve on the traditional insolvency probability bound given by Equation (1), without imposing any further distributional assumptions, by drawing on the one-sided Chebyshev inequality instead;[8] we state this result in the following

Proposition 1. *If roa is a random variable with finite mean μ_{roa} and variance σ_{roa}^2, an (improved) upper bound of the bank's probability of insolvency p is given by*

$$p(roa \leq -car) \leq \frac{1}{1+Z^2} < 1 \tag{2}$$

where the Z-score Z is defined as $Z \equiv \frac{car+\mu_{roa}}{\sigma_{roa}} > 0$.

Proof. See the Appendix.

We could refer to the measure $\left(1+Z^2\right)^{-1}$ characterized by Equation (2) as the improved insolvency probability bound; it is straightforward to see that it is consistently tighter than the traditional insolvency probability bound given by Equation (1), and is also naturally bounded below one.[9] In particular, we can state

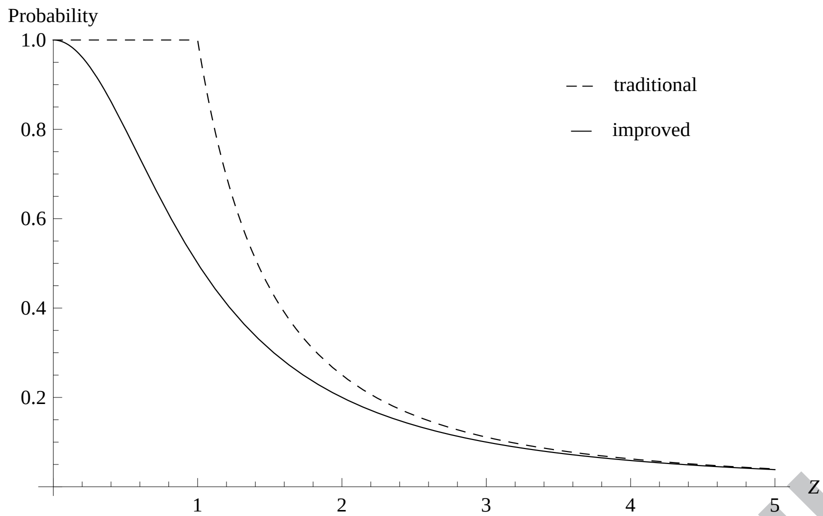
Corollary 1. *The traditional insolvency probability bound provides a less effective upper bound of the probability of insolvency than the improved measure given in Proposition 1; the difference between the traditional and improved measures $D(Z)$ has a maximum value of* 0.5 *at $Z = 1$, with $\lim_{Z\to\infty} D(Z) = \lim_{Z\to 0} D(Z) = 0$.*

being widely used in cross-sectional, but increasingly also in panel studies (see Lepetit and Strobel 2013).

[8]Note that if the moment generating function of the random variable roa were known, one could draw on the even more effective Chernoff bounds (see Ross, 1997, p. 415); however, as our aim is to construct a simple, robust bank insolvency risk measure, we do not pursue this further here.

[9]This is in contrast to the traditional insolvency probability bound, which needs to be bounded at one for $0 < Z < 1$, as $Z^{-2} > 1$ with $\lim_{Z\to 0} Z^{-2} = \infty$ in this case.

Figure 1. Plot of traditional vs. improved insolvency probability bounds, as function of Z-score



Proof. See the Appendix.

We thus observe that the lack of effectiveness in the proxying of a bank's probability of insolvency encountered when relying on traditional insolvency probability bounds is particularly large in the region that is arguably the most relevant in this context, i.e. for banks with relatively low Z-scores and thus at significant risk of becoming insolvent. This is illustrated in Figure 1, which also shows that the difference between the traditional and improved measures does get (fairly rapidly) smaller the larger the Z-score, and thus the more remote the chance of the bank becoming insolvent. The practical relevance of this lack of effectiveness when relying on traditional insolvency probability bounds will be illustrated further using real data in Section 3.1.

Rather intriguingly, we can further note

Corollary 2. *The traditional insolvency probability bound satisfies* $Z^{-2} \geq \frac{p(roa \leq -car)}{1-p(roa \leq -car)}$, *i.e. it gives an upper bound of the odds of insolvency.*

Proof. This follows from rearranging equation (2).

While the traditional insolvency probability bound gives a less effective upper bound of the probability of insolvency based on our refinement, it does in fact provide an upper bound of the odds of insolvency, a closely related risk measure; we could thus, more appropriately, refer to the measure Z^{-2} as the insolvency odds bound.[10] Furthermore, our results also imply a refined probabilistic interpretation of the more commonly used simple Z-scores Z as such, as stated in the following

Corollary 3. *The Z-score satisfies* $Z \leq \left(\frac{p(roa \leq -car)}{1-p(roa \leq -car)}\right)^{-\frac{1}{2}}$*, i.e. it gives a lower bound of the inverse square root of the odds of insolvency.*
Consequently, the log of the Z-score satisfies $\ln\left(\frac{p(roa \leq -car)}{1-p(roa \leq -car)}\right) \leq -2\ln(Z)$*, i.e. it is negatively proportional to an upper bound of the log odds*[11] *of insolvency.*

Proof. This follows from Corollary 2.

Amongst these alternative, if intrinsically related insolvency risk measures, the Z-score or its log-transformation are in widespread use in the empirical literature, whereas the improved insolvency probability bound and the (reinterpreted) insolvency odds bound do not seem to be commonly used so far. Notwithstanding any such implementation decision, the particular probabilistic interpretations of the different insolvency risk measures discussed might prove useful when formulating empirical models and hypotheses and discussing results more generally.

3. Further implications

3.1. Economic significance

We now illustrate that the lack of effectiveness arising (in line with Corollary 1) when a bank's probability of insolvency is proxied using traditional insolvency probability bounds is of practical relevance in real data. For this, we calculate Z-scores, traditional and improved insolvency probability bounds, and, in the first instance, the resulting (relative) difference

[10]Note that it thereby sheds some of the practical limitations, such as the need to be bounded at one, attached to its previous interpretation as a probability bound.
[11]The term log odds was introduced by Barnard (1949).

Table 1. Mean of different Z-score measures, relative difference between traditional & improved probability bounds, and hypothetical equity buffers (relative & absolute) equating traditional & improved probability bounds, for OECD commercial, cooperative & savings banks (1998-2012)

	Number of Banks	Mean Z-score	Ln(Z-score)	Traditional Insolvency Probability Bound	Improved Insolvency Probability Bound	Relative Difference (%) of Traditional vs Improved Measure	Relative Hypothetical Capital Ratio Buffer (%) Equating Traditional and Improved Measure	Hypothetical Equity Buffer (million USD) Equating Traditional and Improved Measure
OECD	10949	39.564	3.258	0.0106	0.0092	0.991	0.947	2.391
Commercial	7604	32.221	3.113	0.0120	0.0104	1.123	1.116	3.068
Cooperative	1729	52.737	3.533	0.0067	0.0057	0.670	0.437	0.691
Savings	1616	60.020	3.642	0.0080	0.0068	0.713	0.695	1.023
US	7360	32.740	3.159	0.0096	0.0088	0.934	0.697	0.639
Commercial	6619	32.433	3.156	0.0090	0.0083	0.888	0.618	0.433
Cooperative	8	59.376	3.232	0.0130	0.0125	1.305	0.641	51.977
Savings	733	35.222	3.183	0.0154	0.0129	1.342	1.409	1.936
EU15	2497	53.671	3.545	0.0111	0.0087	0.919	1.656	6.156
Commercial	584	23.847	2.736	0.0418	0.0314	3.348	6.775	25.198
Cooperative	1270	60.027	3.745	0.0017	0.0017	0.173	0.091	0.349
Savings	643	68.206	3.884	0.0019	0.0018	0.186	0.098	0.329

Source: Own calculations using BvD Bankscope data.

between the traditional and improved measures using a dataset of OECD commercial, cooperative and savings banks, extracted from BvD Bankscope and covering the period 1998–2012.[12]

We note from Table 1 that the average relative difference between the traditional and improved measures for our OECD sample is 0.991%, being highest for commercial banks at 1.123% overall; it is particularly high for EU15 commercial banks at 3.348% and US cooperative and savings banks at 1.305% and 1.342%, respectively. However, for the banks with the lowest 10% of Z-scores, i.e. banks with a more pronounced risk of becoming insolvent, we can see from Table 2 that the average relative difference between the traditional and

[12] We clean for outliers/erroneous data by discarding the lowest/highest 0.75% of *roa* values and any *car* values lying outside the range of 0 to 100%, retain for each bank the longest contiguous run of observations, conditional on it covering at a minimum the period 2004-2009 (to allow our later crisis/pre-crisis split), and end up with data for 10949 banks.

Table 2. Mean of different Z-score measures, relative difference between traditional & improved probability bounds, and hypothetical equity buffers (relative & absolute) equating traditional & improved probability bounds, for OECD commercial, cooperative & savings banks with lowest 10% Z-scores (1998-2012)

	Number of Banks	Mean						
		Z-score	Ln(Z-score)	Traditional Insolvency Probability Bound	Improved Insolvency Probability Bound	Relative Difference (%) of Traditional vs Improved Measure	Relative Hypothetical Capital Ratio Buffer (%) Equating Traditional and Improved Measure	Hypothetical Equity Buffer (million USD) Equating Traditional and Improved Measure
Banks with lowest 10% Z-scores								
OECD	1095	4.767	1.489	0.0816	0.0678	7.509	8.189	17.679
Commercial	895	4.782	1.496	0.0790	0.0661	7.273	8.266	19.338
Cooperative	92	4.820	1.490	0.0888	0.0704	8.834	6.279	7.083
Savings	108	4.601	1.437	0.0970	0.0790	8.330	9.174	12.953
US	812	4.892	1.535	0.0643	0.0565	6.158	5.080	4.175
Commercial	717	4.941	1.551	0.0593	0.0530	5.823	4.430	2.299
Cooperative	2	4.556	1.515	0.0486	0.0463	4.862	2.377	206.061
Savings	93	4.523	1.415	0.1036	0.0836	8.770	10.156	14.294
EU15	130	4.170	1.248	0.1793	0.1324	14.195	29.962	93.468
Commercial	106	3.820	1.139	0.2120	0.1549	16.630	36.340	113.507
Cooperative	12	5.991	1.778	0.0301	0.0292	3.014	1.572	5.197
Savings	12	5.437	1.673	0.0387	0.0370	3.870	2.022	4.728

Source: Own calculations using BvD Bankscope data.

improved measures rises substantially to 7.509% for the OECD sample and is now highest for cooperative banks at 8.834% overall; it is particularly high for EU15 commercial banks at 16.630% and US savings banks at 8.770% in this case. Lastly, we can also observe from Table 3 that, as expected, the average relative difference between the traditional and improved measures has gone up throughout when comparing these measures for the pre-crisis (1998-2006) and crisis (2007-2009) periods, respectively.

As a complementary way of highlighting the economic significance of the difference between the traditional and improved measures, we can further ask what hypothetical equity buffers (i.e. differentials) would lead the traditional and improved measures to be identical for a given bank.[13] We see from Table 1 that the average relative hypothetical capital ratio

[13]We derive the hypothetical capital-asset ratio car_h that equates the traditional and improved insolvency probability bounds, in the sense that the hypothetical Z-score $Z_h = (car_h + \mu_{roa})/\sigma_{roa}$

buffer (or alternatively, average absolute hypothetical equity buffer) equating the traditional and improved measures for our OECD sample is 0.947% (\$2.391m) and is highest for commercial banks at 1.116% (\$3.068m) overall; it is particularly high for EU15 commercial banks at 6.775% (\$25.198m) and US savings banks at 1.409% (US cooperative banks at \$51.977m). For the banks with the lowest 10% of Z-scores, Table 2 shows that the corresponding average relative hypothetical capital ratio buffer (average absolute hypothetical equity buffer) rises substantially to 8.189% (\$17.679m) for the OECD sample and is now highest for savings banks at 9.174% (commercial banks at \$19.338m) overall; it is particularly high for EU15 commercial banks at 36.340% (\$113.507m) and US savings banks at 10.156% (US cooperative banks at \$206.061m) in this case.

Overall, while one could argue that the traditional insolvency probability bound gives a "conservative" measure of a bank's probability of insolvency as viewed from a regulator's perspective, the improved insolvency probability bound is clearly the more appropriate measure in the sense of being more effective to an economically significant degree, particularly for banks with higher levels of insolvency risk, and should thus be preferred.

3.2. Forecasting performance

In addition to the effectiveness issue outlined above, it is interesting to look at some forecasting properties of the traditional as compared with the improved insolvency probability bound measure. We focus on the recent financial crisis to highlight this issue, examining how well those two measures calculated over a pre-crisis sample (covering the years 1998-2006) were able to forecast the corresponding realized ones calculated over the crisis period (defined as the years 2007-2009). In order to assess forecasting performance in this context, we use the coefficient of variation of the root mean squared error (*CV(RMSE)*), calculated for both

satisfies $(1 + Z_h^2)^{-1} = Z^{-2}$ for $Z \geq 1$ and $(1 + Z_h^2)^{-1} = 1$ for $Z < 1$; this gives $car_h = -\mu_{roa} + \sigma_{roa}\sqrt{Z^2 - 1}$ for $Z \geq 1$ and $car_h = -\mu_{roa}$ for $Z < 1$. Relative hypothetical capital ratio buffers are then defined as $car/car_h - 1$, and absolute hypothetical equity buffers as $(car - car_h)\,TA$, with total assets TA.

Table 3. Mean, difference in relative difference and forecasting performance of traditional vs improved insolvency probability bound measures, between pre-crisis and crisis periods, for OECD commercial. cooperative & savings banks (1998-2009)

	Number of Banks	Mean					Coeff. of Variation of RMSE, Forecast of Crisis Value with Pre-Crisis, Traditional Measures	Coeff. of Variation of RMSE, Forecast of Crisis Value with Pre-Crisis, Improved Measures
		Traditional Insolvency Probability Bound, Pre-Crisis Period	Traditional Insolvency Probability Bound, Crisis Period	Improved Insolvency Probability Bound, Pre-Crisis Period	Improved Insolvency Probability Bound, Crisis Period	Difference Crisis & Pre-Crisis in Relative Difference (%) of Traditional vs Improved Measure		
OECD	10945	0.0059	0.0106	0.0051	0.0092	0.4616	3.7488	3.2292
Commercial	7600	0.0062	0.0120	0.0055	0.0104	0.5773	3.5450	3.0543
Cooperative	1729	0.0064	0.0067	0.0054	0.0057	0.0380	2.6126	2.2924
Savings	1616	0.0038	0.0080	0.0032	0.0068	0.3705	5.6299	4.7886
US	7360	0.0042	0.0096	0.0039	0.0088	0.5392	3.7488	3.2292
Commercial	6619	0.0039	0.0090	0.0037	0.0083	0.5186	3.5450	3.0543
Cooperative	8	0.0004	0.0130	0.0004	0.0125	1.2661	2.6126	2.2924
Savings	733	0.0072	0.0154	0.0057	0.0129	0.7173	5.6299	4.7886
EU15	2493	0.0065	0.0111	0.0053	0.0087	0.4241	3.7488	3.2292
Commercial	580	0.0231	0.0420	0.0179	0.0315	1.7241	3.5450	3.0543
Cooperative	1270	0.0016	0.0017	0.0016	0.0017	0.0138	2.6126	2.2924
Savings	643	0.0012	0.0019	0.0012	0.0018	0.0618	5.6299	4.7886

Source: Own calculations using BvD Bankscope data. Pre-crisis period is defined as 1998-2006, crisis one as 2007-2009.

the traditional and improved probability bound measures pb as

$$CV(RMSE) = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(pb_i^{precrisis} - pb_i^{crisis}\right)^2}}{\frac{1}{n}\sum_{i=1}^{n} pb_i^{crisis}}$$

where the n banks are indexed by i. The results are given in Table 3: we observe that the coefficient of variation of the RMSE is consistently lower when using the improved insolvency probability bound measures compared with using the traditional ones, irrespective of country grouping or bank type. Therefore, the improved insolvency probability bound is the more accurate measure also in this forecasting context, and should thus be preferred over the traditional one.

3.3. *Log of Z-score*

From a practical implementation point of view, Laeven and Levine (2009) and Houston et al. (2010) advocate the use of the log of the Z-score over the simple Z-score on the basis that the latter's distribution is heavily skewed, whereas the former's is not. Figure 2 confirms this observation for our dataset of OECD commercial, cooperative and savings banks, showing also that both the traditional and the improved insolvency probability bounds have distributions with degrees of skewness even higher than those of the simple Z-scores.[14]

On a related note, it is instructive to examine the respective ranges of the different insolvency risk measures considered in this context. The improved insolvency probability bound lies in the interval $[0, 1)$, while the insolvency odds bound and thus, from Corollary 3, the Z-score are meaningfully defined on the interval $[0, \infty)$; these might thus require the use of limited dependent variable techniques when the insolvency risk measures are used as dependent variables in relevant empirical analysis. The log of the Z-score, on the other hand, has a meaningful probabilistic interpretation on the interval $(-\infty, \infty)$, i.e. the domain of all real numbers, as a consequence of our refinement.

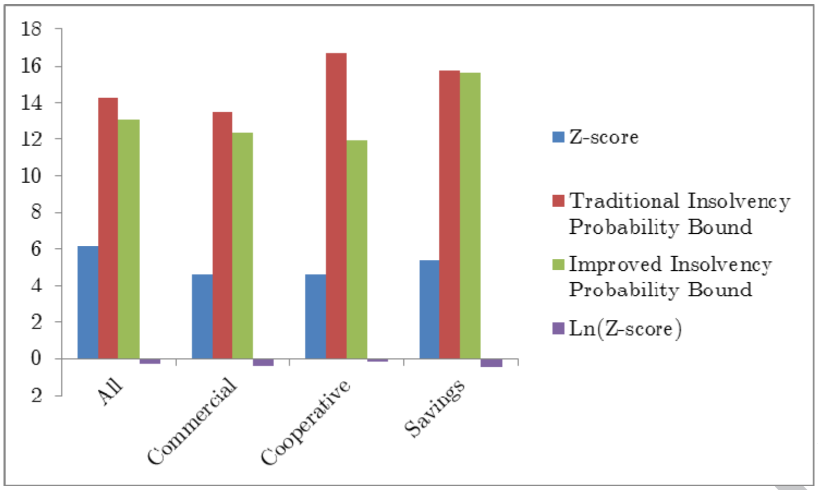
Overall this makes the log of the Z-score an unproblematic insolvency risk measure to use in standard regression analysis, both as dependent and independent variable; clearly, this would lend support to its emerging use in the literature, albeit with a now more solidly founded probabilistic interpretation as a risk measure that is negatively proportional to a bank's log odds of insolvency.

4. Conclusion

We re-examine the probabilistic foundation of the traditional link between Z-score measures and banks' probability of insolvency, providing an improved measure of that probability without imposing further distributional assumptions. The traditional measure of the probability of insolvency thus provides a less effective upper bound of the probability of insolvency,

[14]Note that skewness of Z-score measures is not a problem in itself, but could complicate inference in regression analysis.

Figure 2. Skewness of different Z-score measures, calculated for OECD commercial, cooperative & savings banks (1998-2012)



but can in fact be meaningfully reinterpreted as a measure capturing the odds of insolvency instead. We obtain analogous refined probabilistic interpretations of the commonly used simple and log-transformed Z-score measures. In particular, the log of the Z-score is shown to be negatively proportional to the log odds of insolvency, and thus meaningfully defined on the domain of all real numbers. As a consequence, it emerges from our refinement as an attractive and unproblematic insolvency risk measure to use (even as a dependent variable in standard regression analysis), giving now more rigorously founded support to its emerging use in the literature.

Appendix

A. Proofs

Proof of Proposition 1: This is an application of the one-sided Chebyshev inequality (see Ross, 1997, p. 414, or previously, Feller, 1971, p. 152): it states that for a random variable X with finite mean μ and variance σ^2, it holds for any $a > 0$ that $P\{X \leq \mu - a\} \leq \frac{\sigma^2}{\sigma^2+a^2}$. Setting $X = roa$ and $a = car + \mu_{roa}$, and dividing both numerator and denominator of the right hand side of the inequality by σ^2_{roa}, we obtain Equation (2), with $\lim_{Z\to 0}\left(1+Z^2\right)^{-1} = 1$.

Proof of Corollary 1: This follows from Equations (1) and (2). The difference between the traditional and improved measures simplifies as $D(Z) = Z^{-2} - (1+Z^2)^{-1} = (Z^4+Z^2)^{-1}$ for $Z \geq 1$, and as $D(Z) = 1 - (1+Z^2)^{-1} = \frac{Z^2}{1+Z^2}$ for $Z < 1$ (noting footnote 9), implying $\max_Z D(Z) = 0.5$ at $Z = 1$ and $\lim_{Z\to\infty} D(Z) = \lim_{Z\to 0} D(Z) = 0$.

B. ROE-based Z-score

An alternative, return-on-equity based Z-score measure was first proposed in Goyeau and Tarazi (1992) (in the special context of normal return distributions); we can provide a similarly refined probabilistic interpretation for such a measure that allows for non-normal return distributions, analogously to the discussion of the more commonly used ROA-based measure in the main text.

In line with our approach in Section 2, we can equivalently define bank insolvency as a state where $roe \leq -1$, with roe the bank's return on equity; this allow us to then state

Proposition 2. *If roe is a random variable with finite mean μ_{roe} and variance σ_{roe}^2, an upper bound of the bank's probability of insolvency p is given by*

$$p(roe \leq -1) \leq \frac{1}{1+Z_e^2} < 1 \tag{3}$$

where the (alternative) Z-score Z_e is defined as $Z_e \equiv \frac{1+\mu_{roe}}{\sigma_{roe}} > 0$.

Proof. This is analogous to the proof of Proposition 1, setting $X = roe$ and $a = 1 + \mu_{roe}$ and dividing both numerator and denominator of the right hand side of the inequality by σ_{roe}^2; again, $\lim_{Z_e\to 0} (1+Z_e^2)^{-1} = 1$.

It is straightforward to see that this alternative ROE-based Z-score measure behaves and can be utilized analogously to the more commonly used ROA-based measure discussed in the main text; for conciseness, we only state the most practically relevant corollary to Proposition 2 as

Corollary 4. *The log of the (alternative) Z-score Z_e satisfies* $\ln\left(\frac{p(roe\leq -1)}{1-p(roe\leq -1)}\right) \leq -2\ln(Z_e)$, *i.e. it is (also) negatively proportional to an upper bound of the log odds of insolvency.*

Proof. This follows analogously to Corollary 3.

Such an ROE-based Z-score measure (or particularly, in the light of Section 3.3, its log transformation) might in some respects be more appropriate than the more commonly used ROA-based measure, as it is by construction unaffected by potentially spurious variability in total assets (this aspect could be particularly relevant in the, now increasingly common, construction of time-varying Z-score measures).

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Highlights

> Re-examine link between Z-score measures and banks' probability of insolvency.
> Improve on measure of that probability without further distributional assumptions.
> Log of Z-score is shown to be negatively proportional to the log odds of insolvency.