Mathematical Modeling and Performance Study of Fischer-Tropsch Synthesis of Liquid Fuel over Cobalt-Silica

Nima Moazami, Hamid Mahmoudi, Pooria Panahifar, Kiyarash Rahbar, Athanasios Tsolakis, Miroslaw L Wyszynski

School of Mechanical Engineering, College of Engineering and Physical Sciences, The University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

Abstract

A numerical one-dimensional pseudo-homogeneous mathematical model of a fixed bed reactor for Fischer-Tropsch (FT) synthesis was developed over a simulated nitrogen-rich syngas (33% hydrogen, 17% carbon monoxide and 50% nitrogen (volume basis)), on a cobalt-silica catalyst. An algorithm was developed and the MATLAB codes were written in order to predict the product selectivity (H$_2$O, CO$_2$ and hydrocarbons i.e. CH$_4$, C$_2$, C$_3$, C$_4$ and C$_5$+) and syngas conversion (CO and H$_2$). In order to predict the kinetic parameters, the global search optimization subroutine (from MATLAB Global Optimization) was used. The model was fitted with experimental data at five different operating conditions with respect to conversion and selectivity. Discrimination between the model and the experiments was determined by the mean absolute relative residuals percentage (MARR %) and the value was 13.29%. The Effects of operating conditions such as reaction temperature, total pressure, flow rate and H$_2$/CO molar ratio were investigated on the catalytic performance of the cobalt-silica for synthesis of liquid fuel. The model was studied in the range of 200-260 °C, 1-25 bar, reduced gas flow rate (per unit mass of catalyst) of 2.4-3.6 NL g$^{-1}$ h$^{-1}$ and H$_2$/CO = 1.75-2.75 (mole basis).

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Keywords: Fixed bed reactor; Fischer-Tropsch synthesis; mathematical modeling; renewable energy; kinetic study; biofuel production

* Corresponding author. Tel: +44 121 414 4159; mobile: +44 7968 157 909
E-mail address: M.L.Wyszynski@bham.ac.uk (Miroslaw L. Wyszynski)
1. Introduction

Fischer-Tropsch (FT) synthesis comprises a system of catalytic processes that can be defined as the means to produce renewable, carbon-neutral and sustainable transportation fuels such as gasoline, diesel, jet fuel and other chemicals from synthesis gas. Converting biomass to liquid via FT synthesis has been gaining increasing attention from industry and academia because of the absence of aromatic compounds, sulfur, their high cetane number and its capability to produce globally friendly chemicals and ultra-clean fuels. In general, the synthetic biodiesel has 22% more energy per gallon and 80% less combustion emissions; compared to petroleum diesel [1, 2]. Among different solid catalysts, cobalt is considered the most favorable catalyst for the production of long-chain hydrocarbons due to its high selectivity to linear paraffins, high activity, and high resistance to deactivation [3]. To enhance the performance of the complete FT process, the development of engineering mathematical description of the reactor and kinetics model is necessary, in which detailed selectivity, conversion and product distribution can be presented. A few examples of modeling and kinetic study of a fixed bed FT reactor over cobalt catalyst were reported in the present study [4-10]. No-one has carried out mathematical modeling of a fixed bed reactor with the details of the kinetic model on Co-SiO₂ using a simulated nitrogen-rich syngas.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_j)</td>
<td>pre-exponential factor of reaction ('j'), [mol pa((m+n)) g(_{cat})(^{-1}) s(^{-1})]</td>
</tr>
<tr>
<td>(C_i)</td>
<td>concentration of species ('i'), [mol m(^{-3})]</td>
</tr>
<tr>
<td>(d_p)</td>
<td>average particle diameter, [m]</td>
</tr>
<tr>
<td>(E_j)</td>
<td>activation energy of reaction ('j'), [J mol(^{-1})]</td>
</tr>
<tr>
<td>(M_m)</td>
<td>molecular weight of mixture, [g mol(^{-1})]</td>
</tr>
<tr>
<td>(m)</td>
<td>partial order of the reactant with respect to hydrogen, [-]</td>
</tr>
<tr>
<td>(n)</td>
<td>partial order of the reactant with respect to carbon monoxide, [-]</td>
</tr>
<tr>
<td>(p_i)</td>
<td>partial pressure of species ('i'), [bar]</td>
</tr>
<tr>
<td>(p_T)</td>
<td>total pressure, [bar]</td>
</tr>
<tr>
<td>(R_j)</td>
<td>rate of reaction ('j'), [mol g(_{cat})(^{-1}) s(^{-1})]</td>
</tr>
<tr>
<td>(R_g)</td>
<td>universal gas constant, 8.314 [J mol(^{-1}) K(^{-1})]</td>
</tr>
<tr>
<td>(S_i)</td>
<td>selectivity of species ('i'), [%]</td>
</tr>
<tr>
<td>(T)</td>
<td>reaction temperature, [K]</td>
</tr>
<tr>
<td>(u_s)</td>
<td>superficial fluid velocity, [m s(^{-1})]</td>
</tr>
<tr>
<td>(x)</td>
<td>conversion, [%]</td>
</tr>
<tr>
<td>(\beta)</td>
<td>volume fraction of active site of the solid particles, [-]</td>
</tr>
<tr>
<td>(\delta)</td>
<td>explicit temperature dependence factor of the pre-exponential constant, [-]</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>void fraction, [-]</td>
</tr>
<tr>
<td>(\rho_f)</td>
<td>density of the fluid, [kg m(^{-3})]</td>
</tr>
<tr>
<td>(\rho_B)</td>
<td>density of the bulk catalyst, [kg(_{cat}) m(^{-3})]</td>
</tr>
<tr>
<td>(v_{ij})</td>
<td>stoichiometric coefficient of component ('i') in reaction ('j'), [-]</td>
</tr>
</tbody>
</table>

2. Experimental setup

A seamless stainless steel single micro-structured vertical fixed bed reactor was used with a tube length of 52.83 cm, outer diameter of 1.90 cm and wall thickness of 0.16 cm. An in-house spherical Co catalyst and commercial silica support (Fuji Silysia Chemical Ltd) had the following specifications: silica support bulk density of 0.38 g ml\(^{-1}\); cobalt catalyst with particle diameter; weight, surface area, water content and
pore volume of 0.15 mm, 2.0 g, 257 m² g⁻¹, 0.6 wt% and 1.20 ml g⁻¹, respectively. In this process, a simulated N₂-rich syngas bottle with 33% H₂, 17% CO and 50% N₂ (volume basis) was employed to feed into the reactor inlet. Fig. 1 indicates the Process Path Flow (PPF) of the experimental setup. The reactor was fixed in a tube furnace (with the temperature ranging from 50 to 1100 °C) in order to produce the heat zone. A cast iron jacket was installed between the furnace and reactor to provide the uniform wall temperature. A calibrated smart mass flow controller (Bronkhorst Ltd) was employed to regulate the flow rate of the syngas. The catalytic reactor bed was purged by N₂ bottle and then the catalyst was activated by employing the H₂ gas bottle. The active catalyst was put on stream for 12 hours to determine the conversion of CO, selectivity of CH₄, CO₂, and other HC products over this period of reaction time. The liquid products were separated from the gaseous stream by two liquid/gas separators. The changes in the CO₂ and CO mole fraction were monitored on-line by using a modified CO analyzer (AVL Digas 440). Gas Chromatogram Flame Ionization Detector (GC-FID) (HP 5890) was employed to analyze the HC₁-HC₈ on-line. Gas Chromatogram-Mass Spectrometry (GC-MS) (PerkinElmer™) was also used to analyze the liquid products off-line.

![Diagram of the experimental setup](image)

**Fig. 1.** Schematic diagram of the experimental setup in the present work.

### 3. Development of the reactor model and FT synthesis kinetic

The reaction mechanism of FT synthesis is very complex, whereby thousands of species participate in a highly coupled mode. For this reason, it is not possible to establish a reaction equation for every species by the conventional method used in reaction kinetics studies. In this study, the innumerable chemical compounds, with carbon number higher than five (C₅⁺), in the reaction system are lumped by C₆.0₅H₁2.3₆. In order to derive the reaction kinetics model, the production rate of each species was assumed to be of the power-law form, as a function of the partial pressure of CO and H₂ components. Reaction Eqs. (1) to (8) are the proposed chemical reactions for FT synthesis over Co-SiO₂ in the present study. Table 1 represents the reaction kinetic parameters estimated by the model on the basis of experimental data.
\[
\begin{align*}
\text{CO} + 3\text{H}_2 & \rightarrow \text{CH}_4 + \text{H}_2\text{O} \quad (1) \\
2\text{CO} + 4\text{H}_2 & \rightarrow \text{C}_2\text{H}_4 + 2\text{H}_2\text{O} \quad (2) \\
2\text{CO} + 5\text{H}_2 & \rightarrow \text{C}_2\text{H}_6 + 2\text{H}_2\text{O} \quad (3) \\
3\text{CO} + 7\text{H}_2 & \rightarrow \text{C}_3\text{H}_8 + 3\text{H}_2\text{O} \quad (4) \\
4\text{CO} + 9\text{H}_2 & \rightarrow \text{n-C}_4\text{H}_{10} + 4\text{H}_2\text{O} \quad (5) \\
4\text{CO} + 9\text{H}_2 & \rightarrow \text{i-C}_4\text{H}_{10} + 4\text{H}_2\text{O} \quad (6) \\
6.05\text{CO} + 12.23\text{H}_2 & \rightarrow \text{C}_{6.05}\text{H}_{12.36} (\text{C}_5^+) + 6.05\text{H}_2\text{O} \quad (7) \\
\text{CO} + \text{H}_2\text{O} & \rightarrow \text{CO}_2 + \text{H}_2 \quad (8)
\end{align*}
\]

Table 1 Adjusted kinetic parameters of FT synthesis over Co/SiO₂

<table>
<thead>
<tr>
<th>Reaction equation</th>
<th>Eq. 1</th>
<th>Eq. 2</th>
<th>Eq. 3</th>
<th>Eq. 4</th>
<th>Eq. 5</th>
<th>Eq. 6</th>
<th>Eq. 7</th>
<th>Eq. 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_j )</td>
<td>1.04</td>
<td>-2.13</td>
<td>2.21</td>
<td>1.58</td>
<td>-1.03</td>
<td>-1.82</td>
<td>1.62</td>
<td>2.5</td>
</tr>
<tr>
<td>( m_j )</td>
<td>-0.27</td>
<td>0.59</td>
<td>-1.42</td>
<td>-0.8</td>
<td>1.5</td>
<td>2.12</td>
<td>-0.59</td>
<td>-2.05</td>
</tr>
<tr>
<td>( \delta_j )</td>
<td>-0.86</td>
<td>0.01</td>
<td>-0.71</td>
<td>-0.13</td>
<td>1</td>
<td>1</td>
<td>-0.21</td>
<td>1</td>
</tr>
<tr>
<td>( a_j )</td>
<td>6.56 \cdot 10^{-1}</td>
<td>4.54 \cdot 10^{-2}</td>
<td>1.14 \cdot 10^{-3}</td>
<td>1.19 \cdot 10^{-6}</td>
<td>1.84 \cdot 10^{-1}</td>
<td>6.59 \cdot 10^{-9}</td>
<td>1.98 \cdot 10^{-4}</td>
<td>3.78 \cdot 10^{-3}</td>
</tr>
<tr>
<td>( E_j )</td>
<td>8.35 \cdot 10^{-4}</td>
<td>6.50 \cdot 10^{-4}</td>
<td>4.98 \cdot 10^{-4}</td>
<td>3.49 \cdot 10^{-4}</td>
<td>2.77 \cdot 10^{-4}</td>
<td>2.57 \cdot 10^{-4}</td>
<td>2.36 \cdot 10^{-4}</td>
<td>5.88 \cdot 10^{-4}</td>
</tr>
</tbody>
</table>

A one-dimensional single-phase pseudo-homogeneous mathematical model was developed to simulate a single fixed bed FT reactor. A steady state operation was assumed so that there was not any change over the time including catalytic activity, selectivity and stability. The model describes the axial profiles of radially averaged concentrations, partial pressure, feed conversion and selectivity accounting for twelve species at each operating condition. The heat transfer in this model was assumed to be isothermal. The partial pressure gradient with respect to the axial distance along the reactor length was calculated by conservation of species in Eq. (9). The continuity equation was employed to demonstrate the gas velocity profile (Eq. (10)). The chain rule was applied to the ideal gas law in order to compute the fluid density changes (Eq. (11)) in the axial dimension. The overall pressure drop (Eq. (12)) was calculated by the classical Ergun law for the fluid flow. The CO conversion was calculated by Eq. (13) and the selectivity of CO₂ and hydrocarbon products were determined by Eqs. (14) to (16). Also, Eq. (17) is the classic Arrhenius equation which is developed in order to determine the rate of consumption CO and H₂, as well as production of species. The error obtained between the model and experiential data was calculated by Eq. (18).

\[
\frac{u_s}{R_g T} \frac{d\rho_f}{dz} = \rho_g \frac{\sum_{j=1}^{N_R} v_{ij} R_j}{R_g T} - \left( \frac{\rho_T}{R_g T} \frac{du_s}{dz} \right) \quad (9)
\]

\[
\frac{du_s}{dz} = -\frac{d\rho_f}{dz} \quad (10)
\]

\[
\frac{d\rho_f}{dz} = \frac{\frac{d\left( \frac{\rho_T M_m}{R_g T} \right)}{dz}}{\frac{M_m}{R_g T}}, \quad \frac{1}{dz} \quad (11)
\]

\[
\frac{dp_T}{dz} = -\frac{1}{\varepsilon^3} \frac{36(25/6) u_s^2 \rho_f}{Re \frac{d\rho_f}{dz}} \quad (12)
\]

\[
x_i = \frac{C_{\text{input},i} - C_{\text{output},i}}{C_{\text{input},i}} \times 100 \quad (13)
\]
4. Results and discussion

4.1. Model prediction

Five sets of experimental conditions at different reaction temperature, inlet total pressure and GHSV were performed to determine the accuracy of the mathematical model and developed kinetic equations. Fig. 2 shows the predicted results and the corresponding measured data by the experiment for the proposed FT species CO, CO₂, CH₄, C₂, C₃, C₄ and C₅+. A parity plot (Fig. 3) indicates the accuracy of the prediction in terms of conversion and selectivity (Eq. (13) - (16)). The discrimination of the results between the model and experiments was based on the mean absolute relative residuals percentage (MARR %) error (Eq. (16)). Among the different techniques, the global search method was utilized from MATLAB Global Optimization to minimize the MARR, by adjusting the kinetic parameters. The model was fitted against the experiments satisfactorily with MARR = 13.29%.

![Fig. 2. Comparison of the results between model predictions and measured data with respect to CO conversion and selectivity of CO₂, CH₄, C₂, C₃, C₄ and C₅+. Exp. No. 1 at 230 °C, 15 bar and 2.4 NL gcat⁻¹ h⁻¹; Exp. No. 2 at 230 °C, 20 bar and 3 NL gcat⁻¹ h⁻¹; Exp. No. 3 at 230 °C, 25 bar and 3.6 NL gcat⁻¹ h⁻¹; Exp. No. 4 at 245 °C, 10 bar and 2.4 NL gcat⁻¹ h⁻¹; Exp. No. 5 at 255 °C, 10 bar and 3 NL gcat⁻¹ h⁻¹.](image-url)
Fig. 3. Parity plot of experimental values versus calculated values for a cobalt-silica catalyst. FT synthesis species including CO conversion, CO₂, CH₄, C₂-C₄ and C₅⁺ selectivity accounting for all experimental conditions.

4.2. Effects of operating conditions

4.2.1. Effects of H₂/CO feed molar ratio and reaction temperature on conversion and selectivity

The impacts of reaction temperature and the H₂/CO molar ratio on syngas conversion, selectivity of HCs and CO₂ were demonstrated in Figs. 4 (a), (b), (c) and (d), assuming the temperature and H₂/CO ratio in the range of 200 to 257 °C and 1.75 to 2.75 (mole basis), respectively. As it can be seen, the temperature has positive effect on the CO and H₂ conversions (generally syngas conversion); whereas H₂/CO molar ratio has negative impact on total conversion. An increase in the H₂/CO molar ratio from 1.75 to 2.75 reduces the CO conversion and dwindles the CO partial pressure gradient. Decrement of CO partial pressure causes a lower concentration of adsorbed CO, so that more H₂ can be adsorbed and dissociated. As it was expected, the lower concentration of CO leads to a decrease in CO conversion from 47.23 to 36.89 % at 207 °C and from 98.55 to 62.99 % at 257 °C. Decreasing of CO concentration leads to loss of catalytic activity and therefore it causes a decrease in heavy hydrocarbon formation. Therefore it can be concluded that decreasing the H₂/CO molar ratio is favorable due to its effect on conversion of syngas. In contrast, when H₂/CO ratio decreases, the CH₄ selectivity increases and its effect on C₅⁺ formation is not considerable. Since the objective is to minimize the CH₄ formation while maximizing CO, H₂ conversion, and C₅⁺ selectivity, therefore further investigation on the effects of other operating conditions were carried out to improve the rate of production and conversion.
Fig. 4. (a) Effects of the H\textsubscript{2}/CO feed stream molar ratio and the reaction temperature on catalytic performance based on CO conversion and CH\textsubscript{4} selectivity at \( p = 10 \) bar and GHSV = 3 NL g\textsubscript{cat}\(^{-1}\) h\(^{-1}\). (b) \( \text{H}_2 \) conversion and \( \text{C}_{5+} \) selectivity under the same conditions in (a). (c) Effects of the reaction temperature on catalytic performance based on CO and \( \text{H}_2 \) conversion, CH\textsubscript{4}, \( \text{C}_{5+} \) and CO\textsubscript{2} selectivity at \( p = 15 \) bar, GHSV = 3 NL g\textsubscript{cat}\(^{-1}\) h\(^{-1}\) and \( \text{H}_2/\text{CO} = 2/1 \) (mole basis). (d) Effects of the reaction temperature on \( \text{C}_2-\text{C}_4 \).

4.2.2. Effects of total pressure on conversion and selectivity

FT synthesis can operate at both low and high pressure. At atmospheric pressure, the hydrocarbons are typically in the gaseous state. However, by increasing the total pressure they tend to condense into a liquid; whereas it has a negative influence on CO and \( \text{H}_2 \) conversion and this attribute was satisfactory outlined in Figs. 5 (a) and (b). An increase in total pressure can be favorable due to its effects on the enhancement of liquid fuel productions. CO and \( \text{H}_2 \) conversion reduced by 29.2% and 26.5% respectively with an increase of total pressure from 1 to 25 bar. Following changes in total pressure caused a
decrement of CO$_2$ and C$_2$-C$_4$ selectivity by 6.5% and 8.5% respectively and an increase of 10.3% in C$_{5+}$ selectivity. It was clear that the reduction of lighter biofuels’ selectivity was due to an increase in the fraction of the liquid products (C$_{5+}$).

Fig. 5. (a) Effects of the total pressure on catalytic performance based on CO and H$_2$ conversion, CH$_4$, C$_{5+}$, and CO$_2$ selectivity at 227 °C, GHSV = 3 NL g$_{cat}^{-1}$ h$^{-1}$ and H$_2$/CO = 2/1 (molar basis), (b) Effects of the total pressure on C$_2$-C$_4$ selectivity.

Fig. 6. (a) Effects of the Gas Hourly Space Velocity (GHSV) on catalytic performance based on CO and H$_2$ conversion, CH$_4$, C$_{5+}$, and CO$_2$ selectivity at 227 °C, p = 15 bar and H$_2$/CO = 2/1 (molar basis), (b) Effects of the GHSV on C$_2$-C$_4$ selectivity.

4.2.3. Effects of GHSV on conversion and selectivity

Figs. 6 (a) and (b) show the impact of GHSV in the range of 1.8-3.6 NL g$_{cat}^{-1}$ h$^{-1}$. Changes in the GHSV had a substantial impact on the CO and H$_2$ conversion because the flow rate corresponded to the inverse of the residence time. Decreases in residence time or enhances of flow rate cause a strong
decrement of CO and H₂ conversion. However, the flow rate did not affect the molecular weight of the hydrocarbons (i.e. CH₄, C₂-C₄ and C₅+) that were produced using a cobalt-silica catalyst. The results indicate that at GHSV = 1.8 NL g⁻¹ h⁻¹, the same reaction temperature and total pressure, the optimal catalyst showed a CO conversion of 86.1% and overall selectivity of 76.8% for heavy hydrocarbons (C₅+).

5. Conclusion

A numerical one-dimensional pseudo-homogeneous fixed bed FT synthesis was modeled on a Co-SiO₂ catalyst over simulated N₂-rich syngas. The results predicted by the model were in agreement satisfactorily with the experiments in terms of conversion and selectivity, with the mean absolute relative residuals (MARR) of 13.29%. The results show an increase of the reaction temperature by 57 °C enhanced the catalyst activity in terms of CO and H₂ conversion by up to 50%; whereas its effect on the synthetic bioliquid selectivity was relatively negative, so that the reduction of 20.8% was observed. This reduction was due to an enhancement of the methane selectivity from 7.8 to 27.8%; since an increase in reaction temperature causes a decrease in CO hydrogenation, an increase of the degree of carbidization of the surface and willingness of the surface species to desorb instead of propagating to heavier hydrocarbons. The increased total pressure from 1 to 25 bar resulted in a 6.5 and 8.5% reduction of CO₂ and C₄ selectivity, respectively and an increase of 10.3% in C₅+ selectivity. It was pointed out that at GHSV = 1.8 NL g⁻¹ h⁻¹ a high CO and H₂ conversion were achieved (86.1 and 93.7%, respectively) while C₅+ total selectivity was enhanced up to 76.8%. An increase in H₂/CO molar ratio from 1.75 to 2.75 reduces the CO conversion and decreases the CO partial pressure gradient. Decrement of CO partial pressure and its conversion rate causes an increase of H₂ adsorption and dissociation. Consequently, CO conversion was raised from 36.89 to 47.23% at 207 °C and from 62.99 to 98.55% at 257 °C, by decreasing the molar ratio from 2.75 to 1.75.

References

Biography

Nima Moazami was born in Tehran, Iran, in 1988. He received his MEng degree in Mechanical Engineering by achieving First-Class Honors from the University of Birmingham, Birmingham, UK, in 2011. He has been awarded The University of Birmingham School Full-Scholarship to fund his Ph.D. research programme with supervision from Prof. Miroslaw L. Wyszynski since November 2011.