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Numerical studies on the effect of plan irregularities in the progressive collapse of steel structures

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This research examines the effect of plan irregularities on the progressive collapse of steel structures. The performance of four structures located in regions with different seismic activity designed in accordance with AISC (2010) and ASCE7 (2010) is determined. The plans of the first and second structure are irregular, whilst those of the third and fourth structures are regular.

The collapse patterns of the four buildings are examined and compared under seven loading scenarios using non-linear dynamic and static analyses. In the non-linear dynamic analyses, node displacements above the removed columns and the additional force on the columns adjacent to them are discussed. Furthermore, the strength and capacities of the columns are compared to determine their susceptibility to collapse. In the non-linear static analyses, the pushdown curve and yield load factor of the structures are obtained after column removal.

The results indicate that an irregular structure designed in site class C seismic zone, collapses in most of the column removal scenarios. Moreover, when comparing regular and irregular structures designed in site class E seismic zone, the demand force to capacity ratio (D/C) of the columns in the irregular structures is on average between 1.5 and 2 times that of the regular ones.

Keywords: progressive collapse, irregularity in plans, steel building, non-linear dynamic analysis, non-linear static analysis, pushdown

1. Introduction

In past years, blast load inside and around buildings have led to significant casualties and damage to structures due to progressive collapse (ASCE, 2005). Potential risks and abnormal loads which may lead to failure include plane crash, erroneous design or construction, fire, gas explosion, occasional overload, vehicle impact and blast (National Institute of Standard and Technology [NIST], 2007). Nonetheless, since the risk associated to the occurrence of those events is low, buildings are not designed to

withstand abnormal loads nor their effects on structures are closely examined - hence constructions remain susceptible to various scales of damage. Mitigation measures to prevent progressive collapse however there exist. Those are provided by the Unified Facilities Criteria UFC (2009) and the GSA (2003, 2013), both addressing the Alternative Path Method (APM) which to date is the most common approach to combat progressive collapse. The correct parameterisation of the procedure is however still under scrutiny. For example, Powel (2005) compared linear static, non-linear static and non-linear dynamic analyses and found that using a load factor of 2 in static analyses can produce very conservative results. Similarly, Ruth, Marchand, and Williamson (2006) analysed 2D and 3D steel frames to show that using a load factor of 2 for non-linear static analyses may be conservative. It was then found that a factor of ~ 1.5 is more accurate to capturing dynamic effects inferred from quasi-static analyses and that a load factor of 2 seems more appropriate for high ductility structures, provided the behaviour of the materials is not elastic-perfectly plastic and the materials exhibit hardening over a wide range of strains after yielding. The authors of the study hence recommended to use a load factor of 2 for ductile structures and one of 1.5 for other buildings.

Over the last decades, the assessment of the sensitivity or insensitivity, to local damage, has also been widely researched. Gerasimidis and Baniotopoulos (2011) examined the problem of disproportionate collapse in steel moment frames and compared the APM with a numerical approximation based on the β -Newmark and linear Hilbert–Hughes–Taylor procedures. Following, a parametric study considering irregular steel frames subject to vertical geometric irregularity was reported in Gerasimidis, Bisbos, and Baniotopoulos (2012) whilst Gerasimidis, Bisbos, and

Baniotopoulos (2013) discussed the sensitivity of structures to local damage and introduced the notion of partial damage to structural elements.

In terms of lateral stability, Khandelwal, El-Tawil, and Sadek (2009) analysed the progressive collapse of seismically designed steel-braced frames using explicit transient dynamic simulations. The study used the APM on previously designed 10-storey prototype buildings and observed that eccentrically braced frames are much less prone to progressive collapse than its concentrically braced version. In 2012, Chen, Peng, Ma, and He investigated the effectiveness of horizontal bracing on a steel moment-resisting frame and concluded that displacements and rotation angles in the model with bracings were much smaller than those observed when bracing was not present. More recently, Kim and Park (2014) studied the progressive collapse-resisting capacity of special truss moment frames considering arbitrary column removal scenarios. It was pointed out that structures designed according to the AISC seismic provision collapsed as a result of plastic hinge formation at highly stressed regions, once a column was suddenly removed. Furthermore, Gerasimidis and Baniotopoulos (2014) studied the impact of various strengthening techniques for reducing progressive collapse in 2D steel moment frames whilst Gerasimidis, Deodatis, Kontoroupi, and Ettouney (2015) conducted a progressive collapse analysis of a tall steel frame following the removal of a corner column, according to the APM.

The present paper builds on previous research and focus on the impact of plan irregularities on structural stability evaluated at two distinct seismic regions, which creates risk scenarios that have not received adequate attention from scholars. Hence the spread of damage induced by various column removal scenarios on four building prototypes is examined and discussed throughout.

2. Model structures

Four 15 m tall steel structures with plan irregularities were selected for the present investigation. Intermediate Steel Moment Frames were pre-designed according to the AISC (2010) and ASCE (2010) to study progressive collapse scenarios in structures showing plan irregularities. These are five-storey buildings modelled in ETABS and subject to dead load of 520 kg/m^2 and live load of 192 kg/m^2 , as per the referred guidelines.

The first two structures, shown in Figure 1a, have plan irregularities and are assumed in site class, C and E seismic regions, respectively. Structures 3 and 4 are regular in plan and have each six bays of 4 m wide (Figure 1b). These are also assumed in site class C and E, correspondingly. Further details of the four structures are provided in Table 1 whilst sections of structural members for the regular and irregular structures are given in Tables 2 and 3, respectively.

Figure 1. Model structures, (a) Plan of structures 1 and 2, (b) Plan of structures 3 and 4.

Table 1. Analysis model structures

Structure	Seismic zone	Type of Soil	Regularity	Number of storeys
Structure 1	C	very dense soil and soft rock	Irregular	5
Structure 2	E	soft clay soil	Irregular	5
Structure 3	C	very dense soil and soft rock	Regular	5
Structure 4	E	soft clay soil	Regular	5

Table 2. Detail of sections used in irregular structures

Floor	Structure 1						Structure 2					
	Column (Box)		Beam (PG)				Column (Box)		Beam (PG)			
	b	t	b _f	t _f	b _w	t _w	b	t	b _f	t _f	b _w	t _w
First	200	12	150	8	250	8	200	20	150	15	250	8
								15	150	12	250	8
									150	10	250	8
Second	200	10	150	8	250	8	200	15	150	15	250	8
									150	12	250	8
									150	10	250	8
Third	200	10	150	8	250	8	200	12	150	12	250	8
									150	10	250	8
									150	8	250	8
Fourth	200	10	150	8	250	8	200	10	150	8	250	8
Fifth	200	10	150	8	250	8	200	10	150	8	250	8

Table 3. Detail of sections used in regular structures

Floor	Structure 3						Structure 4					
	Column (Box)		Beam (PG)				Column (Box)		Beam (PG)			
	b	t	b _f	t _f	b _w	t _w	b	t	b _f	t _f	b _w	t _w
First	200	12	150	8	250	8	200	15	150	12	250	8
	200	10					200	12	150	10		
Second	200	10	150	8	250	8	200	15	150	12	250	8
							200	12	150	10		
							200	10				
Third	200	10	150	8	250	8	200	12	150	10	250	8
							200	10	150	8		
Fourth	200	10	150	8	250	8	200	10	150	8	250	8
Fifth	200	10	150	8	250	8	200	10	150	8	250	8

3. Numerical modelling

The 3D model structures were numerically analysed with OpenSees. Non-linear analyses were run considering a simple bi-linear material model with post-yield stiffness of 2% of the initial stiffness. Non-linear beam-column elements were used for

modelling the cross-sectional areas as precisely as possible. The plastification over element length and cross-sections were also considered, whereas large displacements effects were also accounted for by the employment of the co-rotational transformation of the geometric stiffness matrix. The dynamic behaviour caused by sudden column removal was not a factor in the load reversal because, in structures subjected to earthquake loads, using a complicated hysteretic model is unnecessary. The fraction of damping was assumed to be 5% which is usually the case for structures with large deformations.

4. Analysis method for progressive collapse

Following the GSA (2013) guidelines, load combinations including 120% of dead load plus 50% of the total live load were gradually applied within a time frame of 5 s. Then, and in order to account for non-linear dynamic effects, the load was maintained steady for the following 2 s. After the 7 s sequence, when gravity load effects are considered to be fully transferred to the structure, a pre-selected column was suddenly removed from the model and the structural response was examined.

In parallel, non-linear static analyses were performed, following the GSA (2003, 2013) recommendation for using a dynamic amplification factor of ~ 2 . That, in order to reflect a ratio of 2 between the load that is applied to the spans that are adjacent to the removed column with respect to that applied on other spans. In this case, vertical loading is applied by following a step-wise increase until the maximum amplified loads are attained or the structure collapses. This vertical pushover analysis procedure, which is often called the 'pushdown analysis method', accounts for non-linear effects which approximate the non-linear dynamic response whilst providing a reliable estimation of the elastic and failure limits of the subject structure.

Derived from the non-linear static analyses, the effective imposed load plotted against the node displacement of the removed column indicates the capacity of a structure against progressive collapse. If the load value is divided by the standard gravity load, the vertical axes of the pushdown capacity curve are converted into dimensionless load factors, as in Eqn. (1). This standardises the load ratio and makes it easier to establish generic observations. The load factor calculated in this way have thus been used herein as a criterion for assessing structural collapse. Namely, if the load factor corresponding to the displacement causing material yield is higher than 1, the structure can withstand the removal of a column, otherwise the structure will collapse.

$$Load\ Factor = \frac{Load}{Nominal\ gravity\ load} \quad (1)$$

5. Verification of dynamic analysis

The inelastic performance of a single degree of freedom system can be obtained by numerically integrating its equation of dynamic equilibrium whose general form is given by Eqn. (2).

$$p(t) = m\ddot{u} + c\dot{u} + f_s(u, \dot{u}) \quad (2)$$

$$\text{with } u = u(0), \dot{u} = \dot{u}(0)$$

In Eqn. (2) m is the mass, c is damping coefficient, and $f_s(u, \dot{u})$ is the restoring force of the system. If the effective force $p(t)$ is an arbitrary and/or complex function of time (t), then solving single degree of freedom system motion equation analytically is not impossible. In order to solve the differential equation of dynamic equilibrium, time step-by-step numerical methods, such as those discussed in Chopra (1995), may be used. In this case the Newton-Raphson algorithm was selected. In this method, the

external force $p(t)$ is divided into separate and consecutive forces $p_i = p(t_i)$, $i = 0 \sim N$. Single degree of freedom system response, including motion, velocity and acceleration, $u_i, \dot{u}_i, \ddot{u}_i$, were thus determined in separate points in time t (called the i -th time). The discrete response values satisfy Eqn. (3), accordingly.

$$p_i = m\ddot{u}_i + c\dot{u}_i + (f_s)_i \quad (3)$$

For a linear elastic system, the restoring force satisfies $(f_s)_i = ku_i$. However, for a non-linear and inelastic system, the value of the restoring force depends on the rate of motion. Using the numerical method quoted above, the values of $u_{i+1}, \dot{u}_{i+1}, \ddot{u}_{i+1}$ in the $(i+1)^{th}$ time were determined. The dynamic response obtained in this way thus satisfies Eqn. (4), being the dynamic motion of the system controlled by Eqs. (5).

$$p_{i+1} = m\ddot{u}_{i+1} + c\dot{u}_{i+1} + (f_s)_{i+1} \quad (4)$$

$$\ddot{u}_{i+1} = \ddot{u}_i + [(1 - \gamma)\Delta t]\ddot{u}_i + (\gamma\Delta t)\ddot{u}_{i+1} \quad (5.a)$$

$$u_{i+1} = u_i + (\Delta t)\dot{u}_i + [(0.5 - \beta)(\Delta t)^2]\ddot{u}_i + [\beta(\Delta t)^2]\ddot{u}_{i+1} \quad (5.b)$$

In Eqs. (5) β and γ define the rate of acceleration in a time step. Common values for these constants are $\gamma = \frac{1}{2}$ and $\frac{1}{6} \leq \beta \leq \frac{1}{4}$. Eqs. (4) and (5) were combined to determine $u_{i+1}, \dot{u}_{i+1}, \ddot{u}_{i+1}$ in $(i+1)$ following the known values of $u_i, \dot{u}_i$, and $\ddot{u}_i$ estimated for time i .

In order to calibrate the quasi-static analyses used here, a case study drawn from Chopra (1995) consisting of one degree freedom elasto-plastic system with a mass of 253.3 kg, 5% damping and yield deformation of $u_y = 7.5$ mm, was analysed with OpenSees as well as with the algorithm outlined above. Figure 2a shows the load

applied to the model whilst Figure 2b shows the discrepancy between the two procedures. The differences encountered seem to be caused by a numerical error in Chopra (1995) when estimating the tangent stiffness. This explains why the curves in Figure 2b coincide in the first half of the analysis but diverge in the second half.

Figure 2. A case study analysis a) Load applied to the model b) Displacement in dynamic analysis

Following, the two models were loaded with the time-varying load shown in Figure 3a. That load represents the load transfer process recommended by GSA (2003, 2013), hence it is the one used to run the full analyses of the multi-storey buildings described in sections 2-4. The dynamic performance obtained for the single degree of freedom system for the initial 5 s is illustrated in Figure 3b.

Figure 3. Validation of the progressive collapse analysis a) Load b) Displacement

6. Analysis results

As outlined above, in this investigation, the potential collapse of the structures listed in Table 1 is studied under the scenarios set out in Table 4 and Figure 4. In all cases, the column removed correspond those located in the ground floor, as that induced the most critical conditions concerning structural stability. Additionally, a range of column-removal scenarios have been identified in order to induce meaningful configurations of potential failure. In each of these scenarios, a column is suddenly removed and the response of the structure is examined through non-linear dynamic and static analyses, as described above. The columns selected for removal are shown in Figure 4.

Figure 4. Location of the columns removed for each of the four structures

Table 4. Column removal scenarios for each of the four structures

Number	location of removal column			Scenario notation
	Storey	Frame	Pier	
1	1	1	A	S1F1PA
2	1	1	B	S1F1PB
3	1	1	D	S1F1PD
4	1	4	A	S1F4PA
5	1	4	D	S1F4PD
6	1	4	F	S1F4PF
7	1	7	A	S1F7PA

Figure 5 shows the pushdown capacity curve reflecting the removal scenarios 1 and 3 for each of the four structures, following non-linear static analyses. The vertical axis of these curves represents the load factor dimensionless parameter given by Eqn. (1). It can be seen in Figure 5 that the load factor of irregular structures is lower than the load factor in regular ones for either column-removal scenario. This was to some degree expected given the fact that irregular buildings do not exhibit simple alternate load paths hence extra-forces tend to be present. Load factors of regular structures are greater than 1 which suggests there is some extra-capacity to withstand progressive collapse. Additionally, structure 1 and 3, which are designed in site class C, show less capacity than structure 2 and 4, assumed in site class E.

Figure 5. Pushdown capacity curve of all the structures a) Scenario 1 b) Scenario 3

Figure 6 shows the pushdown curves derived from non-linear static analyses represented in bi-linear form covering scenario 5 for structure 2. This curve can be used to identify the yielding load factor defined as the ratio between applied load and the identified yield capacity.

Figure 6. Pushdown curve and its bilinear curve for the fifth scenario for the second structure

In Figure 7, the yield load factor is shown for all the structures and scenarios. It can be seen that structure 4, which is a regular structure designed in site class E, has the highest yield load factor. On the other hand, among all scenarios, scenario 5 which is

related to the structure's central column, has a greater yield load factor and a reduced collapse potential.

Figure 7. Yield load factors for all the structures and scenarios

On the other hand, non-linear dynamic analyses were used to calculate the peak displacement of the node above a removed column. Figure 8 shows the results obtained for scenarios 1 and 3 across all structures. As it can be inferred from the results, node displacements in structures 1 and 2 represent structural collapse of the region around the removed column. In contrast, the node displacement remains constant after 7 s from the removal in structures 3 and 4 which reveals a robust structural performance following the potential failure of the target column.

Figure 8. Vertical displacement of removal point, a) Scenario 1, b) Scenario 3

Another key aspect for assessing structural performance under progressive collapse is the force taken by columns that are adjacent to the removed column. In Figure 9 the demand to capacity (D/C) ratio of the columns adjacent to the corner columns in scenario 1 across all the structures is given. It can be seen in this figure that the D/C ratio associated to adjacent columns is around 1 in scenario 1 related to structures 3 and 4. This suggests that columns adjacent to the target-removal one may not be exposed to total damage as alternative load paths do not seem to directly redistribute to those spans. However, in structures 1 and 2 the strength demand for both adjacent columns in the progressive collapse analysis is between 1.2 and 1.8 times the column capacity, which indicates that these columns would have been damaged after the collapse of the target column.

a) Column 1B

Figure 9. The demand force to capacity ratio (D/C) of the adjacent columns in Scenario 1, a) Column 1B, b) Column 2A on the ground floor

Figure 10 shows the D/C ratio of adjacent columns in scenario 3. Structures 3 and 4 exhibit D/C of 0.6 and 0.73, respectively. Therefore, these columns are not exposed to collapse under that column-removal scenario, but they would be damaged if belonging to structures 1 and 2. In Table 5, node displacement and maximum D/C ratio of adjacent columns is given for all the scenarios and structures.

Figure 10. The demand force to capacity ratio (D/C) of the adjacent columns in Scenario 3, a) Column 1C, b) Column 2D on the ground floor

Table 5. Node displacement and maximum D/C ratio of adjacent columns for all the scenarios and structures

Scenario	Structure 1		Structure 2		Structure 3		Structure 4	
	Node Displacement (mm)	D/C	Node Displacement (mm)	D/C	Node Displacement (mm)	D/C	Node Displacement (mm)	D/C
S1F1PA	Fail	1.77	Fail	1.44	97	1.03	72.6	0.96
S1F1PB	Fail	1.29	86	0.92	56.5	0.71	19.4	0.57
S1F1PD	Fail	1.5	Fail	1.23	64.4	0.73	21	0.6
S1F4PA	Fail	1.15	84.9	0.87	56.3	0.71	19.3	0.56
S1F4PD	91	1.05	84	0.83	55.7	0.68	19.1	0.49
S1F4PF	Fail	1.31	87.8	0.94	62.6	0.71	20	0.58
S1F7PA	Fail	1.67	Fail	1.38	97	1.03	72.6	0.96

7. Conclusion

In this study, four steel structures were designed in site class C and E according to the AISC (2010) and ASCE7 (2010). The effect of plan irregularities and type of seismic regionalisation on progressive collapse have been analysed under various column

removal scenarios. The results of the analyses reveal that in cases where the structural plans were similar, the structure designed in a region in site class E seismic risk has less collapse potential. Moreover, the potential for progressive collapse was identified to be higher for buildings with plan irregularities and or in site class C. It was also seen that the displacement of the node above the removed column and the D/C ratio of the columns adjacent to the one removed could provide a fair indication of the risk of overall collapse of structures.

The comparison of structures 2 (irregular) and 4 (regular) revealed that, on average, the D/C ratio in the irregular structures is between 1.5 and 2 times larger than that of the regular structures. Moreover, in the scenario of the removal of columns in structure 4, the yield load factor is significantly higher than that in structure 2, indicating the high capacity of structure 4 to resist progressive collapse.

By comparing the dynamic behaviour of structures 3 and 4 it is concluded that designing in site class E seismic region results in a 25% decrease in the displacement of the node above the corner columns and a 65% decrease in the displacement of the node above the middle columns. Finally, the D/C ratio of the columns in the structure in site class E decreases by between 10 and 20% in comparison to that of the structure located in site class C.

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