Monte Carlo Based Method for Managing Risk of Scheduling Decisions with Dynamic Line Ratings

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Abstract—Dynamic line ratings have been shown as an attractive alternative to conventional congestion management methods that can potentially improve wind integration. However, the modelling of dynamic line ratings is dependent on effectively modelling the risk of thermal overload which usually has a high amount of uncertainty. This paper uses a sample average approximation method to model the uncertainty in risk function and determine how scheduling decisions are affected. It also presents a sensitivity analysis to determine the level of uncertainty in the risk function that can be managed and how the sampling process should be adjusted. Test cases indicate that there is a high level of confidence in scheduling decisions for sample sizes less than 100. A larger sample size can maintain the high level of confidence if there is a greater uncertainty associated with the risk function.

Index Terms—dynamic line rating, sample average approximation, Monte Carlo simulation.

I. INTRODUCTION

Network congestion makes large-scale integration of wind a challenging task. Due to insufficient capacity being available during periods of congestion generation, curtailment is inevitable and this leads to under-utilization of wind power output. In some areas, there are incentives such as production tax credits (PTC) to offset the risk of curtailment by allowing wind producers to bid negative prices in the electricity market [1]. However, for long term competitiveness and viability of wind power, reducing the risk of curtailment is critical. Conventional congestion management methods involving redispatch or generation management [2],[3] can potentially lead to wind power curtailment if there is an insufficient network capacity. Network based measures [4],[5] are more likely to manage congestion without curtailing wind but is dependent on congestion being localized and the network topology allowing power flows to be diverted through non congested areas.

Recently a number of sources [6-10] have proposed dynamic line ratings (DLR) as an alternative to alleviating network congestion. DLR takes advantage of a smart grid infrastructure to relax transmission line thermal constraints thus temporarily releasing latent network capacity of short to medium length lines. This is possible since traditional line thermal ratings are calculated under worst case assumptions for ambient weather and temperature conditions which rarely occur in practice. IEEE Standard 738-2012 [11] defines the relationship between temperature and ampacity including methods for calculating dynamic ampacity iteratively. Utilities have traditionally used multiple thermal limits for different weather conditions but DLR assigns thermal limits that are more in line with real time conditions.

Studies in [7],[8] have modelled the effects of DLR by replacing the thermal constraints in the optimal dispatch problem by a penalty function that accounts for the risk of thermal overload when DLR is implemented. It has thus been shown that DLR can provide a significant increase in the normal and emergency operational flexibility of power transmission systems and compared to the more traditional static rating and can potentially alleviate network congestion due to short periods of high wind power output. The benefit of DLR over conventional congestion management approaches is that it can potentially release latent capacity dynamically rather than relying on generation curtailment and demand reduction in congested parts of a network thus defers investments and improves the power supply security.

The studies in [7],[8] model the risk based penalty function as a deterministic function of line current in excess of the rating. The advantage of a risk based penalty function is that it includes the likelihood as well as severity of outages as compared to using the costs of outages which only account for the severity, excluding the effects of asset de-rating. Accounting for the severity while ignoring likelihood assumed a worst case scenario, which is already the case with existing static line ratings. According to IEEE Std. 738-2012, the line ampacity is determined by ambient conditions which often have a high degree of variability. Thus, the variability needs to be accounted for in scheduling and planning decisions.
This paper proposes a sampling based method to account for the variability in the risk based penalty function of dynamic line ratings. The parameters in the penalty function proposed in our previous work in [7],[8] are replaced by random variables and realizations of the penalty function are obtained. A sample average approximation (SAA) method is used to determine the level of uncertainty in the solution to the optimal dispatch problem due to the variability of the risk function. A sensitivity analysis is also carried out to determine the effect of increasing the variability of the cost function on the optimal solution. The key contribution of the paper is that the proposed technique allows the uncertainty associated with dynamic line ratings to be accounted for making scheduling decisions in real-time to improve the scheduling confidence while mitigating risks associated with traditional scheduling decisions.

II. DYNAMIC LINE RATING

The formulation of DLR cost is presented in [7],[8] and a brief outline is repeated in this section for the convenience. The cost of DLR is part of the overall objective function as shown in (1).

\[ f(x) = C_g(P_g) + C_w(P_w) + C_{DLR} + C_{congestion} \]  

(1)

where \( C_g(P_g), C_w(P_w), C_{DLR} \) and \( C_{congestion} \) represent cost of conventional generation, cost wind (including reserves), cost of dynamic ratings, and cost of congestion respectively. The total cost of DLR represents the penalty for temporarily relaxing the line thermal constraint. The cost is due to the risk of thermal overload which includes the likelihood of exceeding the maximum allowable line temperature and the severity. The severity relates to an outage due to thermal overload and the associated cost of unsupplied energy. The risk is expected to be negligible for lower levels of DLR. However, as the level of DLR increases, the sensitivity of the penalty function to dynamic overloading must increase thus, suggesting an exponential penalty function. Instead, it is modelled using a quadratic function as given in (2) since it can approximate the exponential function accurately for low levels of DLR, and the relative ease of calculating the Jacobian and Hessian matrices for quadratic functions.

\[ C_{DLR} = \sum_{p=1}^{N} \sum_{q=1}^{N} C_{OLp} \left( \sum_{k=1}^{N} h_{pq,k} a_{pq,k} \right)^2 \]  

(2)

where \( p-q \) represents a line from bus \( p \) to bus \( q \). The cost of violating the constraint is proportional to the magnitude by which the actual line flow exceeds the line capacity. The constraints in (3) complement the expression for \( C_{DLR} \) in (2) to account for the cost of uncertainty in stochastic line rating.

\[ a_{pq,k} \geq s_{\text{max},pq,k} - S_{\text{ch},pq} \]

\[ a_{pq,k} \geq 0 \]  

(3)

The thermal capacity of line \( p-q \) is a discrete random variable where each discrete value (represented by index \( k \)) of \( s_{\text{max},pq,k} \) has corresponding probability \( h_{pq,k} \). The term \( a_{pq,k} \) (with per unit cost \( c_{OLp} \)) represents the amount by which the actual line flow exceeds the discrete line capacity in the \( k \)th ordered pair and it corrects any violation in the constraint \( S_{\text{ch},pq} > s_{\text{max},pq,k} \). Thus, \( (h_{pq,k}, a_{pq,k}) \) represents the probability distribution of dynamic line rating and the average value of \( a_{pq,k} \) for \( k \) represents the expected dynamic line rating.

The cost of DLR is based on the expected value of dynamic line rating which includes both the amount of DLR \( (d_{pq,k}) \) and the time for which it is implemented \( (h_{pq,k}) \). \( h_{pq,k} \) is an array of relative frequencies associated with each value of \( a_{pq,k} \). If the time for which DLR is implemented varies, the value of \( a_{pq,k} \) associated with the specific value of \( h_{pq,k} \) for that time will change. If the time for a specific amount of DLR is varied, it changes the probability distribution (specifically a change in probability for that level of DLR) and hence the expected value of DLR.

The parameter \( c_{OLp} \) defines the cost function and it is modelled as a fixed value in [7],[8]. However, it is expected to vary since the risk is dependent on factors such as temperature and wind speed. To account for the variation, this paper models \( c_{OLp} \) as a random variable.

III. SAMPLE AVERAGE APPROXIMATION

The function \( C_{DLR} \) is replaced by the expected value of \( C_{DLR} \) which is obtained by repeatedly sampling \( C_{DLR} \) for different values of \( C_{OLp} \) and finding the average of those samples as \( C_{DLR,N} \) for \( N \) samples. It is assumed that each sample is independent and identically distributed.

\[ C_{DLR,N}(x) = \frac{1}{N} \sum_{i=1}^{N} C_{DLR}(x, \xi_i) \]  

(4)

where \( x \) represents the vector of all the control variables such that \( x = \{a_{pq,k}\} \) \( (h_{pq,k} \) is constant). \( \xi_i \) represents the vector that contains all the parameter with uncertainty. In this instance \( \xi_i = \{c_{OLp}\} \) such that it contains DLR cost parameter for each line in the system. The function \( C_{DLR} \) is as defined in (2). In the objective function of optimization (1), the function in (2) can be replaced by the SAA function in (4). Thus, if the original objective function is denoted \( f(x) \), it can be replaced by the SAA approximation \( f_N(x) \).

If \( f_N(.) \) converges uniformly to \( f(.) \) then it is well established that for a given sample size \( N \), the optimal solution, \( x_N \) and the optimal value, \( f_N(x_N) \) are random variables with normal distribution (multivariate normal distribution in the case of the optimal solution \( x_N \)). \( f_N(x_N) \) is approximately \( \text{Normal}(v^*,\sigma'(x_N)/N) \) and \( x_N \) is \( \text{Normal}(x^*,H'/\psi H^{-1}) \). The true values of optimal solution and optimal value are given by \( x^* \) and \( v^* \) respectively. \( H \) represents the hessian matrix of the objective function \( f(x) \). \( \psi \) is the asymptotic covariance matrix of \( \sqrt{N} \left[ \nabla f_N(x) - \nabla f(x) \right] \).

Thus, it should be possible to determine a \((1-\alpha)\) confidence region for the optimal solution in terms of a tolerance \( \epsilon \). It can then be stated with a probability \((1-\alpha)\) that any estimator of the optimal solution based on \( N \)
samples, will be within a tolerance of $\varepsilon$ from the true solution. The value of $\varepsilon$ can be expressed as a percentage and be made smaller for a more accurate solution by choosing a larger value of $N$ as long as uniform convergence characteristics are satisfied. It should be noted that for a multivariate normal distribution the confidence region is an ellipsoid. Thus, the tolerance $\varepsilon$ is defined in terms of the boundaries of the ellipsoid. As the worst case scenario, one may only consider the largest axis of the ellipsoid which represents the furthest distance a possible solution can be from the true solution.

IV. CASE STUDIES

All case studies were performed on the IEEE 14 bus system [12] with integrated wind turbines on buses 6 and 8 as shown in Fig. 1.

![Image of Modified IEEE 14 bus system with integrated wind farms](image)

Figure 1. Modified IEEE 14 bus system with integrated wind farms.

All the data used is the same as [8]. The variable $c_{OLp}$ was assumed to be Normal $(0.4, \sigma^2)$. It is also possible to use any other distribution or change the parameters. A sample of the optimum solution and the corresponding optimum value for different values of samples size and $\sigma$ is shown in Fig. 2. The SAA function was determined for 1000 different samples and the corresponding histogram is plotted.

![Histograms of optimum value](image)

Figure 2. Distribution of optimum value based on 1000 simulations (a) $N = 20, \sigma = 0.3$, (b) $N = 150, \sigma = 0.3$, (c) $N = 20, \sigma = 1.0$, (d) $N = 150, \sigma = 1.0$.

A. Optimal value

All the distributions in Fig. 2 pass the Kolmogorov-Smirnov (K-S) test for fitting probability distribution functions. The null hypothesis is rejected at a significance level of 0.05 which indicates that a normal distribution is suitable. Each element of $x$ also passes the K-S test which indicates a multivariate normal distribution. The lines on either side represent the 95% confidence limits and the dotted line in the middle represents the mean (true value).

Thus, the uncertainty associated with locational marginal price (LMP) estimates increases if the uncertainty associated with DLR cost is higher. However, choosing a larger sample size can potentially reduce the uncertainty.

B. Optimal Solution

Fig. 3 shows the mean value of different control variables in the power system for $N = 20$ and $\sigma = 0.3$ where $\sigma$ is the standard deviation of $c_{OLp}$. The variation in the 95% confidence interval for different levels of uncertainty in the DLR cost for different sample sizes is also shown as error bars. At bus 1, the generation is unaffected by the variation in DLR cost. However, at bus 2, a slightly higher variation occurs. This is also evident from Fig. 4(a) and 5(a).

![System parameters for N = 20 and \sigma = 0.3](image)

Figure 3. System parameters for $N = 20$ and $\sigma = 0.3$ with 95% confidence margin shown (a) Real power generation, (b) reactive power generation, (c) bus voltage, (d) voltage angle, (e) LMP.
According to Fig. 4(c) and 5(c), at generator buses the voltage appears to be less affected by any variation in DLR cost than load buses. For example, Bus 2 has a value of $\varepsilon$ approximately 100 times less than Bus 10. This is because of the voltage control capability of generator buses being able to maintain the voltage.

V. SENSITIVITY ANALYSIS

Fig. 4 and 5 show the value of $\varepsilon$ as a percentage of the true (mean) value for different values of $\sigma$ and sample size ($N$). This demonstrates the effect of uncertainty in the DLR cost function on the optimal solution and the adjustment in sample size required to maintain the optimal solution at the desired level. Voltage and real power generation at generator buses have a low value $\varepsilon$ and are relatively unaffected by any change in $\sigma$ or sample size. Value of $\varepsilon$ for generation at Bus 8 reaches as high as nearly 2% when $\sigma = 1$ but increasing sample size reduces the value to under 0.01%.

However, for all other parameters such as bus voltage at load buses, reactive power generation and voltage angle, increasing the standard deviation of $c_{OLp}$ leads to a higher value of $\varepsilon$. To maintain the same value of $\varepsilon$, the sample size needs to be increased. For example in Fig. 4(c) the voltage at bus 8 has a 95% confidence limit that is 0.12% of the mean for $\sigma = 0.3$. When $\sigma$ increases to 1, the 95% confidence limit triples to 0.32% of the mean. If the sample size is then increased to 150 (Fig. 5(c)), the confidence limit reduces to 0.13%. Thus, it is likely that the DLR cost function has uniform convergence properties which guarantee an optimum solution if the sample size is sufficiently large.

While there is a decrease in confidence intervals when the sample size is increased, it should be noted that the 95% confidence intervals are overall low (less than 1% for all cases) for this particular system. However, for larger systems the tolerance is likely to be higher and the improvement by using larger sample sizes is expected to be more significant. These results could also indicate that when congestion in a smaller part of the network is considered, the uncertainty in DLR cost does not necessarily affect the scheduling decisions significantly.

The proposed framework can be used in a real time scheduling framework for short scheduling periods ranging from a few minutes up to 30 minutes. For short scheduling periods, it is possible to estimate the risk of thermal overload with a relatively low level of certainty when dynamic line rating is implemented by applying the method given in IEEE Standard 738 – 2012. If the average value of the risk function is considered for scheduling purposes, there is 50% probability that the actual risk function could be higher than this. The probability of the risk function being within a
certain tolerance of the mean can be estimated since a normal distribution is assumed for the function parameters. If the risk of underestimating the thermal overload is to be reduced, a different point in the envelope of risk functions should be chosen. For example, if the upper 90% confidence bound of the risk function is chosen, it will ensure that there is only a 10% probability of underestimating the risk function and overestimating is more likely. The uncertainty associated with the risk can be reduced further if accurate measurement and forecasting of wind speeds and temperature are available. In a smart grid framework, it is expected that there would be adequate monitoring of ambient weather and temperature conditions and availability of accurate forecasts.

VI. CONCLUSION

This paper has proposed a sample average approximation method for accounting for the uncertainty in the risk of thermal overload in scheduling decisions involving dynamic line ratings. Test cases on IEEE 14 bus system showed that if the average DLR risk function is estimated for sample sizes as low as 20 then there is a 95% probability of being within 1% tolerance of the optimal solution. If there is a greater uncertainty in the DLR cost function, a larger sample size may be used to maintain the tolerance at the desired level. It appears that the DLR cost function has uniform convergence properties which always guarantee a high likelihood that the estimated solution is closer to the optimum solution within an adequate number of samples.

In a smart grid framework, availability of accurate, real time forecasts would ensure that uncertainties are minimized and scheduling decisions closely approximate the true requirement. In that context, the proposed method could be used in real time scheduling decisions in congested and high-risk networks with integrated wind plants.

REFERENCES