LABOR MARKET DYNAMICS WITH SEARCH FRICTIONS AND FAIR WAGE CONSIDERATIONS*

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Abstract

Fairness considerations in wage setting can improve the ability of the Diamond-Mortensen-Pissarides search and matching model to account for U.S. labor market dynamics. Firms’ production is influenced by workers’ effort input, which depends on whether workers consider the employment relation as fair. A typical worker’s effort is determined in a comparison of individual current wage with wage norms, including the outside option, the individual past wage, and the wage level in the steady state. The fairness considerations in the search framework give rise to endogenous real wage rigidity, and realistic volatilities of unemployment, vacancies, and labor market tightness.

JEL Classification: E24, E32, J64.

Keywords: Search and matching frictions; Fair wage; Real wage rigidity; Unemployment volatility

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*We thank Thomas Laubach, Mirko Wiederholt, Ester Faia, and Stéphane Moyen for helpful discussions and suggestions. Wang acknowledges financial support from "The Fundamental Research Funds of Shandong University" (grant no. 2014HW003). Any remaining errors are our own.

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I INTRODUCTION

In the last two decades, the Diamond-Mortensen-Pissarides search and matching model (DMP model henceforth) has been extensively used to model the labor market.\(^1\) However, as pointed out in Shimer (2005), the DMP model produces much smaller volatilities of unemployment and labor market tightness, but more volatile wages compared with the U.S. data, known as the Shimer puzzle. This paper contributes to the resolution of the Shimer puzzle by proposing effort and fairness concerns in wage determination in a search framework. According to the fair wage hypothesis à la Akerlof (1982) and Akerlof and Yellen (1990), fair-minded workers compare individual current wage with wage norms to evaluate the level of firms’ generosity and reciprocate by providing more effort; firms unilaterally set wage taking into account of workers’ effort decision.

This fair wage consideration, which replaces wage determination by Nash bargaining in the standard DMP model, is the key element that improves the model performance. When deciding effort input, workers employ the aggregate wage in the economy, individual past wage, and the steady state wage rate as reference norms. In response to a positive technology shock, the marginal product of labor increases. Firms find it beneficial to hire more workers and raise wages, in line with wage hikes in other firms. However, as workers refer to past wage in effort consideration, an increase in current wage raises workers’ reference wage level in the following period. The higher future reference wage generates a negative impact on future effort and consequently on future output. The opposite dynamic effects lower the benefit of a marginal change in wage, and firms are therefore reluctant to make large wage adjustments. Meanwhile, the steady state wage is not varying with current economic conditions, acting like an “anchor” of reference wage norm. As a result, firms tend to set wage in line with its level in the long-run equilibrium. Wage rigidity arises

endogenously as a result of effort considerations.

As labor productivity increases and wage responds only moderately, firms’ surplus resulting from hiring responds strongly to the state of technology. In sticky wage models, the difference between labor productivity and wage is named "the fundamental surplus" in Ljungqvist and Sargent (2015) (LS henceforth), which governs the amount of resources used to post vacancies. We find that the procyclical fundamental surplus that is sensitive to technology in our model gives rise to a volatile amount of resources devoted to hiring over business cycles. As a consequence, the fair wage model produces high volatilities of vacancies, unemployment and labor market tightness, consistent with empirical evidence.

The current study is connected to the literature proposing potential solutions to the Shimer puzzle. LS show that a small fundamental surplus ratio (fundamental surplus relative to labor productivity) is the common channel that generates large volatilities of vacancy and tightness in several variants of the DMP models.2 Our model also benefits from a small fundamental surplus ratio, but we find that wage rigidity increases the sensitivity of fundamental surplus to a technology shock and improves the model performance in generating volatile tightness and vacancy. Meanwhile, the literature proposing wage rigidity to solve the Shimer puzzle presents divided results.3 On the one hand, Rudanko (2009) and Costain and Jansen (2010) show that wage rigidity is not synonymous with high volatilities of unemployment and vacancies. On the other hand, Kennan (2010) incorporates private information of productivity and improves the empirical performance of the DMP model to account for the labor market dynamics. Our model falls in the second strand

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2Hagedorn and Manovskii (2008), Hall (2005), Wasmer and Weil (2004), and Hall and Milgrom (2008).

3There has been a debate on the relevant measure of wage rigidity and whether wage rigidity in the aggregate level is the right answer to the Shimer puzzle. Pissarides (2009) argues that the volatile wages of new matches influence the job creation, and wage rigidity in on-going jobs is not relevant. However, the higher flexibility of new hires’ wages is challenged by Gertler and Trigari (2009), who show that the wages of new hires are not more cyclical after controlling for compositional effects. Hence the empirical evidence on the degree of wage rigidity of new hires is controversial.
of literature by showing that wage rigidity from fair wage setup helps to solve the Shimer puzzle.

The rest of the paper is organized as follows. Section II reviews the fair wage hypothesis and its development. Our model is introduced in Section III and calibrated to the U.S. economy in Section IV. We explore the mechanism of the model in Section V. Section VI reports impulse responses to a technology shock and statistics moments of the model. Section VII presents robustness checks and Section VIII concludes.

II Literature on fair wage hypothesis

Originally illustrated by Akerlof (1982), the partial gift exchange model raises the point that by offering workers the gift of a wage rate above some reference norms, the firm anticipates workers’ higher effort in work in return. The "exchange of gifts" is argued to be voluntary and cannot be determined in an incomplete labor contract. Effort provided by workers affects labor productivity and firms’ output. Therefore, firms find it desirable to elicit the optimal level of effort through wage setting. According to the fair wage hypothesis, what matters to a worker is not only the level of individual current wage, but also whether it is "fair" in comparison with some reference level. Fehr, Goette, and Zehnder (2009) provide a survey of abundant laboratory and field evidence that supports fairness concerns in employment relations.

Danthine and Donaldson (1990) make the first attempt to incorporate the gift exchange framework in a Real Business Cycle (RBC henceforth) model to explain the wage-employment puzzle. Their model does not generate rigid wage as past wage is not included in the reference wage norm. Collard and de la Croix (2000) modify the effort function in Danthine and Donaldson (1990), em-

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4 According to Fehr and Falk (1999), an incomplete labor contract implies "the obligations of the employer and employee are not specified in each possible state of the world" (p. 109).
phasizing the role of past wage in reference norms. They show that the intertemporal wage comparison in effort considerations improves the ability of RBC model to account for the labor market fluctuations. In a New Keynesian setup, Danthine and Kurmann (2004) generate rigid wages and improve the ability of internal propagation of shocks when fair wage plays a part. These papers, in the absence of search frictions, do not study cyclical fluctuations of vacancy and tightness. In contrast, this is a main theme of our paper which incorporates fair wage considerations into a search and matching model.

In a game-theoretical model, Eliaz and Spiegler (2013) combine reference-dependence wage setting motivated by reciprocity considerations with search and matching frictions to model downward wage rigidity. Their study also sheds light on the Shimer puzzle, as the volatility of labor market tightness is higher. However, the model doesn’t provide any quantitative results due to its qualitative approach.

III THE MODEL

In an RBC framework, we modify the DMP model by incorporating fairness considerations in labor relations. To keep the analysis of labor market simple and transparent, our model abstracts from capital accumulation and frictions on other markets following Galí and van Rens (2014).

3.1 Households

The economy is populated by a large number of households, uniformly distributed on the unit interval $[0, 1]$. Each household is thought of as a very large family, containing a continuum of infinitely lived members represented by the unit interval. Some of the household members are employed, while others are unemployed and searching for jobs. Households are identical in that
employment is assumed to be randomly allocated, and the fraction of employed members is equal across households. Each period, household members pool together their income and enjoy equal amount of consumption no matter they are employed or not.

The representative household \( j \) derives utility from consumption and disutility from providing effort in work. The household chooses consumption and effort to maximize the expected discounted lifetime utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t(j) - n_t(j)G(e_t(j)) \}
\]

(1)

where \( c(j) \) is the level of consumption of household \( j \), \( n(j) \) is the fraction of household members who are employed and \( e(j) \) is the effort provided by a working member in the household.

The specification of household’s utility differs from the standard setup in the RBC literature, which features consumption-leisure trade-off and intertemporal labor supply. Consistent with the fair wage literature,\(^5\) we assume that household members supply labor inelastically, and the disutility of working is replaced by utility loss from providing effort. The choice of effort and consumption is separable to ensure that effort is independent of wealth. The disutility of effort stems from the difference between the effort level provided by worker \( (e_t(j)) \) and an evaluation of firm’s generosity \( (g(w_t(j), \cdot)) \):\(^6\)

\[
G(e_t(j)) = (e_t(j) - g(w_t(j), \cdot))^2
\]

(2)

while the dot represents wage norms that the worker refers to. Workers’ perception of firm’s generosity \( (g(w_t(j), \cdot)) \) increases in individual current wage and decreases in wage norms.

In line with the partial gift exchange hypothesis, we can view \( g(w_t(j), \cdot) \) as the gift offered by


\(^6\)This line of argument follows Danthine and Kurmann (2010).
the firm. A higher individual wage than reference wage norms implies a generous gift from the firm. Utility maximization and separability between consumption and effort dictate that $e_t(j) = g(w_t(j), \cdot)$ as the optimal choice. Intuitively, to reward firm’s good will, the worker chooses to provide an effort level that is appropriate for the wage offer as a gift back. Though workers dislike effort, they are willing to provide effort to the extent that they feel well treated by the firm. The satisfaction from returning firm’s favor offsets the disutility of providing effort per se, and the optimal decision on effort brings no adverse effect on the utility level ($G(e_t(j)) = 0$).

Similar with Collard and de la Croix (2000), the worker evaluates his wage according to the following equation:  

$$g(w_t(j), \cdot) = \gamma_0 + \gamma_1 \log \frac{w_t(j)}{n_t w_t} + \gamma_2 \log \frac{w_t(j)}{\sqrt{w_{t-1}(j)/\bar{w}}}$$

(3)

where $\gamma_0$ is a scale parameter, and $\gamma_1$ and $\gamma_2$ are both positive. $n_t$ is the employment rate in the economy, and $w_t$ is the aggregate wage level. $w_{t-1}(j)$ is the individual wage in period $t - 1$ and $\bar{w}$ is the steady state wage rate.

$n_t w_t$ can be interpreted as a measure of outside option, or the average earning a worker gets if he quits the current job. The comparison with aggregate wage reflects the idea that the worker cares about not only his absolute wage level, but also his relative income level. If the worker’s wage is higher than the average level, he is more motivated to work because he feels like reciprocating the generous wage offer with high effort. If his own wage is lower than the aggregate wage in the economy, the worker is discouraged to work hard because he feels being treated unfairly. The important role of relative income in effort determination is supported by Clark et al. (2010) among

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$^7$Section 7.3 considers a generalized power effort function employed in de la Croix, et al. (2009) rather than logarithmic form. Section 7.4 replaces steady state wage in equation (3) by a weighted average of past wages.
Moreover, the worker compares his current wage rate with the mean of individual past wage\textsuperscript{8} and the wage level in the steady state.\textsuperscript{9} The comparison with the past wage reflects the role of wage changes in determining worker’s effort level. Research reporting firms’ reluctance of wage cuts in consideration of workers’ morale includes Fehr and Falk (1999) and Bewley (1998) among others. Elsby (2009) points out that downward wage rigidity also gives rise to a compression of wage increase, as firms realize that the wage increase is irreversible to some extent. In our model, a wage raise is considered as a reward and improves the worker’s intrinsic motivation in work. By contrast, a wage cut is interpreted as a punishment, which hurts workers’ morale and dampens their effort input.

Meanwhile, the reference to the steady state wage reflects the worker’s concern of the relative level of wage to its long-run equilibrium. Holding everything else equal, if the current wage is higher than the steady state wage level, the worker is more willing to work hard. In a similar vein, characterizing the wage determination in a demand game, Hall (2005) adopts a constant wage rule, which lies in the bargaining set but is insensitive to employment conditions. He interprets the constant wage rate as a social consensus or a focal point.\textsuperscript{10}

In evaluating the wage offer, the worker makes the comparison in two dimensions. On the

\textsuperscript{8} Though no individual past wage is available for workers who just get employed in period \( t \), we assume that a typical new hire has a hypothetical level of past wage, which the firm has full knowledge of. For example, a new worker forms the hypothetical reference level according to his past wage when he was employed, or the past wage rate of the position he fills. Therefore, we assume no distinction between the reference wage norms for the new hires and existing workers.

\textsuperscript{9} Another perspective on this term is to consider a worker who is an adaptive learning agent, using the lagged-expectation as workers’ reference point (Kőszegi and Rabin, 2006; Eliaz and Spiegler, 2013). In period \( t - 1 \), the worker forms the expectation of period \( t \) wage level based on the most recent individual wage (\( w_{t-1}(j) \)) and the long-run average wage level (\( \bar{w} \)). A realized wage level \( w_t \) higher than the expected wage is viewed as a pleasant surprise and motivates the worker to work hard. A wage level falling short of his expectation is viewed as a disappointment and suppresses worker’s effort.

\textsuperscript{10} The key difference of Hall (2005) from our model is that Hall’s wage rate has no impact on labor productivity.
one hand, the comparison with current aggregate wage level is a contemporary comparison, and
the reference wage norm varies with current productivity and labor market conditions. On the
other hand, the comparison with individual past wage and average wage is dynamic and history-
dependent, and the intertemporal link provides a backward-looking channel of wage determination.
This feature can potentially explain the strong auto-correlation of wages in the U.S. data.\footnote{Using an HP filter with smoothing parameter $10^{-5}$, Silva and Toledo (2009) calculate the degree of auto-correlation of wages is 0.907 based on U.S. data between 1951 and 2003.} Since
workers’ perception of fairness depends on how wage compares to its past and average level,
firms tend to smooth the wage path in order to avoid adverse effects on effort, which leads to
strong dependence of wage on its past level and endogenous wage persistence. The intuition of
this formulation is similar with habit formation of consumption (Fuhrer, 2000): the response of
consumption to various shocks is gradual and sluggish when the utility of a consumer does not
only depend on its current consumption level, but also how it compares to his past consumption
history.

The budget constraint of the representative household is

\begin{equation}
    c_t(j) \leq w_t(j)n_t(j) + \Pi_t(j) \tag{4}
\end{equation}

\( w_t(j)n_t(j) \) is labor income earned by the employed members, and \( \Pi_t(j) \) denotes profits earned by
the household as the owners of firms.

The representative household chooses consumption and effort to maximize the expected dis-
counted life-time utility (1), while the disutility of effort is defined in (2), subject to the budget
constraint (4). The optimization problem is also subject to the nonnegativity conditions \( c_t(j) \geq\)
0, \( e_t(j) \geq 0 \). First-order conditions with respect to consumption and effort are

\[
1/c_t(j) = \lambda_t
\]

\[
e_t(j) = \gamma_0 + \gamma_1 \log \frac{w_t(j)}{n_tw_t} + \gamma_2 \log \frac{w_t(j)}{\sqrt{w_{t-1}(j)\sqrt{w}}}
\]

where \( \lambda_t \) is the Lagrange multiplier associated with the budget constraint. Workers provide effort according to equation (6).

### 3.2 Search and matching frictions

The labor market is subject to search and matching frictions. The total labor force is normalized to 1. In period \( t \), a fraction \( n_t \) of the labor force is employed, and the remaining fraction \( u_t \) is unemployed: \( u_t = 1 - n_t \). A fraction \( \rho \) of employment relationships are terminated exogenously in each period. In order to form new employment relationships, firms need to post vacancies \( (v_t) \), which incurs vacancy-posting costs \( \kappa \). There is a constant return to scale matching technology, which pairs unemployed workers with vacancies to generate new matches, \( m_t = \overline{m} v_t^\nu u_t^{1-\nu} \). \( \overline{m} \) is a scale factor, representing the state of matching technology. \( \nu \in (0, 1) \) is the elasticity of matches with respect to vacancies. Labor market tightness \( (\theta) \) is defined as the ratio of vacancies to unemployment, \( \theta_t = v_t/u_t \). The matching probability for vacancies is \( q_t = m_t/v_t = q(\theta_t) \), which is a decreasing function of the labor market tightness. Firms are less likely to fill their vacancies in a tighter labor market. The job finding rate for unemployed workers is \( s_t = m_t/u_t = \theta_t q(\theta_t) \), and it increases with \( \theta_t \). Job-seekers are more likely to find jobs in a tighter labor market.
The law of motion of aggregate employment is:

\[ n_t = (1 - \rho)n_{t-1} + m_t \]  

(7)

Employment in period \( t \) is employment from last period net of separation, plus new matches \( m_t \) in the same period.

3.3 Firms

There are a large number of identical firms on the unit interval \([0, 1]\). The production function of a representative firm \( i \) is

\[ y_t(i) = A_t[e_t(i)n_t(i)]^{1-\alpha} \]  

(8)

where \( A_t \) represents the aggregate technology, and \( e_t(i) \) is effort provided by firm \( i \)'s workers. \( A_t \) is assumed to follow a stationary stochastic process \( \log(A_t) = \rho_a \log(A_{t-1}) + \varepsilon_t \), with \( \rho_a < 1 \). \( \varepsilon_t \) is a Gaussian white noise with \( E(\varepsilon_t) = 0 \) and \( E(\varepsilon_t^2) = \sigma_a^2 \). Labor input \( (n_t(i)) \) is augmented by effort \( (e_t(i)) \) supplied by workers, and therefore workers’ effort affects the production level directly. Due to the presence of effort, equation (8) differs from the standard production function, which normally ignores the effort margin of labor input.

After the firm and a potential worker meet, the firm makes a take-it-or-leave-it wage offer. The labor contract between the firm and workers is incomplete in the sense that the firm cannot contract on workers’ effort input. However, the firm understands that workers supply effort according to equation (6), and elicits the desired effort level through wage setting. Therefore, the firm makes the optimal wage decision given workers’ effort function.
The representative firm $i$ chooses \{${v}_t(i), n_t(i), w_t(i)$\}$_{t=0}^{\infty}$ to maximize its expected discounted profit flow, subject to the law of motion of employment and household’s effort decision.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t (\lambda_t/\lambda_0)[y_t(i) - w_t(i)n_t(i) - \kappa v_t(i)]$$

s.t. $n_t(i) = (1 - \rho)n_{t-1}(i) + v_t(i)q(\theta_t)$

$$e_t(i) = \gamma_0 + \gamma_1 \log\frac{w_t(i)}{\bar{w}} + \gamma_2 \log\frac{w_t(i)}{\sqrt{w}}$$

The first-order optimality conditions are as follows:

$$w_t(i) : (1 - \alpha)\frac{y_t(i)}{e_t(i)}\frac{\gamma_1 + \gamma_2}{w_t(i)} = n_t(i) + (1 - \alpha)\frac{\gamma_2}{2} \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}(i)}{e_{t+1}(i)w_t(i)}$$ (9)

$$v_t(i) : \kappa/q(\theta_t) = \chi_t$$ (10)

$$n_t(i) : \chi_t = (1 - \alpha)\frac{y_t(i)}{n_t(i)} - w_t(i) + \beta(1 - \rho)E_t \frac{\lambda_{t+1}}{\lambda_t} \chi_{t+1}$$ (11)

$\chi_t$ is the Lagrange multiplier on the law of motion of employment, representing the marginal value of a worker.

Equation (9) gives the optimal wage under fairness considerations. The left-hand side of the equation is the marginal benefit of an increase in wage, and the right-hand side is the marginal cost. When the firm raises its wage, the marginal effect on effort level is $(\gamma_1 + \gamma_2)/w_t(i)$, which is influenced by the size of $\gamma_1 + \gamma_2$. The marginal influence of a change in effort on current output is measured by $(1 - \alpha)[y_t(i)/e_t(i)]$. Thus the increase in wage leads to higher current production, measured by the left-hand side of equation (9). This is an outcome resulting from reciprocity: workers devote more effort when individual wage is higher, and output is higher as a result. On the negative side, the firm increases its wage bill, captured by the first term on the right hand side.
In the meantime, the firm also takes into account the adverse effect of a higher wage on future production: a higher wage today raises worker’s reference compensation level in the next period. Holding everything else constant, a higher wage in period $t$ leads to lower effort in period $t + 1$, and dampens the level of output in period $t + 1$, captured by the second term on the right hand side. Firms understand that a higher current wage makes it more difficult to provide incentives for workers in the future. The intertemporal view of wage setting discourages the firm from making large wage changes.

Equation (10) gives the optimal decision for vacancy-posting. It equates the expected cost to fill one vacancy ($\kappa/q(\theta_t)$) to the marginal value of the worker ($\chi_t$). Equation (11) shows that the marginal value of an additional worker is the marginal product of an additional worker net of wage payment, plus the discounted future value of the worker if the employment relationship survives in the next period with probability $1 - \rho$.

Combining equation (10) and (11) to eliminate the Lagrange multiplier, we arrive at the job-creation condition:

$$\kappa/q(\theta_t) = (1 - \alpha)(y_t/n_t) - w_t + \beta(1 - \rho)E_t(\lambda_{t+1}/\lambda_t)[\kappa/q(\theta_{t+1})]$$

(12)

It states that the expected cost to fill one vacancy ($\kappa/q(\theta_t)$) equals the benefits of an additional worker, including the current benefits (marginal product of labor net of wage payment) and savings from avoiding future vacancy-posting. When the level of technology increases, it is more profitable for firms to post vacancies as the marginal gain of hiring an additional worker is larger than wage payment. Labor market becomes tighter and the vacancy-filling rate ($q(\theta_t)$) drops. Firms continue to post vacancies until the rising expected cost to fill a vacancy exhausts all surplus. The
equilibrium in the labor market is restored as a consequence. This equation is the key condition that governs firm’s job creation decision, which we will examine carefully in Section V.

The model is closed by imposing the resource constraint: $y_t = c_t + k_t v_t$. It states that output is either consumed or used to post vacancies.

IV CALIBRATION

The model is calibrated to match features of U.S. economy at the quarterly frequency. We set the discount rate $\beta$ to 0.99, implying a real interest rate of 1 percent per quarter at the steady state. We set $1 - \alpha = 2/3$ to match the labor’s share in aggregate output. On the labor market front, the quarterly separation rate is 0.1, corresponding to average monthly job separation rate of about 3.4 percent as in the Job Openings and Labor Turnover Survey. We set the matching elasticity with respect to vacancies ($\nu$) to 0.5, which falls in the plausible range proposed by Petrongolo and Pissarides (2001). Following den Haan, Ramey and Watson (2000), the quarterly matching probability of the firm is 0.7 in steady state. The steady state unemployment rate is 5.7%, matching the average unemployment rate in the U.S. between 1951 and 2003. The steady state values of matches and vacancies are determined endogenously.

Coefficients in the effort function (6) are crucial in determining the labor market dynamics. By combining the firm’s optimal wages equation (9) and job-creation condition (12) in steady state, we get an equation relating the parameters in the effort function with the vacancy posting cost:12

$$\gamma_1 + (1 - \beta) \gamma_2 = 1 - \frac{1 - \beta(1 - \rho) \bar{k}}{1 - \alpha} \frac{\bar{n}}{\bar{q} \bar{y}}$$

(13)

12Details on calibration can be found in appendix A.
The value of vacancy-posting cost plays an important role in determining $\gamma_1$ and $\gamma_2$. Following Andolfatto (1996), we set the total vacancy posting cost, $\kappa v$, to 1% of GDP.\textsuperscript{13} Given this value, the combination of $\gamma_1$ and $\gamma_2$ is calculated according to equation (13). We set $\gamma_1 = 0.42$, $\gamma_2 = 1.11$. $\gamma_0$ is a scale factor; its value is chosen to match a steady state effort level of 1 with a steady state employment level of 94.4%. Therefore, for coefficients in the effort function, we have one degree of freedom in parameterizations. We experiment with different values of $\gamma_2$ in section 7.2.

For the technology shock, we set the persistence parameter $\rho_a = 0.95$ following den Haan, Ramey and Watson (2000). The standard deviation of the shock is chosen to match the cyclical volatility of labor productivity in the U.S. between 1951 to 2003, and $\sigma_a = 0.0064$ as a result.

V MECHANISM

Following LS, we analyze the determinants of the elasticity of labor market tightness with respect to technology through the lens of fundamental surplus for the fair wage model. In addition, we study the endogenous propagation mechanism in the fair wage model. For both purposes, we use the standard DMP model as a benchmark.

5.1 The role of fundamental surplus

We define labor productivity $l \equiv (1 - \alpha) y/n$, and the difference between labor productivity and wage as the fundamental surplus, $fs(A) \equiv l(A) - w(A)$.\textsuperscript{14} Labor productivity, wage, and the fundamental surplus are functions of technology. The steady state version of the job-creation

\textsuperscript{13}Section 7.1 tests the model robustness using the value of vacancy posting cost employed in Hall and Milgrom (2008).

\textsuperscript{14}The fundamental surplus takes different forms depending on model setups. Refer to LS for more details.
condition that is present in both the fair wage and DMP model is

\[ \Omega \kappa / q(\theta) = fs(A), \]

(14)

where \( \Omega \equiv 1 - \beta (1 - \rho) \). Differentiating this equation yields the elasticity of tightness with respect to technology.\(^{15}\)

\[ \epsilon(\theta, A) = \frac{1}{1 - \nu} \epsilon(f s, A) \]

(15)

\( \epsilon(f s, A) \) is the elasticity of fundamental surplus with respect to technology.

The elasticity of tightness is a constant multiple of the elasticity of fundamental surplus. First, \( 1/(1 - \nu) = 2 \) with \( \nu = 0.5 \) in our calibration,\(^{16}\) which plays a limited role in generating an elastic labor market tightness. Second, the elasticity of labor market tightness depends crucially on and is increasing in \( \epsilon(f s, A) \). If fundamental surplus is sensitive to a technology change, the amount of resources used for vacancy posting is more responsive as well, leading to volatile tightness, vacancies, and unemployment.

Since \( fs(A) = l(A) - w(A) \), we can calculate \( \epsilon(f s, A) \) as:

\[ \epsilon(f s, A) = \frac{l}{l - w}(\epsilon(l, A) - \epsilon(w, A) \cdot \frac{w}{l}) \]

(16)

\( \epsilon(f s, A) \) depends on the the fundamental surplus ratio \( ((l - w)/l) \), and the difference between the elasticities of labor productivity and wage. We next analyze these two parts respectively in both the DMP model and our model.

\(^{15}\)Detailed derivation can be found in Appendix B.

\(^{16}\)It is consistent with the value used in the literature, i.e. 0.3 – 0.7; see Petrongolo and Pissarides (2001).
5.2 The fundamental surplus ratio

Consistent with LS, the fundamental surplus ratio plays a dominant role in determining the cyclical performance of labor market tightness. Everything else being equal, a smaller fundamental surplus ratio boosts the responsiveness of both fundamental surplus and labor market tightness with respect to a technology shock, according to equation (15) and (16). For a given level of labor productivity, the fundamental surplus ratio is smaller if wage is higher.

In the standard DMP model, the wage, determined in Nash bargaining to divide the match surplus, is relatively lower than labor productivity. The wage rule is \( w = (1 - \eta)z + \eta(l + k\theta) \),\(^{17}\) where \( \eta \) denotes workers’ bargaining power and lies between 0 and 1. \( z \) is the unemployment benefit and calibrated as 40% of labor productivity in Shimer (2005). As wage is a weighted average of labor productivity and unemployment benefit, a relatively large gap between labor productivity and wage, or a high fundamental surplus, arises. As carefully examined in LS, Hagedorn and Manovskii (2005) significantly improve the performance of DMP model with a higher wage level and a lower fundamental surplus ratio by revising the calibration of unemployment benefits and bargaining power.\(^{18}\) However, their model implies an implausibly high elasticity of unemployment with respect to nonmarket activity.

In our model, we abandon Nash bargaining and therefore impose no constraint on the relation between wage and unemployment benefits. The job creation condition, equation (14), shows that the presence of search costs drives a wedge between labor productivity and wage and generates the fundamental surplus. As the search costs vanish (\( \kappa = 0 \)), the fundamental surplus disappears as our model collapses to a standard fair wage model similar with Collard and de la Croix (2000).\(^{19}\)

\(^{17}\)Derivation of wage from Nash bargaining is in Appendix C.

\(^{18}\)They elevate the wage level to 97.6% of labor productivity by setting \( z = 95.5\% \cdot l \) and \( 1 - \eta = 0.052 \).

\(^{19}\)The equality between labor productivity and wage is the optimal condition of labor demand not only in fair wage...
In the search framework, the fundamental surplus ratio is determined by the level of search costs. When the vacancy-posting cost is lower relative to labor productivity in our calibration, a smaller fundamental surplus ratio arises, amplifying the response of tightness.\textsuperscript{20}

### 5.3 The role of wage rigidity

A higher degree of wage rigidity increases the sensitivity of fundamental surplus to technology shock and amplifies the response of tightness. When technology improves, both labor productivity and wage increase as a result. Only if wage responds less in magnitude, or the fundamental surplus is more procyclical, the firm can utilize more resources to create vacancies in good time, leading to highly procyclical vacancies and tightness. Consider the case that wage is flexible and responds to technology by the same magnitude as labor productivity, $\epsilon(w, A) = \epsilon(l, A)$. By combining equation (15) and equation (16), the elasticity of tightness with respect to productivity is $\epsilon(\theta, A) = \frac{1}{1-\nu} \epsilon(l, A)$. As labor productivity responds one-for-one with respect to technological change, $\epsilon(l, A)$ equals 1. With our calibration of $\nu = 0.5$, it follows that $\epsilon(\theta, A) = 2$. Therefore, the amplification effects of a small fundamental surplus ratio are completely eliminated when wage is flexible. However, there is also an upper bound on the effect of wage rigidity on the labor market tightness; the second term in equation (16) reaches a maximum of $\epsilon(l, A)$ when wage is invariant to changes in technology $\epsilon(w, A) = 0$. Therefore, the amplification effects of wage rigidity itself is limited. For a given level of fundamental surplus ratio and unit elasticity of labor productivity, the elasticity of tightness with respect to technology falls in the range $\left[\frac{1}{1-\nu}, \frac{1}{1-\nu} \frac{l}{l-w}\right]$, and it increases model but also in a Walrasian labor market, thus it is not unusual in the literature.\textsuperscript{20}\textsuperscript{2} The steady state value of fair wage is 98.5% of labor productivity, consistent with quarterly job-finding rate of 0.7 and total vacancy-posting cost ($\kappa v$) at 1% of output. This leads to the level of fair wage higher than Nash wage under common calibration in DMP models.
in the degree of wage rigidity.

In the standard DMP model, the wage under Nash bargaining is as flexible as labor productivity, leading to an elasticity of tightness close to the lower end of the range. On the contrary, in the fair wage model, the degree of wage rigidity is governed by the importance of past wage and steady state wage in the reference wage norms. If $\gamma_2$ is larger, more emphasis is placed on past and steady state wages. The resulting wage responds only weakly to the current economic conditions, giving rise to a higher degree of wage rigidity.

The above analysis suggests that the level and rigidity of wage are key determinants of the responsiveness of tightness, which our model successfully generates with fair wage. However, the performance of DMP model is almost unchanged if one increases wage rigidity by lowering workers’ bargaining power $\eta$. This does not suggest that the role of wage rigidity can be ignored; it results from counterbalancing effects of two forces. On the one hand, wage under Nash bargaining gets less responsive to changes in labor productivity with a lower $\eta$, and thereby the fundamental surplus is more responsive to shock, casting positive effects on the variability of tightness. On the other hand, the steady state wage level is lower as a smaller weight is given to the labor productivity. This increases the fundamental surplus ratio and tends to suppress the elasticity of tightness. These two effects cancel each other out and leaves the elasticity of tightness almost unchanged.

---

21The success of Hall (2005) in generating volatile labor market tightness can also be viewed through the lens of equation (15) and (16). Hall (2005) stipulates a constant wage inside the bargaining set, which is invariant to the current labor market conditions. $\epsilon(w, A)$ is zero and the fundamental surplus is very sensitive to technology shock. Also, the wage level is 96% of labor productivity, leaving the fundamental surplus a small fraction of output. High elasticity of labor market tightness is reproduced through these two channels. Compared with Hall (2005), our model employs an intermediate strategy – moderately rigid wage and a smaller fundamental surplus – to achieve this target.
5.4 Endogenous propagation mechanism

This section shows that the fair wage model endogenously produces wage persistence in response to shocks. For illustration, in this subsection, we consider an i.i.d positive technology shock with zero autocorrelation, i.e., $\rho_a = 0$. Figure 1 plots the impulse response of wage, tightness and unemployment in both models and the path of i.i.d. technology shock. The results in our model (called and labeled as "fair wage model") are contrasted with the results in a DMP model.\textsuperscript{22} The wage level in our fair wage model increases in the impact period due to the increase in labor productivity. In the second period, though the shock vanishes, bringing fair wage immediately back to the steady state level would produce large swings in effort because of the reference to past wage in fairness concerns. As a result, the firm prefers to cut wage gradually, which consequently generates endogenous propagation of the shock. On the contrary, if wage is determined by Nash bargaining in each period, it mainly reflect the labor productivity in each period. Therefore, the wage path mimics that of the technology shock: the wage hike only lasts for one period and wage returns to the steady state level as soon as the shock disappears. In addition, the high persistence of fair wage gives rise to persistent deviation of tightness and unemployment from the steady state level.

\textsuperscript{22}The wage rule in the DMP model is derived in Appendix C. Unemployment benefit is 40% of average labor productivity; worker’s bargaining power is 0.6. The calibration of other parameters is consistent with baseline calibration of the fair wage model.
VI RESULTS

6.1 Impulse response

Figure 2 shows the impulse response of key labor market variables to a positive one-standard deviation technology shock over 30 quarters. In response to the positive technology shock, labor productivity increases in both models. In the impact period, the increase of fair wage is about half of the magnitude of the rise in labor productivity. On the contrary, the Nash wage in the DMP model moves one-to-one with changes of labor productivity. In our model, the comparison with the past wage and steady state wage level not only suppresses large changes of wage, but also smooths the wage path. Fair wage displays a hump-shaped response: wages continue to rise for two periods before returning to its steady state level. Higher wages compared to wage references
boost workers’ morale and effort level. Effort jumps as wages increase, and is consistently above the average level. Procyclical effort indicates that firms utilize labor more intensively by means of wage incentives. Collard and de la Croix (2000) also produce procyclical effort and they suggest it is consistent with procyclical measurement error of Solow residual (Hall (1990)) or the assumption of labor hoarding (Burnside et al. (1993)). Firms find it profitable to increase the labor input as a result of the mild increase in wage in our model. Vacancies increase by 10%, and unemployment rate drops by 5% as a result of the intensive hiring. Labor market tightness increases by 15%, as a joint result of the significant drop in unemployment and the surge in vacancies. This is consistent with the empirical observation that tightness is highly procyclical and very volatile, confirming our analysis in Section V. On the contrary, in the DMP model, the large increase in wage mitigates firms’ incentives to hire. As a result, the change in vacancy and unemployment is only marginal. The response of labor market tightness is suppressed, only about one-fifth of the change in our model.

6.2 Statistical moments

This section presents the standard deviation and degree of autocorrelation of several key variables in our model. We contrast them with data and the counterpart in the DMP model. The first row of table 1 reports the standard deviation of key labor market variables calculated using the U.S. data from 1951 to 2003 and reproduced from Shimer (2005) and Silva and Toledo (2009). In the data, the standard deviation of unemployment and vacancies are of similar magnitude (about 0.20), while labor market tightness is twice as volatile as unemployment (0.382). The second row presents the
corresponding statistics in our fair wage model. The model generates the volatility of unemployment (0.175), vacancies (0.211) and tightness (0.383) of similar magnitudes as in the data, with a reasonable degree of wage rigidity. The third row shows the statistics in the DMP model. Wages are as volatile as labor productivity, and much more volatile than the data suggests. Moreover, the DMP model produces volatility of unemployment, vacancies, and tightness of only a fraction of those in the data. Over all, results show that our model can generate volatile unemployment, vacancies, and tightness, with moderately rigid wages as presented in the data.

Table 2 reports the quarterly autocorrelation of labor market variables and the correlation between unemployment and vacancy in the U.S. data and models. Both models can match the per-
sistence of key variables and the negative correlation between $u$ and $v$ well.

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s(\theta)$</th>
<th>$w$</th>
<th>$y/n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.118</td>
<td>0.014</td>
<td>0.020</td>
</tr>
<tr>
<td>Fair wage model</td>
<td>0.175</td>
<td>0.211</td>
<td>0.383</td>
<td>0.191</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>DMP model</td>
<td>0.027</td>
<td>0.032</td>
<td>0.058</td>
<td>0.030</td>
<td>0.020</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Table 2:

Autocorrelation of key labor market variables

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s(\theta)$</th>
<th>$w$</th>
<th>$y/n$</th>
<th>$corr(u, v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.936</td>
<td>0.940</td>
<td>0.941</td>
<td>0.908</td>
<td>0.907</td>
<td>0.878</td>
<td>-0.894</td>
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<tr>
<td>Fair wage model</td>
<td>0.961</td>
<td>0.851</td>
<td>0.913</td>
<td>0.913</td>
<td>0.967</td>
<td>0.927</td>
<td>-0.968</td>
</tr>
<tr>
<td>DMP model</td>
<td>0.975</td>
<td>0.914</td>
<td>0.949</td>
<td>0.949</td>
<td>0.950</td>
<td>0.949</td>
<td>-0.978</td>
</tr>
</tbody>
</table>

VII Robustness

The section checks the robustness of our model with an alternative calibration of vacancy posting cost, different parameterizations of effort function (6), a generalized effort function, and an alternative lag structure of reference wages.

7.1 Alternative value for vacancy posting cost

As illustrated in the Mechanism, the calibration of fundamental surplus depends crucially on the level of vacancy-posting cost, $\kappa$. Therefore, it is important to check whether our main quantitative results are robust to an alternative calibration of the vacancy posting cost. We now follow Hall
and Milgrom (2008) and set the ratio of the expected vacancy posting cost to the steady state wage to 14%. Table 3 reports the standard deviation of labor market variables under this alternative calibration. The volatility of labor market tightness and unemployment increases compared with the baseline results due to a smaller fundamental surplus ratio under this calibration.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( u )</td>
<td>( v )</td>
<td>( \theta )</td>
<td>( s(\theta) )</td>
<td>( w )</td>
</tr>
<tr>
<td>U.S. data</td>
<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.118</td>
<td>0.014</td>
</tr>
<tr>
<td>Fair wage model (Baseline calibration)</td>
<td>0.175</td>
<td>0.211</td>
<td>0.383</td>
<td>0.191</td>
<td>0.015</td>
</tr>
<tr>
<td>Fair wage model (Alternative value for ( \kappa ))</td>
<td>0.184</td>
<td>0.221</td>
<td>0.402</td>
<td>0.201</td>
<td>0.015</td>
</tr>
</tbody>
</table>

### 7.2 Different parameterizations of \( \gamma_2 \)

The degree of wage rigidity depends on the weight of past and steady state wage rates in the effort function. This part checks the robustness of our model to an alternative value of \( \gamma_2 \), which governs the relative importance of wage norms in effort considerations. We vary the value of \( \gamma_2 \) in the interval \([0.91, 1.31]\) around the baseline value of 1.11 with a step of 0.1. A larger \( \gamma_2 \) implies less attention to the comparison of individual wage with aggregate current wage, but more attention to past wage and the steady state wage. This brings about a smaller and smoother response of wages and a larger response of quantities of labor. Table 4 confirms our analysis; the standard deviation of the quantities (labor market tightness, unemployment and vacancy) increases while that of wages decreases as \( \gamma_2 \) increases. If we reduce the value of \( \gamma_2 \) further, the performance of our fair wage model approaches the DMP model. Actually, when \( \gamma_2 = 0.17 \), the standard deviation of key labor...

---

23 According to equation (13), we keep the ratio of \( \gamma_1/\gamma_2 \) the same as the baseline calibration.

24 The linear combination of \( \gamma_1 \) and \( \gamma_2 \) is a constant related to model parameters according to equation (13). When \( \gamma_2 \) becomes larger, \( \gamma_1 \) is smaller.
market variables is almost identical in both models. However, it does not imply that our model nests the DMP model. That is because the time-varying effort over business cycles in our fair wage model cannot be captured by the DMP model.

Table 4:

<table>
<thead>
<tr>
<th>$\gamma_2$</th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s(\theta)$</th>
<th>$w$</th>
<th>$y/n$</th>
</tr>
</thead>
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<tr>
<td>0.91</td>
<td>0.138</td>
<td>0.167</td>
<td>0.302</td>
<td>0.151</td>
<td>0.016</td>
<td>0.020</td>
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<tr>
<td>1.01</td>
<td>0.156</td>
<td>0.188</td>
<td>0.342</td>
<td>0.171</td>
<td>0.016</td>
<td>0.020</td>
</tr>
<tr>
<td>1.11</td>
<td>0.175</td>
<td>0.211</td>
<td>0.383</td>
<td>0.191</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>1.21</td>
<td>0.195</td>
<td>0.234</td>
<td>0.426</td>
<td>0.216</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>1.31</td>
<td>0.216</td>
<td>0.259</td>
<td>0.472</td>
<td>0.236</td>
<td>0.014</td>
<td>0.020</td>
</tr>
</tbody>
</table>

7.3 Power effort function

In the baseline model, we employ a logarithmic effort function, which is conventional in the literature. We now check whether our results are robust to a generalized specification of the effort function, called "power effort function" following de la Croix, de Walque and Wouters (2009). The power functional form allows for different degrees of substitutability between wage norms in the effort function. Specifically, we consider the following effort function

$$e_t(j) = \{\gamma_1[w_t(j)/(w_t n_t)]^\tau + \gamma_2[w_t(j)/(\sqrt{w_{t-1}(j)\sqrt{w}})]^\tau + \tau \gamma_0\}/\tau$$

(17)

It can be shown that this power function collapses to the logarithmic function in the baseline model when $\tau$ approaches 0. Table 5 reports the statistical moments of the fair wage model with this power effort function and varying values for $\tau$. We can see that the performance of our model is insensitive to this alternative specification.
7.4 Wage lags in the effort function

In the baseline model, we employ past wage and steady state wage as reference norms in effort determination. We now show similar results can be obtained by replacing steady state wage in the effort function with a long enough lag structure of past wages. We denote by $w_h^i(t)$ wage history faced by worker $i$ at time $t$. The worker compares his current wage with $w_h^i(t)$ to choose his effort level.

$$e_t(i) = \gamma_0 + \gamma_1 \log \frac{w_t(i)}{w_t} + \gamma_2 \log \frac{w_t(i)}{w_h^i(i)}$$  \hspace{1cm} (18)

where $w_h^i(t) = \prod_{k=1}^{k_{max}} w_{t-k}^{\mu(1-\mu)^{k-1}}(i)$. $\mu$ measures the weight assigned to past wages at different points in time.

Given effort function (18), optimal wage decision is more complicated as firm now compares current individual wage with wages back to $k_{max}$ periods, in addition to the most recent wage. For illustration, we choose $k_{max} = 8$ and $\mu = 0.15$. Table 6 reports standard deviation of key labor market variables in this setup, and the quantities of labor (unemployment, vacancies, and tightness)

---

Table 5:

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$u$</th>
<th>$v$</th>
<th>$\theta$</th>
<th>$s(\theta)$</th>
<th>$w$</th>
<th>$y/n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.1752</td>
<td>0.2105</td>
<td>0.3826</td>
<td>0.1913</td>
<td>0.0153</td>
<td>0.020</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1752</td>
<td>0.2104</td>
<td>0.3825</td>
<td>0.1913</td>
<td>0.0153</td>
<td>0.020</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1752</td>
<td>0.2103</td>
<td>0.3825</td>
<td>0.1912</td>
<td>0.0153</td>
<td>0.020</td>
</tr>
<tr>
<td>0.2</td>
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<td>0.2103</td>
<td>0.3824</td>
<td>0.1912</td>
<td>0.0153</td>
<td>0.020</td>
</tr>
<tr>
<td>0.25</td>
<td>0.1751</td>
<td>0.2102</td>
<td>0.3823</td>
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<td>0.020</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1751</td>
<td>0.2101</td>
<td>0.3822</td>
<td>0.1911</td>
<td>0.0153</td>
<td>0.020</td>
</tr>
</tbody>
</table>

---

25 The ratio of $\gamma_1$ to $\gamma_2$ is calibrated to be consistent with the baseline calibration to keep the relative weights on aggregate current wage and wage history unchanged. The value of other parameters stay unchanged.
is slightly less volatile than the baseline fair wage model.

Table 6: Standard deviation: wage history

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>θ</th>
<th>s(θ)</th>
<th>w</th>
<th>y/n</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. data</td>
<td>0.190</td>
<td>0.202</td>
<td>0.382</td>
<td>0.118</td>
<td>0.014</td>
<td>0.020</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.175</td>
<td>0.211</td>
<td>0.383</td>
<td>0.191</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>Wage history</td>
<td>0.153</td>
<td>0.196</td>
<td>0.345</td>
<td>0.172</td>
<td>0.016</td>
<td>0.020</td>
</tr>
</tbody>
</table>

**VIII CONCLUSIONS**

A labor market search and matching model incorporating fair wage considerations can reproduce moderately rigid wages, volatile unemployment and labor market tightness, comparable with time-series evidence on U.S. labor market. For future research, it would be interesting to pursue along two dimensions. First, it is interesting to study the implications of fair wage for the dynamics of firms’ marginal cost and the persistence of inflation in sticky wage models. Second, Fehr et al. (2009) propose to account for puzzling empirical evidence of minimum wage legislation from the gift-exchange perspective based on experimental evidence (Falk et al. 2006). In light of these findings, it is promising to analyze the economic effects of minimum wage laws in a general equilibrium fair wage model.
References


den Haan, W. J. and Ramey, G. and Watson, J. "Job Destruction and Propagation of Shocks,"


Appendix

A Calibration and steady state values of variables

The steady state level of employment, matches, vacancies, and labor market tightness are calibrated to match steady state unemployment rate ($\bar{u} = 5.7\%$), separation rate ($\rho = 0.1$) and job-finding rate ($\bar{q} = 0.7$), using the following steady state conditions $\bar{n} = 1 - \bar{u}$, $\bar{m} = \rho \bar{n}$, $\bar{v} = \bar{m} / \bar{q}$, and $\bar{\theta} = \bar{v} / \bar{u}$. Effort level in the steady state is normalized to 1. Output level is given by $\bar{y} = \bar{n}^{1-\alpha}$.

The total vacancy-posting cost is set to be 1% of output in the steady state, $\kappa \bar{v} = 1\% \bar{y}$, or $\kappa = \frac{1\%}{\bar{v}}$. Given the resource constraint, the steady state consumption is $\bar{c} = \bar{y} - \kappa \bar{v}$. The steady state of real wage can be calibrated from the job-creation condition, equation (12), $\bar{w} = (1 - \alpha) \frac{\bar{y}}{\bar{n}} - (1 - \beta (1 - \rho)) \frac{\bar{v}}{\bar{q}}$. Therefore we have derived the relationship between the steady state value of real wage and the parameters in the effort function. Meanwhile, from the first order condition of real wage, equation (9), we get $\bar{w} = (1 - \alpha) \frac{\bar{y}}{\bar{n}} (\gamma_1 + \gamma_2 - \frac{\beta}{2} \gamma_2)$. Combining the last two equations, we obtain

$$\gamma_1 + (1 - \frac{\beta}{2}) \gamma_2 = 1 - \frac{1 - \beta (1 - \rho)}{1 - \alpha} \frac{\kappa \bar{n}}{\bar{q} \bar{y}}$$

This equation gives us one constraint on the parameterizations of the effort function. For a given value for $\gamma_2$, we have $\gamma_1 = 1 - \frac{1 - \beta (1 - \rho)}{1 - \alpha} \frac{\kappa \bar{n}}{\bar{q} \bar{y}} - (1 - \frac{\beta}{2}) \gamma_2$, and $\gamma_0$ can be calculated to match the...
steady state levels of effort and employment rate, $\gamma_0 = \bar{e} + \gamma_1 \ln \bar{n}$

**B  Derivation of the elasticity equation (15)**

In steady-state, $\lambda_{t+1} = \lambda_t = \lambda$, $\theta_{t+1} = \theta_t = \theta$, the job creation condition (12) becomes $\Omega \kappa/q(\theta) = fs(A)$, with $\Omega \equiv 1 - \beta(1 - \rho)$. In order to obtain the elasticity of tightness with respect to technology, we implicitly differentiate this equation.

$$\frac{d\theta}{dA} = -\frac{fs'(A)}{\Omega \kappa q(\theta)/q^2(\theta)}$$

Therefore the elasticity is calculated as

$$\epsilon(\theta, A) = \frac{d\theta}{dA} \frac{A}{\theta} = -\frac{fs'(A)}{\Omega \kappa q(\theta)/q^2(\theta)} \frac{A}{\theta}$$

Simplify the above equation using the relation that $q'(\theta)/q(\theta) = \nu - 1$. We get

$$\epsilon(\theta, A) = \frac{1}{1-\nu} \frac{fs'(A)A}{fs(A)} = \frac{1}{1-\nu} \epsilon(fs, A)$$

**C  The Nash bargaining of wage**

In the Nash bargaining, workers and firms split the match surplus. The value of an employed worker $V^N_t$ and an unemployed worker $V^U_t$ are:

$$V^N_t = w_t + \beta E_t \lambda_{t+1}/\lambda_t \{[1 - \rho(1 - s_{t+1})]V^N_{t+1} + \rho(1 - s_{t+1})V^U_{t+1}\}$$

$$V^U_t = z + \beta E_t \lambda_{t+1}/\lambda_t \{s_{t+1}V^N_{t+1} + (1 - s_{t+1})V^U_{t+1}\}$$

Here we assume $z$ is the unemployment insurance or the value of home production. Therefore, the surplus of an additional worker for the household is $S^H_t = V^N_t - V^U_t$.

Let $V^F$ and $V^O$ denote the value of a filled job position and a vacancy to the firm.

$$V^F_t = (1 - \alpha) y_t/n_t - w_t + \beta(1 - \rho) E_t \lambda_{t+1}/\lambda_t V^F_{t+1}$$

The surplus of an additional worker for the firm is $S^F_t = V^F_t - V^O_t$. Free entry implies $V^O_t = 0$.

Assume that Worker’s bargaining power is $\eta$. The outcome of wage bargaining is determined in the sharing rule, $\eta S^F_t = (1 - \eta) S^H_t$. Substituting $V^F_t$ and $V^H_t$ into the above equation and solving for the wage, one gets the Nash bargained wage, $w_t = (1 - \eta) z + \eta((1 - \alpha) y_t/n_t + k E_t \lambda_{t+1}/\lambda_t \theta_{t+1})$, with $k \equiv \beta(1 - \rho) \kappa$. Therefore in the steady state, we have

$$w = (1 - \eta) z + \eta((1 - \alpha) y/n + k \theta)$$