

# Identification of patterns in control charts for processes with statistically correlated noise

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Identification of patterns in control charts for processes with statistically correlated noise

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## Abstract

In real industrial scenarios, if the quality characteristics of a continuous or batch production process are monitored using Shewhart control charts, there could be a large number of false alarms about the process going out of control. This is because these control charts assume that the inherent noise of the monitored process is normally, independently and identically distributed, although the assumption of independence is not always correct for continuous and batch production processes. This paper presents three control chart pattern recognition systems where the inherent disturbance is assumed to be stationary. The systems use the first-order autoregressive (AR(1)), moving-average (MA(1)) and autoregressive moving-average (ARMA(1,1)) models. A special pattern generation scheme is adopted to ensure generality, randomness, and comparability, as well as allowing the further categorisation of the studied patterns. Two different input representation techniques for the recognition systems were studied. These gave nearly the same performance for the MA(1) and ARMA(1,1) models, while the raw data yielded the highest accuracies when AR(1) was used. The effect of autocorrelation on the pattern recognition capabilities of the developed models was studied. It was observed that Normal and Upward Shift patterns were the most affected.

Keywords: Control chart pattern recognition, stationary processes, dynamic regression, pattern generation, Support Vector Machine

## Nomenclature

AR	Autoregressive
ARMA	Autoregressive moving-average
BPNN	Back-propagation neural network
BESS	Bessel kernel
CCPR	Control chart pattern recognition
CYC	Cyclic pattern
$D_t$	Disturbance magnitude at time $t$
DS	Downward shift pattern
DT	Downward trend pattern
$e_t$	Random error (white-noise) at time $t$
IMA	Integrated moving-average model
IRT	Input representation technique
LAPLA	Laplace kernel
LVQ	Learning Vector quantisation
MA	Moving-average model
ML	Machine learning
$N_t$	Inherent disturbance at time $t$
NIID	Normally, identically and independently distributed
NLM	Nonlinear model
NLM-ARMA	Nonlinear regression model with autoregressive moving-average errors
NORM	Normal pattern
PGS	Pattern generation scheme
$r_1$	Autocorrelation coefficient
RBF	Radial basis function kernel

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3	$S_Y$	Standard deviation of the variable Y
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5	sin	Sine function
6		
7	SSE	Sum of squared error
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9	SVM	Support Vector Machine
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11	SYS	Systematic pattern
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13	TANH	Hyperbolic Tangent kernel
14		
15	US	Upward shift pattern
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17	UT	Upward trend pattern
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19	$Y_t$	Measurement of the quality characteristic under study at time $t$
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21	$Z_t$	Scaled variable at time $t$
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23	$\alpha$	Significance level
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25	$\beta$	Abnormal pattern parameter
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27	$\sigma_N$	Standard deviation of the variable N
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29	$\mu$	Mean value
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31	$\tau$	Time when a break point occurs
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33	$\theta$	Moving-average coefficient
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35	$\phi$	Autoregressive coefficient
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## 1 Introduction

Control charts are graphical tools that monitor and assess the performance of production processes, revealing abnormal (deterministic) disturbances when there is a shift in the process. When the process is operating normally, a “Normal” control chart pattern can be observed (see Western Electric Company 1958). If an assignable cause is affecting the process, the control chart can exhibit one or more of fourteen types of patterns (Western Electric Company 1958), six of these being considered as basic patterns: Upward/Downward Trends (UT/DT), Upward/Downward Shifts (US/DS), Cyclic and Systematic (CYC and SYS) (see Western Electric Company 1958). The remaining eight patterns are either particular cases or combinations of these basic patterns.

The efficient and accurate identification of these simple patterns has been a problem studied by researchers during the last two decades. These researchers have focused mainly on the identification of patterns when the inherent disturbance of the process is purely random, i.e. normally, independently and identically distributed (NIID) (Kazemi et al. 2015; Chompu-Inwai & Thaiupathump 2015; Xanthopoulos & Razzaghi 2014). Unfortunately, the assumption of uncorrelated (independent) observations is not even approximately satisfied in some processes. Examples include chemical processes where consecutive measurements of process or product characteristics are often highly correlated, or automated test and inspection procedures, where every quality characteristic is measured on every unit in time order of production (Montgomery 2009).

Disturbances in control charts can be divided into two categories: stochastic and deterministic (MacGregor 1988). Stochastic disturbances are those related to fluctuation in the performance of the process with time, and in most cases, can be modelled by time series. In this paper, stochastic disturbances are assumed to be inherent to the process. On the other hand, deterministic disturbances are those that are independent of the process and are fully

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3 determined by external variables; these can be caused by faults in machines, tools or  
4 materials or by environmental variables or other factors. They manifest themselves as  
5 abnormal patterns in the control chart (Western Electric Company 1958).  
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10 Very few studies have been performed regarding control chart pattern recognition (CCPR)  
11 when the inherent disturbance of the process is autocorrelated, the first-order autoregressive  
12 model (AR(1)) being the only utilised so far. The aim of this work is to develop new CCPR  
13 systems where the inherent disturbance is modelled by other stationary time series.  
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19 Three time series models for stationary processes are used to model the inherent disturbances:  
20 AR(1), MA(1) and ARMA (1,1). Stationarity implies that the generated time series varies in a  
21 stable manner about a fixed mean (Box et al. 2011). In this work, independence is assumed  
22 between the causes of the inherent disturbance and the causes of patterns in the control chart,  
23 thus retaining the same seven patterns observed in NIID processes. The autocorrelation level  
24 is expected to remain constant during the pattern recognition window and is also assumed to  
25 be unknown.  
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35 The CCPR systems developed comprise a Machine Learning (ML) algorithm named the  
36 Support Vector Machine (SVM), as the pattern recognition system, and a pattern generation  
37 scheme (PGS) for first-order stationary processes. PGSs synthesise patterns for CCPR  
38 systems taking into account the main aim of identifying patterns in control charts: automate  
39 the analysis and identification of patterns in control charts, allowing a full categorisation of  
40 the fault.  
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49 The training of ML algorithms requires large amounts of data which are not readily available.  
50 It is therefore necessary to synthesise patterns that are representative of those to be  
51 recognised during the operation of the CCPR system.  
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The PGS employed in this work is related to that proposed by De La Torre Gutierrez and Pham (2016) where the inherent noise is NIID. As the assumption of independence is not fulfilled in autocorrelated<sup>1</sup> processes such as those that are continuous, the PGS must be capable of generating patterns of this type, taking into account the real variance of the process. Under the assumption of independence between stochastic and deterministic disturbances, the dynamic regression model named Nonlinear regression model with autoregressive moving-average errors (NLM-ARMA) is able to represent the data generated by the PGS instead of the nonlinear regression model proposed by De La Torre Gutierrez and Pham (2016). This PGS scheme fulfils the conditions needed for the design of CCPR systems previously mentioned. The p-value ( $\alpha$ ) of the NLM-ARMA model fitted to the control chart data determines whether or not to categorise it in its initial class, thus making this PGS robust to variations in the training pattern parameters.

In the proposed PGS, unbiased estimation of common-cause parameters (autocorrelation coefficients, standard deviation and mean value) and pattern parameters is ensured. If the autocorrelation and pattern parameters are not estimated simultaneously, an abnormal pattern can badly affect their estimation, i.e., they can be biased due to the mixing of the effects of the two types of noise. For instance, Figure 1(a) shows a Normal pattern generated from an AR(1) process with autocorrelation coefficient  $\phi=0.5$ . Figure 1(b) shows the same pattern added to a trend of slope  $0.05\sigma_y$ . Using equation (1) for estimating  $\phi$  from Figure 1(a) data, the estimated value of autocorrelation is  $\phi=0.50$ .

$$r_1 = \frac{\sum_{t=1}^{n-1} (Y_t - \bar{Y})(Y_{t+1} - \bar{Y})}{\sum_{t=1}^n (Y_t - \bar{Y})^2} \quad (1)$$

where  $r_1$  represents the estimated value of the autocorrelation coefficient of the AR(1) process.

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<sup>1</sup> In this work, autocorrelation will refer to both autoregressive and moving-average models.



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3 On the other hand, in the control chart shown in Figure 1(b), the estimated value of  $\phi$  is 0.81.  
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5 Thus, the estimation of  $\phi$  is biased by the positive trend.  
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8 Such biasing of parameters when assignable causes are present was studied by Woodall and  
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10 Faltin (1993) and Boyles (2000) who found that if pattern recognition is employed to detect  
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12 assignable causes, estimators of common-cause, in this case the autocorrelation level, are  
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14 highly biased when an abnormal pattern is present.  
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20 Figure 1: Two simple patterns with  $\phi=0.50$

21 In the literature, CCPR models that deal with the recognition and classification of significant  
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23 patterns and that can estimate the corresponding parameters while minimising  
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25 misclassification errors are very rare (Lesany et al. 2013). In this work, it is proposed to  
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27 employ a PGS that is able to separate common-cause and assignable-cause effects and  
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29 estimate them simultaneously. However, the estimation is carried out independently as the  
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31 two disturbances are assumed to be independent due to their different sources. The NLM-  
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33 ARMA model used in the proposed PGS ensures such conditions of parameter estimation  
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35 (Pankratz 1991; Hyndman & Athanasopoulos 2014).  
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39 Once the synthetic patterns are generated by applying the proposed PGS, the next step is to  
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41 train the ML algorithm. In this paper, three factors relating to the input of the CCPR system  
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43 are studied: input representation technique (IRT), PGS and kernel of the pattern recognition  
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45 system. Finding arrangements of these factors that achieve the highest accuracies when the  
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47 inherent noise is autocorrelated is another aim of this research.  
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51 The remainder of the paper is organised as follows. Section 2 reviews the most relevant work  
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53 related to this paper. The proposed CCPR is introduced in section 3. Section 4 presents the  
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55 results obtained. Finally, conclusions and suggestions for future research are given in section  
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## 2 Literature review

This section reviews work related to CCPR for autocorrelated processes, synthesis of patterns for training CCPR systems and estimation of parameters of control charts using CCPR systems.

### 2.1. Monitoring autocorrelated processes

The literature on monitoring autocorrelated processes is extensive. This subsection summarises work that is most relevant to this research.

There is a substantial discussion in the literature on choosing the disturbance model for the inherent noise of the process (Wang & Tsung 2007). Authors such as Zhang and Pollard (1994), Nembhard and Kao (2003) and Hwang (2004) used the AR(1) model to describe the inherent disturbance of some industrial processes, Tsung et al. (1998), Jiang and Tsui (2002) and Wang and Tsung (2007) employed the ARMA (1,1) model. Montgomery et al. (2000) and Jiang and Tsui (2002) modelled the inherent disturbance using an integrated moving-average model (IMA (1,1)).

The most common procedure for monitoring autocorrelated processes consists of plotting the residuals of a fitted time series model. If the fitted model is adequate, these residuals will be NIID and thus traditional control charts can be used to monitor them. Recent research found that monitoring residuals affects the detection of mean shifts and highly depends on the ability to fit time series models and a-priori knowledge of the process (Longnecker and Ryan 1992; Zhang 1997; Lu and Reynolds 1999).

Recent advances have been made by forecast-based monitoring schemes to address this problem. Dyer et al. (2003) proposed a forecast-based monitoring scheme for three stationary processes, AR (1), MA (1) and ARMA(1,1). Alwan (1991) investigated the effect of

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3 autocorrelation on masking the effect of special causes and also studied how run-rules and  
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5 static control limits can increase the number of false alarms. Boyles (2000) studied the  
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7 problem of splitting common-cause signals from assignable-cause signals by means of a  
8  
9 standard estimator for first-order autocorrelated processes.  
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12 For a literature review and broader discussion of Statistical Process Control methods for  
13  
14 monitoring autocorrelated processes, see Psarakis and Papaleonida (2007).  
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## 16 17 2.2. CCPR for NIID processes 18

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20 The design of CCPR systems for processes where the inherent disturbance is NIID is the  
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22 most common problem studied in recent research, as this NIID condition is important for  
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24 monitoring production processes by means of traditional control charts, such as  $X$  and  $\bar{X}$   
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26 charts.  
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29 SVM is used as the recognition algorithm in the proposed CCPR system. Adopting this  
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31 algorithm, authors such as Chinnam (2002) and Xanthopoulos and Razzaghi (2014) achieved  
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33 good pattern recognition accuracies. Wu et al. (2015) developed a binary tree SVM-based  
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35 recognition system, also achieving good recognition accuracies. Khormali and Addeh (2016)  
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37 described a CCPR system where the SVM is enhanced by using a type-2 fuzzy c-means  
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39 algorithm. Other authors such as Lu et al. (2011), Du et al. (2013), Xie et al. (2013) and  
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41 Chompu-Inwai and Thaiupathump (2015) utilised signal processing techniques such as  
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43 Independent Component Analysis and Wavelet Transforms to pre-process the control chart  
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45 data, also with good accuracies.  
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49 Barghash and Santarisi (2004) studied the effect of training parameters on the performance of  
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51 the CCPR system, finding that the values range used during pattern generation greatly affects  
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53 Type 1 and Type 2 errors. Cheng and Cheng (2008) highlighted the importance of the same  
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55 parameter in the generalisability of the CCPR model.  
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3 Guh and Tannock (1999) were the first authors that attempted to obtain relevant information  
4 about the pattern once it had been identified as one of the seven simple types. They designed  
5 an ensemble of subsystems where, in a first step, the pattern is recognised and, in the second  
6 step, the magnitude of the pattern is obtained. The creation of ensembles of subsystems for  
7 pattern identification and posterior magnitude identification seems to be the most common  
8 approach in the work reviewed. For example, see Guh (2003, 2005), Jiang et al. (2009) and  
9 Shaban and Shalaby (2012).  
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### 19 2.3. CCPR for autocorrelated processes

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22 The identification of patterns where the inherent noise is not NIID is an infrequently studied  
23 issue, so few papers dealing with this topic were found. The task of CCPR for autocorrelated  
24 processes can be divided into two: identifying changes in process mean, and identifying  
25 abnormal patterns like those proposed in Western Electric Company (1958). Furthermore, all  
26 CCPR systems developed so far have assumed that the inherent disturbance can be  
27 represented by an AR(1) model. CCPR systems for disturbances modelled by other types of  
28 times series such as MA(1) and ARMA(1,1) are needed.  
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38 The first attempts to apply ML algorithms for CCPR with autocorrelated processes were  
39 intended to detect only mean shifts. Chiu et al. (2001) utilised a back-propagation neural  
40 network (BPNN) to identify mean shifts in AR(1) processes with varying autocorrelation  
41 levels. Hwang (2004) monitored the mean value of an autocorrelated process by means of a  
42 BPNN, comparing the monitoring capability with those achieved using Exponentially  
43 Weighted Moving Average (EWMA), special cause control and other charts. Zobel et al.  
44 (2004) developed a BPNN-based technique for CCPR for recognising process mean shifts,  
45 incorporating a data processing classification algorithm. Hwang (2005) proposed a neural-  
46 network-based identification system for detecting mean shifts and correlation changes. Wu  
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3 and Yu (2010) used a selective network ensemble approach called Discrete Particle Swarm  
4 Optimisation for detecting both mean and variance shifts. Guh (2008) developed a real-time  
5 recognition ensemble with nineteen on-line Learning Vector Quantisation (LVQ) networks  
6 for identifying the simple patterns in control charts, training one on-line LVQ for each  
7 correlation level. Cheng and Cheng (2008) developed a neural network recogniser for  
8 autocorrelated processes, using Haar Discrete Wavelet Transform (HDW) to denoise,  
9 decorrelate and extract features for the CCPR system.

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11  
12 Noorossana et al. (2003) were the first to apply neural networks for detecting and classifying  
13 non-random disturbances. The patterns under study were referred to as *level shifts*, *additive*  
14 *outliers* and *innovation outliers*.

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17 Most recent authors have focused on recognition of simple patterns by on-line ensembles. Lin  
18 et al. (2011) designed an on-line real-time CCPR using SVMs as pattern classifiers, training  
19 one SVM for each autocorrelation level. Yang and Zhou (2015) developed an LVQ-based  
20 ensemble for on-line pattern recognition of the seven simple patterns, providing the  
21 autocorrelation level as additional information about the processes.

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23  
24 Cheng and Cheng (2008), Lin et al. (2011) and Yang and Zhou (2015) used an estimator of  
25 autocorrelation for common-cause charts in their recognition systems. They neglected the  
26 biasing effect on the autocorrelation coefficient caused by the abnormal disturbance (Woodall  
27 and Faltin 1993, Dyer et al. 2003), so training the ML algorithms based on erroneous  
28 autocorrelation coefficients.

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31 This paper proposes a methodology for generating training patterns for CCPR systems taking  
32 into account the possible existence of assignable causes. A recognition model will be  
33 presented for each of the following stationary time series models, AR(1), MA(1) and  
34 ARMA(1,1). The models are independent of the autocorrelation level.

### 3 CCPR for autocorrelated processes and parameter estimation

#### 3.1. PGS for autocorrelated processes

Synthetic patterns generated from PGSs for the purpose of training CCPR systems should fulfil the following conditions:

- ❖ Being generated from a broad range of fully randomised training pattern parameters (slope, shift magnitude, break point position, systematic departure (Hachicha & Ghorbel 2012), amplitude and frequency of cycles). This is to ensure that the CCPR system is able to identify a wide variety of patterns.
- ❖ Having parameters that are statistically significant. This is to ensure that correct decision boundaries for pattern classification are obtained by the ML algorithm. For example, the slope of an UT pattern could be reduced or completely removed by noise. The statistical significance of the slope must be tested and found to be positive for a pattern to be classified as a Trend. If this condition is fulfilled, correct decision boundaries for pattern classification can be obtained by the ML algorithm.
- ❖ Being generated by an automatic and reproducible procedure, so allowing other researchers to generate the same patterns for comparison purposes.

In order to produce synthetic patterns that fulfil these conditions, the proposed PGS comprises the following steps:

- i. Generate one of the seven simple patterns using expressions A5 - A9 from the Appendix and randomly choosing the magnitude  $\beta$  of the pattern.

Thus, the control chart data can be modelled by a NLM-ARMA of the following type:

$$Y_t = D_t + N_t \quad (2)$$

where  $D_t$  represents the magnitude of the abnormal pattern and is given by a nonlinear model (to be defined in later steps), and  $N_t$  represents the inherent disturbance to be modelled by one of the three stationary processes utilised in this work (see Appendix).

- ii. Determine the type of mean change in the pattern. Following the methodology proposed by De La Torre Gutierrez and Pham (2016), the determination of the type of mean change is considered as a problem of selecting between two models, in this case, two NLM-ARMA models when  $Y_t$  is as shown in equation (3). The most likely break point is found by fitting piecewise NLM-ARMA models at time  $\tau = 16, 17, \dots, (n - 15)$  (see Appendix).

$$Y_t = \beta_0 + \beta_1 t + \beta_2 d + \beta_3 \sin\left(\frac{2\pi t}{\beta_4}\right) + \beta_5 (-1)^t + N_t \quad (3)$$

where the  $\beta_1$  to  $\beta_5$  and  $d$  represent the pattern parameters as defined in the Appendix.  $\beta_0$  and  $N_t$  represent the intercept with the y-axis of the regression model and the random error at time  $t$ .

The Bayesian Information Criterion (BIC) is extracted from each fitted model and the one with the minimum BIC value is selected. Such a selected model represents the model with the most likely break point.

- iii. Compare the model representing the most likely break point with the one where no break points are assumed. Equation (4) represents the model to be used for  $Y_t$  when there is no break point. Models (3) and (4) represent two NLM-ARMA nested models; the full model is that with the most likely break point and the reduced model the one that does not consider the existence of break points.

$$Y_t = \beta_0 + \beta_1 t + \beta_3 \sin\left(\frac{2\pi t}{\beta_4}\right) + \beta_5 (-1)^t + N_t \quad (4)$$

The hypotheses for this problem of model selection are:

$$H_0: \text{There are no break points (the reduced model fits better)}$$

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3  $H_1$ : A break point is detected (the full model fits better)  
4

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6 The F-statistic shown in (5) is used to determine which model better fits to the control  
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8 data.  
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$$10 \quad F_{v_2}^{v_1} = (SSE_{full} - SSE_{reduced}) / (SSE_{full} / (n - k - 1)) \quad (5)$$

11  
12 where  $SSE_{full}$  and  $SSE_{reduced}$  represent the Sum of Squared Error (SSE) of the full and the  
13 reduced model, respectively. F represents an F-distribution with one degree of freedom  
14 in the numerator ( $v_1 = 1$ ) and  $n - k - 1$  degrees of freedom in the denominator ( $v_2 =$   
15  $n - k - 1$ ), k being the number of parameters in the full model.  
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- 21  
22 iv. Determine pattern class. Once it has been decided whether (3) or (4) fits the pattern  
23 better, the pattern class is determined by the corresponding  $\beta$  value that is statistically  
24 significant. Three significance levels were utilised  $\alpha=0.01$ , 0.02 and 0.03 in this work.  
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29 Figure 2 summarises the proposed PGS.  
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32 Figure 2: Flowchart of the proposed PGS  
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### 34 3.2. Training of the ML

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37 Three sets of 5,600 training patterns, 800 of each type, were generated using  $\alpha = 0.01$ , 0.02,  
38 0.03 using the proposed PGS and another set was created conventionally (Pham et al. 2006,  
39 Pham and Oztemel 1996).  
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43  
44 The Bees Algorithm proposed by Pham et al. (2006) was used to find the best sets of free  
45 parameters that ensure the minimum misclassification rate with the 5-fold cross-validation  
46 technique.  
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50 The Bees Algorithm is a novel population-based optimisation algorithm inspired by the  
51 foraging activities of bees. The algorithm was chosen due to its simplicity and proven search  
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and optimisation capabilities (Pham and Castellani 2014). The parameters of the Bees Algorithm used in this work are shown below.

Initial population ( $n$ )	30
Number of “best” sites ( $m$ )	5
Number of “elite” sites ( $e$ )	2
Patch size for Cost (C) parameter ( $ngh-c$ )	0.5
Patch size for Kernel parameters ( $ngh-k$ )	0.02
Number of elite bees for the elite sites ( $ne$ )	4
Number of bees for the remaining “best” points ( $nb$ )	2

For more explanation of the algorithm and parameters, see Pham et al. (2006) and Pham and Castellani (2014).

Once the best sets of free parameters are found and the SVMs are trained, the pattern recognition accuracies of the recognition systems can be assessed.

### 3.3. Determination of the best arrangement of input factors

For testing purposes, one hundred sets of 700 patterns, 100 of each type, were generated using the best arrangement, and the pattern recognition accuracies obtained.

The three factors studied in the first step of the analysis are as follows:

- IRT: Standardised raw data and shape features. To standardise the raw data, the control chart data are rescaled using the following expression:

$$Z_t = \frac{Y_t - \bar{Y}}{S_y} \quad (6)$$

where  $Z_t$  represents the scaled variable to be used as input for the training of the SVM,  $\bar{Y}$  is the estimated mean value of the current control chart and  $S_y$  is the estimated standard deviation.

The shape features utilised here are those initially proposed by Pham and Wani (1997) and improved by Gauri (2010). These features have advantages in CCPR problems

where the process to be monitored is NIID. These are reduction in the training time, increase in pattern recognition accuracy and independence from the data scale. The performance of these features has not been assessed in CCPR systems where the inherent disturbance is not NIID. Therefore, this is a factor to be considered during the first step of the analysis.

- PGS. The PGS adopted in this work deals with the correct categorisation of training patterns before they are input to the CCPR system. The performance of the recognition systems trained using patterns generated at three  $\alpha$  levels is assessed, and compared with the recognition achieved without the PGS.
- SVM kernel. Four kernels were tested: Radial Basis Function (RBF), Laplace (LAPLA), Hyperbolic Tangent (TANH) and Bessel (BESS).

## 4 Results

### 4.1. Analysis of the PGS

Three sets of 70,000 random vectors, 10,000 of each pattern type, were initially generated. Each vector  $(Y_1, Y_2, \dots, Y_{60})$  represents a quality characteristic sampled at time  $t_1, t_2, \dots, t_{60}$ . Each set of random vectors was created using one of the three first-order stationary models. The patterns were passed through the PGS. Three significance levels were set, namely,  $\alpha = 0.01, 0.02$  and  $0.03$ . The allocation of the patterns to the different classes is shown in Table 1.

In Table 1, for each of the models, the column "Retained" gives the percentages of patterns for which the final classification by the PGS agrees with the classification when the patterns were initially produced in step (i). The column "Reclassified" shows the percentages of patterns for which the classification was changed after they were passed through the PGS.

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3 The column “Discarded” gives the percentages of patterns rejected by the PGS as not  
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5 recognisable due to the low statistical significance of the parameters that characterise them.  
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7

8 Table 1: Allocation of patterns passed through the proposed PGS (%)  
9

10  
11  
12 It can also be observed that when  $\alpha = 0.01$ , 24.26%, 32.91% and 43.40% of the patterns were  
13  
14 discarded for AR, MA and ARMA processes, respectively. When this  $\alpha$  level was used, the  
15  
16 pattern with the highest reclassification rate was the Normal pattern for the MA and ARMA  
17  
18 models, and the US pattern for the AR model. This was probably due to the initial blend of  
19  
20 inherent noise and abnormal disturbance. With the significance level set to  $\alpha = 0.02$ , 35.96%,  
21  
22 44.09% and 53.89% of the patterns were discarded when the AR, MA and ARMA models  
23  
24 were used, respectively. In the case of  $\alpha = 0.03$ , the percentage of discarded patterns  
25  
26 increased to 44.43%, 51.93% and 60.78% for the aforementioned three processes.  
27  
28  
29

## 30 **4.2. Overall accuracies**

31  
32  
33 To simplify the analysis of the results, it will be carried out in two steps. First, the best  
34  
35 arrangement amongst the input factors for each disturbance model is determined; these  
36  
37 factors are IRT, PGS and kernel of the recognition system. Using the best arrangement  
38  
39 determined in the first step of the analysis, the performance of the CCPR system for a set of  
40  
41 patterns generated with a specific autocorrelation level is then assessed.  
42  
43  
44

45  
46 Figures 3, 4 and 5 show the accuracies found for the three inherent noise models. These  
47  
48 accuracies are disaggregated by IRT, Kernel and pattern type, the PGS being the constant  
49  
50 factor in each plot. In these plots, it can be observed that the three lines corresponding to the  
51  
52 three  $\alpha$  levels are nearly coincident; this indicates that the proposed PGS is robust to changes  
53  
54 in  $\alpha$  values.  
55

56  
57 Figure 3: Accuracies achieved from AR process (%)  
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59  
60

Figure 4: Accuracies achieved from MA process (%)

Figure 5: Accuracies achieved from ARMA process (%)

ANOVA of type 4x2x4 with single, double and triple interactions was utilised to determine which factors were significant for the pattern recognition accuracies. The p-values of each factor are shown in Table 2.

Table 2: P-values of the of the three factors obtained from the ANOVA

Tukey post-hoc tests were used to determine the best arrangement for each disturbance model; results are summarised in Table 3.

Table 3: Best arrangement for AR, MA and ARMA processes

### 4.3. Analysis of the best arrangement of input factors

To assess the performance of the SVM trained with the best combination of factors, one hundred sets of 700 patterns, 100 of each type were generated for each autocorrelation level. The combination used is: PGS with  $\alpha = 0.01$ , Raw data and RBF kernel.

Table 4 gives the accuracies found for the AR and ARMA models disaggregated by pattern type and autocorrelation level. For the case of the AR process, an overall accuracy of 90.03% was found. The US pattern type was the one with the least recognition accuracy. It was also found that the accuracy for all pattern types was lowest with  $\phi \geq 0.70$ .

The accuracies achieved for the MA and ARMA processes are shown in Table 5, disaggregated by pattern type and moving-average level. For the MA process, the US pattern

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2  
3 class and patterns of all types generated with  $\theta \geq 0.70$  yielded the lowest recognition  
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5 accuracies.

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7  
8 To obtain the accuracies for the ARMA process of Table 4, the  $\phi$  level was fixed and the  $\theta$   
9  
10 values were randomised. It can be observed that the overall accuracy for the ARMA process  
11  
12 considering all the  $\phi$  levels is 89.08%, the Normal pattern having the lowest accuracy. Again,  
13  
14 an autocorrelation level of  $\phi \geq 0.70$  produced the least accuracy.

15  
16  
17 Likewise in Table 5, to obtain the accuracies for the ARMA process, the  $\theta$  was fixed and the  
18  
19  $\phi$  levels were randomised. It can be seen that the lowest accuracies were obtained for Normal  
20  
21 patterns and  $\theta \geq 0.70$ .

22  
23  
24 Table 4: Accuracies for AR and ARMA processes by  $\phi$  values (%)

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26  
27  
28  
29 Table 5: Accuracies for MA and ARMA processes by  $\theta$  values (%)

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31  
32  
33 The performance of the proposed CCPR systems was also analysed by type and magnitude of  
34  
35 the pattern. Table 6 presents the accuracies obtained by the CCPR systems of the three  
36  
37 process types disaggregated by pattern type and magnitude. It can be observed that, for the  
38  
39 three process types, the US and DS patterns showed the lowest and the highest accuracies,  
40  
41 respectively. It can also be seen that the recognition accuracies decreased as the pattern  
42  
43 magnitude was reduced.  
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46  
47 Table 6: Accuracies for the three processes by pattern magnitude (%)  
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#### 4.4. Real data application

Figure 6: Thickness of metallic film in the early stages of the development of an electronic device (Box et al. 2009)

To demonstrate the ability of the proposed CCPR system to handle real data, thickness measurements for a very thin metallic film in the early stages of the development of an electronic device data taken from Box et al. (2009) were utilised (see Figure 6). Box et al. (2009) highlighted the existence of an assignable cause that abruptly increased the metallic film thickness after 30 observations.

The CCPR trained with the best arrangement of input factors for the AR model was applied, and an US pattern was identified.

In order further to categorise the pattern recognised by the aforementioned CCPR system, the two NLM-ARMA models proposed in this paper were fitted to the original data. The p-value corresponding to the F-test for nested models was found to be 0.000002; therefore, the full model fitted better the data. The most likely breakpoint was detected at  $\tau = 30$ .

Table 7 shows all the values for the fitted full model. It can be observed that the parameter related to the Shift pattern is statistically significant and greater than zero. Thus like the CCPR, the NLM-ARMA classifies the pattern as an US pattern.

Table 7: ANOVA of the NLM-ARMA model fitted to the metallic film thickness data

## 5 Conclusion

The literature review uncovered very few papers dealing with monitoring of non-NIID production processes, the AR(1) model being the only model studied. First-order stationary models are used to represent many continuous production processes. It is necessary to develop robust CCPR systems that allow the automated identification of abnormal patterns where these types of processes are monitored using control charts.

The proposed PGS fulfils three conditions: generality, comparability and facilitation of the extraction of further information from the pattern. Generality was achieved by employing a broad range of pattern parameters during the initial pattern generation and the total randomisation of parameters, including the break point position and the amplitude of Cyclic patterns. Comparability between different studies is possible as the proposed PGS is a standard technique for producing the same data for training CCPR systems, requiring only the significance level to be set. Further information (for example, the amplitude of cycles) regarding the abnormal pattern identified can be extracted as the decision boundaries were estimated using patterns for which the statistical significance of the parameters was tested. Therefore, the CCPR system can categorise the presented patterns into the class for which the related parameter is the most significant when a NLM-ARMA model is fitted. Furthermore, by fitting this dynamic model to the control chart data, the PGS is able to divide signals into inherent noise and fault signal.

From the first part of the recognition accuracy analysis, of the four different PGS ( $\alpha = 0.01$ , 0.02, 0.03 and conventional PGS), the proposed PGS significantly increased the pattern recognition accuracy for the three models used. The three  $\alpha$  levels achieved nearly the same results, this indicates that the proposed PGS is robust to changes in  $\alpha$  values. It is

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3 recommended to use low significance levels such as 0.01 in order to reduce computational  
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5 efforts and generate fewer patterns.  
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8 Regarding the IRT, the shape features and raw data achieved similar accuracies in the MA  
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10 and ARMA models, but lower accuracies were obtained with shape features when the AR  
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12 process was utilised. Two kernels, RBF and Bessel, showed the greatest recognition  
13  
14 accuracies.  
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17 In the second part of the analysis, it was found that the values of  $\theta$  and  $\phi$  greatly affected the  
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19 pattern recognition accuracy. For the MA and ARMA processes, the lowest accuracy was  
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21 found when  $\theta \geq 0.70$ , and the highest accuracy was achieved when  $-0.5 < \theta \leq -0.3$ . For AR  
22  
23 and ARMA processes, the same behaviour was observed, with the lowest accuracy obtained  
24  
25 when  $\phi \geq 0.70$ , and the highest accuracy when  $-0.5 < \phi \leq -0.3$ . Regarding the patterns, the  
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27 Normal and US types were those that gave the lowest accuracies.  
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31 The implementation of the proposed recognition system on a real production line is  
32  
33 straightforward; the operator only needs to set the observation window to 60, collect data  
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35 from the production variable being monitored and then input it to a CCPR system trained  
36  
37 using the best arrangement of factors determined in section 4.2.  
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40  
41 Future research can proceed in three directions: (i) developing CCPR systems and PGS for  
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43 non-stationary processes such as IMA and ARIMA, (ii) expanding the proposed methodology  
44  
45 to patterns combining two or more pattern types, and (iii) developing PGS and CCPR  
46  
47 systems for multivariate and stationary variables, using different control chart types to  
48  
49 monitor quality characteristics.  
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51



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## 8 Appendix

The first step in producing patterns for the proposed PGS is to generate the inherent noise according to one of the three models. To generate such an inherent disturbance, it is necessary to create a white-noise vector. Following the methodology proposed by De La Torre Gutierrez and Pham (2016), an NIID vector,  $e_t$ , is produced based on the pseudo-random number generator proposed by Matsumoto and Nishimura (1998), i.e.

$$e_t = \text{Normal}(\mu = 0, \sigma_e = 1) \quad (\text{A1})$$

The methodology for generating ARMA (1,1) processes from normal white-noise proposed by Box et al. (2011) was adopted in this work.

Therefore, the inherent noise represented as an ARMA(1,1) process is represented as:

$$N_t = \phi N_{t-1} - \theta e_{t-1} + e_t \quad (\text{A2})$$

where if  $\phi = 0$ , an MA(1) process is obtained; and if  $\theta = 0$ , an AR(1) model is obtained.

Both  $\phi$  and  $\theta$  are in the range (-1, +1).

The standard deviation of this ARMA(1,1) process can be estimated by means of the following expression:

$$\sigma_N = \sqrt{\frac{1 + \theta^2 - 2\phi\theta}{1 - \phi^2}} \sigma_e \quad (\text{A3})$$

The model for disturbances in control charts, assuming independence between the inherent noise and deterministic disturbance, is the following:

$$Y_t = D_t + N_t \quad (\text{A4})$$

where  $Y_t$  represents the process to be monitored,  $D_t$  the deterministic disturbance and  $N_t$  the inherent noise. The seven simple patterns can be generated by using the following expressions:

- Normal (NORM):

$$Y_t = \mu + N_t \quad (A5)$$

- Upward/Downward Trend (UT/DT):

$$Y_t = \mu + N_t \pm \beta_1 t \quad (A6)$$

- Upward/Downward Shift (US/DS):

$$Y_t = \mu + N_t \pm \beta_2 d \quad (A7)$$

- Cyclic (CYC):

$$Y_t = \mu + N_t + \beta_3 \sin\left(\frac{2\pi t}{\beta_4}\right) \quad (A8)$$

- Systematic (SYS):

$$Y_t = \mu + N_t + \beta_5 (-1)^t \quad (A9)$$

Without loss of generality, in this work  $\mu$  is set to zero.

The meaning and range of the pattern parameters used are as described in De La Torre Gutierrez and Pham (2016).

The magnitude shift, slope, systematic departure and cyclic amplitude were kept within in  $6\sigma_N$  control limits in the inspection window. The frequency of the cyclic pattern was determined to show at least four cycles in the inspection window.

The break point position was randomly chosen between  $\tau = 16$  and  $\tau = n - 15$ . This is due the number of parameters and degrees of freedom available during the estimation of the parameters when NLM-ARMA is utilised.

## TABLES

Table 1: Allocation of patterns passed through the proposed PGS (%)

	AR			MA			ARMA		
	Retained	Reclassified	Discarded	Retained	Reclassified	Discarded	Retained	Reclassified	Discarded
$\alpha = 0.01$	64.45	11.29	24.26	57.12	9.97	32.91	46.67	9.92	43.40
NORM	62.70	20.53	16.77	52.17	22.94	24.89	38.08	26.51	35.40
UT	67.57	2.60	29.83	58.79	1.80	39.41	46.25	3.17	50.58
DT	67.65	2.83	29.52	58.78	1.86	39.37	46.16	3.18	50.66
US	59.78	21.43	18.80	57.36	17.85	24.79	54.80	13.62	31.58
DS	59.89	21.27	18.85	57.38	18.10	24.52	54.78	13.46	31.76
CYC	69.63	1.49	28.88	56.94	2.14	40.92	41.95	2.15	55.90
SYS	63.96	8.88	27.16	58.41	5.10	36.49	44.71	7.36	47.93
$\alpha = 0.02$	54.30	9.74	35.96	47.34	8.57	44.09	37.83	8.28	53.89
NORM	48.20	25.52	26.28	38.74	26.16	35.10	26.98	27.29	45.73
UT	53.55	2.51	43.94	45.87	1.76	52.37	34.85	2.72	62.43
DT	53.77	2.68	43.55	45.98	1.81	52.21	34.87	2.77	62.36
US	58.12	14.74	27.15	54.47	12.33	33.20	51.23	8.87	39.90
DS	58.31	14.58	27.11	54.47	12.64	32.89	51.17	8.86	39.96
CYC	56.28	0.96	42.76	45.34	1.39	53.28	31.69	1.47	66.84
SYS	51.90	7.17	40.93	46.52	3.89	49.59	34.01	5.98	60.02
$\alpha = 0.03$	46.85	8.72	44.43	40.48	7.59	51.93	32.01	7.21	60.78
NORM	38.58	27.48	33.94	30.35	26.81	42.84	20.54	26.43	53.02
UT	43.77	2.39	53.84	37.14	1.69	61.18	27.72	2.42	69.87
DT	43.89	2.53	53.58	37.35	1.76	60.88	27.72	2.49	69.79
US	55.78	11.02	33.20	51.55	9.28	39.18	48.00	6.39	45.61
DS	55.93	10.84	33.23	51.41	9.38	39.21	47.93	6.47	45.60
CYC	46.79	0.69	52.52	37.28	0.99	61.73	25.06	1.12	73.82
SYS	43.23	6.09	50.68	38.25	3.22	58.53	27.10	5.16	67.74



Table 2: P-values of the of the three factors obtained from the ANOVA

p-values			
Factor	AR	MA	ARMA
PGS	0.0000	0.0000	0.0000
KERNEL	0.0000	0.0000	0.0000
IRT	0.0000	0.5150	0.0638
PGS*KERNEL	0.0186	0.1800	0.1510
KERNEL*IRT	0.0000	0.0000	0.0000
PGS*IRT	0.0583	0.6680	0.2197
PGS*KERNEL*IRT	0.0000	0.0000	0.0000

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Table 3: Best arrangement for AR, MA and ARMA processes

Disturbance model	AR	MA	ARMA
PGS	$\alpha = 0.01, 0.02, 0.03$	$\alpha = 0.01, 0.02, 0.03$	$\alpha = 0.01, 0.02$
IRT	Raw data	Raw data, Shape features	Raw data, Shape features
Kernel	Bessel/RBF	Bessel/RBF	Bessel/RBF

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Table 4: Accuracies for AR and ARMA processes by  $\phi$  values (%)

$\phi$	OVERALL	NORM	UT	DT	US	DS	CYC	SYS
AR	90.03	83.92	91.08	93.60	79.70	96.64	90.97	94.28
$\phi \leq -0.7$	91.86	77.50	91.49	94.21	93.17	99.82	91.80	95.00
$-0.7 < \phi \leq -0.5$	92.97	80.00	93.51	93.65	96.54	99.78	93.13	94.33
$-0.5 < \phi \leq -0.3$	92.98	82.63	93.93	93.46	94.92	98.52	93.46	93.83
$-0.3 < \phi \leq 0.0$	92.19	87.17	93.07	93.07	87.94	97.44	92.51	94.12
$0.0 < \phi \leq 0.3$	90.47	89.52	91.94	93.20	77.37	95.46	91.69	94.10
$0.3 < \phi \leq 0.5$	89.22	90.24	91.29	93.79	68.76	95.06	91.08	94.31
$0.5 < \phi < 0.7$	87.39	86.13	89.15	93.71	65.27	94.54	88.99	93.91
$\geq 0.7$	83.15	78.15	84.26	93.68	53.65	92.53	85.09	94.66
ARMA	89.08	80.61	89.63	93.17	82.43	95.55	89.44	92.77
$\phi \leq -0.7$	88.22	67.83	88.72	90.76	90.94	99.03	87.40	92.83
$-0.7 < \phi \leq -0.5$	90.26	75.62	89.78	92.16	93.68	98.56	90.33	91.67
$-0.5 < \phi \leq -0.3$	91.75	82.37	92.35	93.95	93.02	97.13	91.26	92.16
$-0.3 < \phi \leq 0.0$	91.64	86.73	92.05	94.08	88.27	96.53	91.63	92.17
$0.0 < \phi \leq 0.3$	91.22	88.64	91.79	94.68	83.17	95.77	91.68	92.81
$0.3 < \phi \leq 0.5$	90.55	88.27	91.74	94.49	79.30	94.97	91.17	93.93
$0.5 < \phi < 0.7$	88.00	84.72	88.42	93.98	72.63	93.83	88.66	93.73
$\phi \geq 0.7$	81.05	70.70	82.15	91.28	58.39	88.60	83.36	92.85

Table 5: Accuracies for MA and ARMA processes by  $\theta$  values (%)

$\theta$	OVERALL	NORM	UT	DT	US	DS	CYC	SYS
MA	91.47	84.86	92.92	94.39	82.66	97.75	93.15	94.59
$\theta \leq -0.7$	91.88	80.39	89.36	94.24	95.90	99.99	88.76	94.52
$-0.7 < \theta \leq -0.5$	92.56	80.75	92.40	93.82	94.15	99.97	92.32	94.54
$-0.5 < \theta \leq -0.3$	93.22	81.64	94.59	94.16	93.18	99.96	94.39	94.60
$-0.3 < \theta \leq 0.0$	92.86	83.07	95.20	94.15	88.41	99.32	95.41	94.43
$0.0 < \theta \leq 0.3$	92.18	86.99	95.13	94.29	80.71	98.14	95.29	94.73
$0.3 < \theta \leq 0.5$	90.41	87.48	93.54	94.84	72.76	96.06	93.65	94.51
$0.5 < \theta < 0.7$	89.85	89.27	92.99	95.07	68.99	94.53	93.33	94.79
$\theta \geq 0.7$	88.83	89.29	90.18	94.52	67.21	94.03	92.03	94.57
ARMA	89.48	81.47	89.98	93.42	83.65	95.71	89.50	92.65
$\theta \leq -0.7$	89.21	74.55	87.04	91.52	96.34	98.40	85.57	91.07
$-0.7 < \theta \leq -0.5$	90.23	77.16	89.67	93.29	93.99	98.13	87.87	91.47
$-0.5 < \theta \leq -0.3$	90.84	80.42	91.20	93.74	89.77	97.55	91.00	92.18
$-0.3 < \theta \leq 0.0$	90.71	80.43	92.67	94.59	85.38	96.66	92.61	92.61
$0.0 < \theta \leq 0.3$	90.34	83.97	92.64	94.07	80.50	96.20	91.66	93.35
$0.3 < \theta \leq 0.5$	89.58	84.75	91.04	94.24	76.37	94.78	91.33	94.56
$0.5 < \theta < 0.7$	88.29	85.62	89.30	93.41	74.22	93.68	89.16	92.65
$\theta \geq 0.7$	86.67	84.84	86.29	92.47	72.66	90.29	86.81	93.30

Table 6: Accuracies for the three processes by pattern magnitude (%)

	Magnitude	ARMA	MA	AR
UT	TOTAL	89.80	92.92	91.08
	$0.25 \leq \beta_1 \leq 0.30$	99.74	100.00	100.00
	$0.20 \leq \beta_1 < 0.25$	99.59	100.00	100.00
	$0.15 \leq \beta_1 < 0.20$	99.25	100.00	99.99
	$0.10 \leq \beta_1 < 0.15$	98.47	99.96	99.72
	$0.05 \leq \beta_1 < 0.10$	88.74	95.77	93.43
	$0.00 < \beta_1 < 0.05$	53.01	61.77	53.35
DT	TOTAL	93.30	94.39	93.60
	$-0.25 \geq \beta_1 \geq -0.30$	99.83	100.00	100.00
	$-0.20 \geq \beta_1 > -0.25$	99.75	100.00	100.00
	$-0.15 \geq \beta_1 > -0.20$	99.38	100.00	99.95
	$-0.10 \geq \beta_1 > -0.15$	98.51	99.99	99.62
	$-0.05 \geq \beta_1 > -0.10$	91.52	95.17	92.26
	$0.00 > \beta_1 > -0.05$	70.82	71.17	69.75
US	TOTAL	83.00	82.66	79.70
	$2.5 \leq \beta_2 \leq 3.0$	96.65	98.64	97.00
	$2.0 \leq \beta_2 < 2.5$	96.80	98.63	96.38
	$1.5 \leq \beta_2 < 2.0$	96.34	98.25	95.07
	$1.0 \leq \beta_2 < 1.5$	92.70	96.10	89.58
	$0.5 \leq \beta_2 < 1.0$	72.72	76.87	70.06
	$0.0 < \beta_2 < 0.5$	42.79	27.47	30.09
DS	TOTAL	95.60	97.75	96.64
	$-2.5 \geq \beta_2 \geq -3.0$	97.99	99.96	99.82
	$-2.0 \geq \beta_2 > -2.5$	97.56	98.94	98.79
	$-1.5 \geq \beta_2 > -2.0$	96.12	98.38	97.94
	$-1.0 \geq \beta_2 > -1.5$	95.40	97.96	97.86
	$-0.5 \geq \beta_2 > -1.0$	94.24	96.50	94.93
	$0.0 > \beta_2 > -0.5$	92.31	94.73	90.48
CYC	TOTAL	89.50	93.15	90.97
	$2.5 \leq \beta_3 \leq 3.0$	96.51	98.50	98.76
	$2.0 \leq \beta_3 < 2.5$	94.05	98.41	98.14
	$1.5 \leq \beta_3 < 2.0$	91.27	96.44	96.93
	$1.0 \leq \beta_3 < 1.5$	90.09	93.33	89.37
	$0.5 \leq \beta_3 < 1.0$	79.85	88.20	83.64
	$0.0 < \beta_3 < 0.5$	85.20	84.00	79.00
SYS	TOTAL	92.70	94.59	94.28
	$2.5 \leq \beta_5 \leq 3.0$	99.98	100.00	100.00
	$2.0 \leq \beta_5 < 2.5$	99.98	100.00	100.00
	$1.5 \leq \beta_5 < 2.0$	99.91	100.00	100.00

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	$1.0 \leq \beta_5 < 1.5$	99.55	100.00	99.97
	$0.5 \leq \beta_5 < 1.0$	96.51	97.22	96.60
	$0.0 < \beta_5 < 0.5$	60.25	70.33	69.10

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Table 7: ANOVA of the NLM-ARMA model fitted to the metallic film thickness data

Parameter	Estimate	Std. Error	t value	p-value
$\phi$	-0.2632	0.1075	-2.4484	0.0144
Intercept	80.4461	2.1699	37.0736	0.0000
Slope (Trend)	0.1750	0.0986	1.7748	0.0760
Shift magnitude (Shift)	23.0429	4.0195	5.7328	0.0000
Amplitude (Cyclic)	2.1916	1.4738	1.4870	0.1370
Frequency (Cyclic)	8.2311	1.0233	8.0436	0.0000
Departure (Systematic)	0.1576	1.3958	0.1129	0.9101

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**FIGURES**

Figure 1: Two simple patterns with  $\phi=0.50$

Figure 2: Flowchart of the proposed PGS

Figure 3: Accuracies achieved from AR process (%)

Figure 4: Accuracies achieved from MA process (%)

Figure 5: Accuracies achieved from ARMA process (%)

Figure 6: Thickness of metallic film in the early stages of the development of an electronic device (Box et al. 2009)

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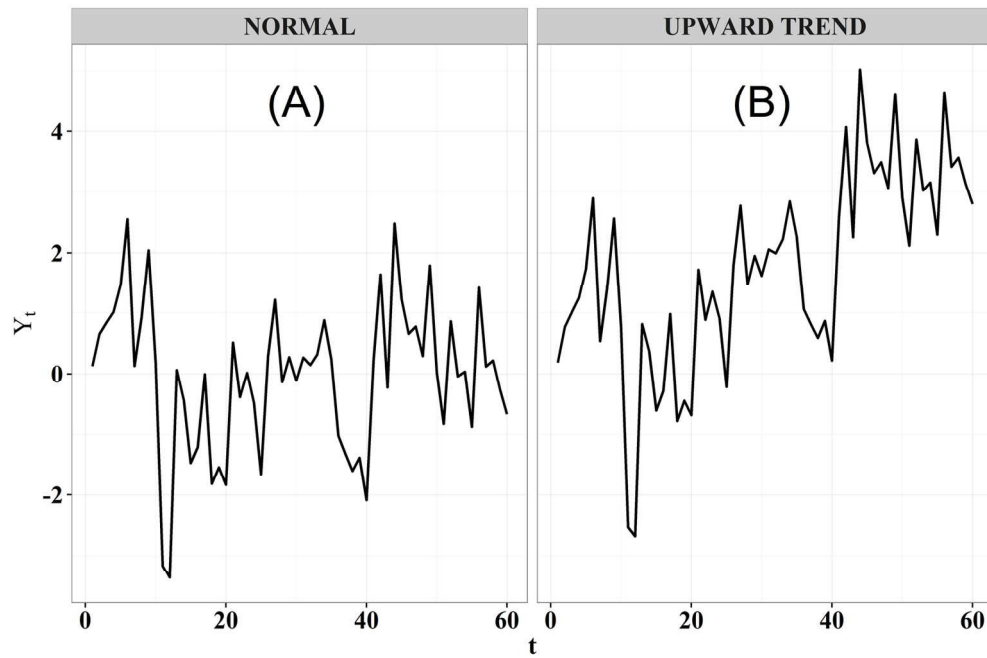


Figure 1: Two simple patterns with  $\phi=0.50$

152x101mm (300 x 300 DPI)

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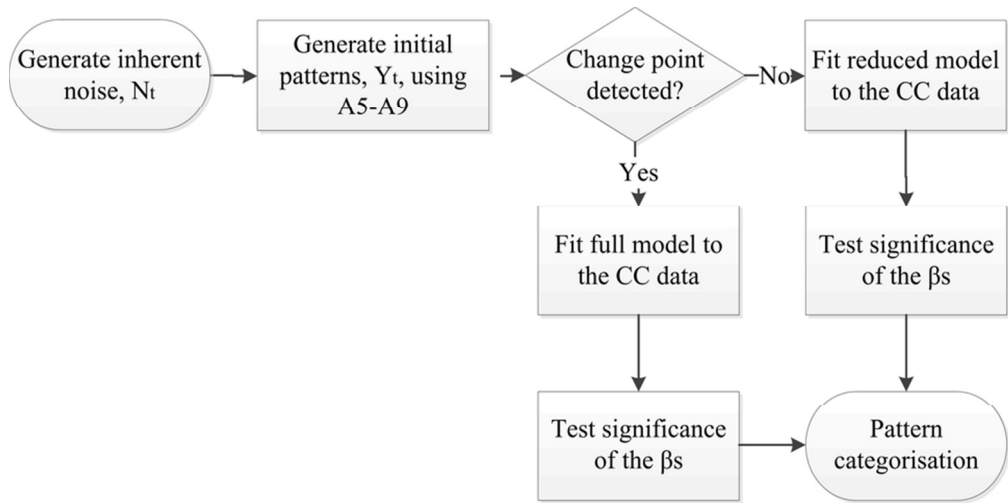


Figure 2: Flowchart of the proposed PGS

81x40mm (300 x 300 DPI)

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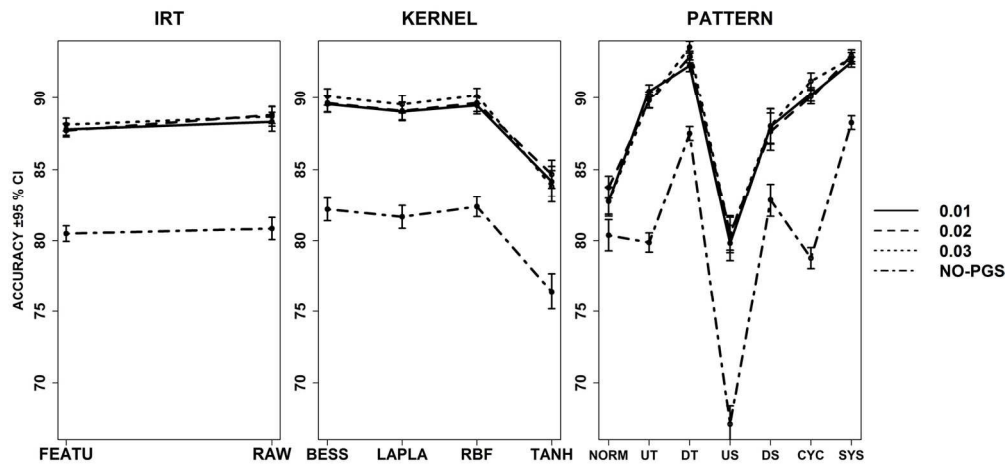


Figure 3: Accuracies achieved from AR process (%)

152x76mm (300 x 300 DPI)

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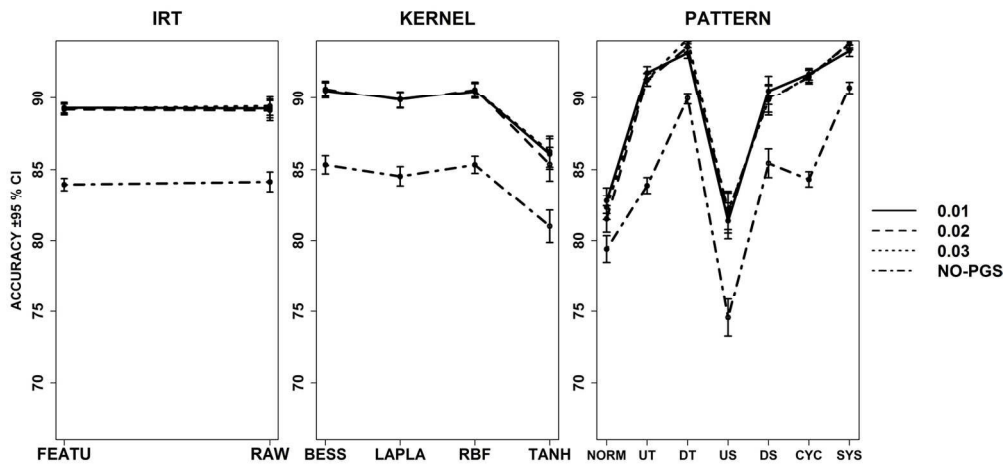


Figure 4: Accuracies achieved from MA process (%)

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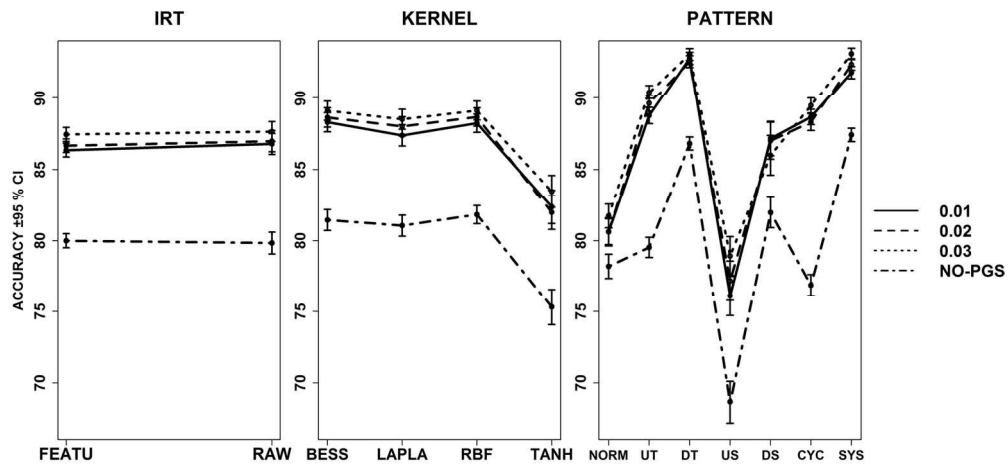


Figure 5: Accuracies achieved from ARMA process (%)

152x76mm (300 x 300 DPI)

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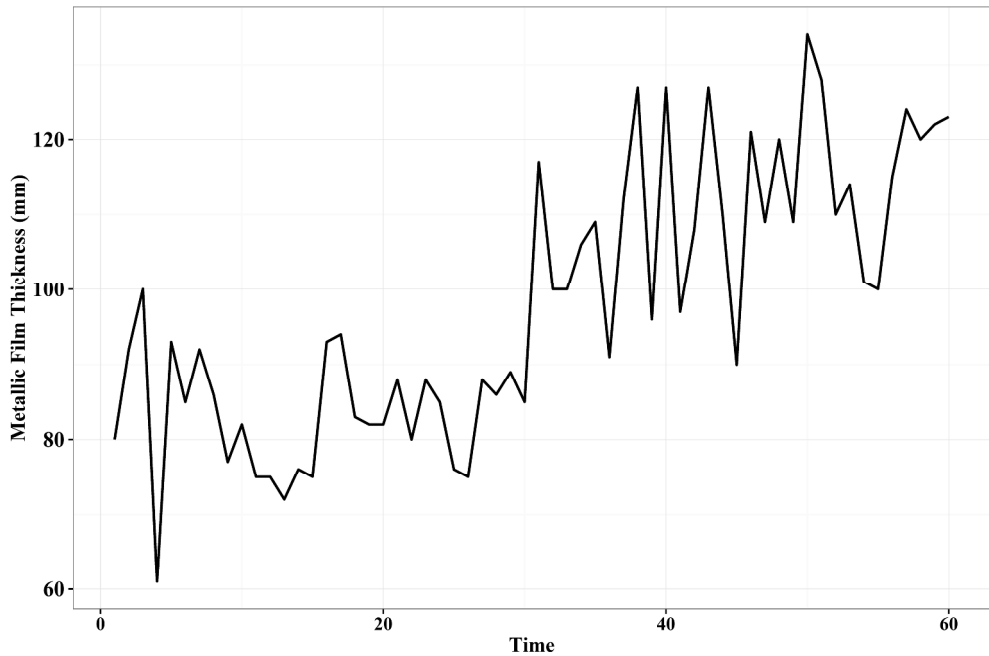


Figure 6: Thickness of metallic film in the early stages of the development of an electronic device (Box et al. 2009)

152x101mm (600 x 600 DPI)

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