Heat generating porous matrix effects on Brownian motion of nanofluid

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<td>Research Article</td>
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<td>Keywords:</td>
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Abstract

In the present study, effect of mounting heat generating porous matrix in a close cavity on the Brownian term of CuO-water nanofluid was studied numerically. Because of presence of heat source in porous matrix, couple of energy equations is solved for porous matrix and nanofluid separately. Thermal conductivity and Viscosity of nanofluid were assumed to be consisting of a static component and a Brownian component that were functions of volume fraction of the nanofluid and temperature. To explain the effect of the Brownian term on the flow and heat fields, different parameters such as heat conduction ratio, interstitial heat transfer coefficient, Rayleigh number, concentration of nanoparticles and porous material porosity were investigated and the obtained results were compared to those of the non-Brownian solution. Results showed that the Brownian term increases the viscosity of nanofluid, so this term affects the velocity and smoothness of streamlines. Furthermore the porous matrix is cooled with higher Nusselt number because of rising thermal conductivity. Besides, the effect of the Brownian term was seen to be greater at low Rayleigh number, low-porosity and small thermal conductivity of the porous matrix. It was further seen that, mounting the porous material into cavity changes the temperature distribution and increases Brownian term effect and heat transfer functionality of the nanofluid. It is noteworthy that due to decrement of thermal conduction in high porosities, the impact of Brownian term drops severely that it is possible to obtain reliable results even in the case of neglecting the Brownian term in these porosities.

Keywords: Nanofluid, Brownian motion, Porous matrix, Dependent internal heat generation, LTNE model.

1. Introduction
Natural heat transfer through cavities saturated with heat generating porous media is widely investigated due to its extensive applications in the fields of thermal and geothermal energy, heat convection management (due to buried atomic wastes), combustion technology, porous catalysts, soil pollution, exothermic reactions inside porous reactors, and fuel cells performance enhancement (Rashidi, 2012; Ioan, 2016; Alsabery, 2016; Sheremet, 2015; Rashad, 2017; Qiang, 2013; Alsabery, 2017; Sheikholeslami, 2015). Likewise, there is a novel concept known as nanofluids introduced by Choi (1995) has gained a great deal of attention in a wide range of studies to enhance heat transfer rate by higher thermal conductivity of nanufluids compared to the base fluid. Many industrial processes such as catalytic chemistry, medicine, biology and environmental applications have adopted the approaches concerned with nanotechnologies.

To calculate the viscosity and thermal conductivity of nanofluid, Koo- Kleinstreuer (2004) correlation was employed. Results indicated that the Nusselt number increases by increasing the Rayleigh number and volume fraction of nanoparticle. Also, it increases by decreasing Hartmann number. In further attempts, skin friction coefficient and the Nusselt number of nanofluid were investigated over a stretching sheet with transverse magnetic field, thermal radiation and buoyancy effects were studied by Rashidi et al. (2014). They show that the skin friction coefficient values of Cu nanofluid are greater than CuO nanofluid. The reduced Nusselt number values of Cu are less than CuO nanofluid.

MHD natural convection in a heat generating porous enclosure saturated with Cu-water nanofluid has been studied by Rashad et al. (2017) numerically. Increment of volume fraction of Cu nanoparticles causes to decrement of nanofluid circulation. Besides variation of magnetic flow direction in porous cavity increases the average Nusselt number. Aminossadati and Ghasemi (2011) presented a numerically investigation of laminar natural convection in a two-dimensional
The natural convection was generated using two pairs of heat source-sink located on the bottom wall of the cavity and CuO-water nanofluid was used acting as working fluid in their study. The main objective of their assessment was to enhance heat transfer at different volume fraction and Rayleigh numbers. The conjugate mixed convection of $\text{Al}_2\text{O}_3$-water inside a double lid-driven square cavity with an inner square solid body has been studied by Alsabery et al. (2018). The Buongiorno’s model has been utilized which shows that applying $\text{Al}_2\text{O}_3$ nanoparticles has an evident enhance of heat transfer in this case. The results show in all conditions except high values of Reynolds and Richardson numbers the dimensions of inner square is adversely proportional to heat transfer increment. Rashad et al. (2018) have assessed entropy generation in MHD natural convection flow according to length and location variation of source and sink. Also it was shown that angels 40, 50 and 300 are the best values for heat transfer in all location of heat source. Mansour et al. (2016) numerically studied MHD natural convection in a cavity saturated by nanofluid. The authors have assessed four different cases based on various arrangements of thermal boundary conditions. Rashidi et al. (2011) studied heat transfer in a porous medium with radiation. Homotopy analysis method was used to achieve a complete analytic solution. The velocity and temperature profiles were illustrated and effect of coupling constant, permeability and radiation parameter on heat transfer of micropolar fluid was investigated. Natural convection heat transfer inside the anisotropic porous cavity saturated with micropolar $\text{Al}_2\text{O}_3$/water nanofluid has been investigated by Ahmed and Rashad (2016). Increment of volume fraction of micropolar nanofluid causes to heat transfer enhancement noticeably. Moreover increment in permeability ratio of porous medium results in lower circulation and velocity. Beckermann et al. (1987) experimentally and numerically studied on an enclosure partially filled porous material. Obtained results denoted that penetration of fluid to porous section which is function of the multiplication of
Darcy and Rayleigh numbers, so this penetration could totally change the flow and thermal fields. Moreover, Second law of thermodynamic was studied in a porous rotating disk with an electrically incompressible nanofluid in a uniform vertical magnetic field by Rashidi et al. (2013). They considered various parameters like volume fraction of suspended nanoparticles, suction and magnetic parameters on velocity, thermal field and entropy generation. This paper established a calculation method for rotating fluidic systems which utilized the second law of thermodynamics.

Considering the studies performed so far, it is well obvious that, natural convection of nanofluid in porous cavity has been among the hot topics considered by researchers in recent years, with its different aspects investigated by far. Nevertheless, considering the diversity of the contributing parameters and governing equations applied to nanofluids and porous substances, the need for further studies to complement previous works is obvious. In this paper, despite the previous works, the effect of mounting the porous matrix with internal heat generation was investigated on the improvement of variable properties of nanofluid. In this way, the assumption of Koo- Kleinstreuer was utilized. This assumption considered that the properties of the nanofluid are equal to the summation of constant and variable properties. The amount of heat generation inside the chamber is a function of temperature. Furthermore, in order to address imperfections of previous studies regarding the effects of the introduction of the porous matrix on increasing Brownian term of nanofluid and achieving the highest Nusselt number, effective range of porosity, Rayleigh number, volume fraction of suspended nanoparticles, thermal conductivity, and thermal convection coefficient of the porous substance were further assessed. Results of Brownian term and non-Brownian states were calculated and compared to each other by isotherms, streamlines and Nusselt number of them.

2. Numerical modeling
2.1 Physical model and governing equation

Figure 1 shows the studied problem consists of a square cavity saturated with CuO-water which its length is L. Non-deformable, isotropic and homogeneous and heat generating square porous matrix by length of L/2 is mounted in center. Meanwhile the generated heat depends on temperature difference. The temperatures of left and right walls are set to temperatures $T_h$ and $T_c$ respectively. Furthermore adiabatic and impermeable assumptions are considered for top and bottom boundaries. Density varies based on Boussinesq approximation. Also thermal conductivity and viscosity of nanofluid depend on the volume fraction and temperature. Thermophysical properties of CuO nanoparticles and base fluid are listed in Table 1. Also, Newtonian nanofluid is considered and the flow regime is laminar and incompressible. The nanoparticles are in thermal equilibrium with base fluid (water), but the nanofluid and porous matrix are not thermal-equilibrium. Because of presence of heat source in porous matrix, couple of energy equations are solved for porous matrix and nanofluid separately. For modeling solid block the porosity is assumed to be 0.005 instead of zero. Also, the dimensions of nanoparticles are very small compared to pore size and suspended nanoparticles are not deposited and agglomerated on the porous material (Nield and Kuznetsov, 2009).

A binary parameter $\delta$ is introduced to combine porous and nanofluid governing equations, which values of zero and one for $\delta$ are used to categorize nanofluid and porous media regions, respectively.

The dimensionless forms of governing equations are as following (Kayhani et al. 2011; Kim at al. 2001):

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$
\[
\left(\frac{\delta}{\varepsilon^2} - (\delta - 1)\right) \left[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right] = - \frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right] + \delta \frac{\mu_{nf}}{\rho_{nf} \alpha_f D a} U \tag{2}
\]

\[
\left(\frac{\delta}{\varepsilon^2} - (\delta - 1)\right) \left[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right] = - \frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right] + \delta \frac{\mu_{nf}}{\rho_{nf} \alpha_f D a} V + \frac{\beta_{nf}}{\beta_f} R a_f P r_f \theta_{nf} \tag{3}
\]

\[
U \frac{\partial \theta_{nf}}{\partial X} + V \frac{\partial \theta_{nf}}{\partial Y} = (\delta(\varepsilon - 1) + 1) \frac{\alpha_{nf}}{\alpha_f} \left[ \frac{\partial^2 \theta_{nf}}{\partial X^2} + \frac{\partial^2 \theta_{nf}}{\partial Y^2} \right] + \delta \theta_s - \theta_{nf} \tag{4}
\]

\[
0 = \delta(1 - \varepsilon) R_k \left \{ \frac{(\rho c)_f}{(\rho c)_{nf}} \right \} \left[ \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} \right] + \delta \theta_s - \theta_{nf} + \delta \frac{\alpha_{nf}}{\alpha_f} q \theta_s \tag{5}
\]

Where \( \varepsilon \) is the porosity of porous material which values between 0 and 1 relate the porosity of solid matrix and for nanofluid equals to 1. By applying the dimensionless parameters, the following terms can be introduced as (Kayhani, 2011; Teamah, 2012):

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{UL}{\alpha_t}, \quad V = \frac{VL}{\alpha_t}, \quad Da = \frac{K}{L^2}, \quad Pr_f = \frac{\nu_f}{\alpha_t}
\]

\[
H = \frac{hL^2}{(\rho c)_{nf} \alpha_f}, \quad R_k = \frac{k_s}{k_f}, \quad P = \frac{pL^2}{\rho_{nf} \alpha_f^2}, \quad \theta_t = \frac{T_c - T_e}{T_h - T_c}, \quad \theta_s = \frac{T_s - T_c}{T_h - T_c}, \quad \text{Ra}_f = \frac{g \beta_t L^3 (T_h - T_c)}{\nu_f \alpha_t}, \quad q = \frac{Q_0 L^2}{(\rho c)_{nf} \alpha_{nf}} \tag{6}
\]

By assuming spherical based porous matrix, permeability of porous media can be defined by

\[
\text{Organ equation (1952)}:
\]

\[
K = \frac{d^2 \varepsilon^3}{175(1 - \varepsilon^2)} \tag{7}
\]

Also the properties of nanofluid can be formulated as below (Khanafer, 2003):

\[ \rho_{nf} = \varphi \rho_p + (1 - \varphi) \rho_f \]  
\[ (\rho c)_{nf} = \varphi (\rho c)_p + (1 - \varphi) (\rho c)_f \]  
\[ (\rho \beta)_{nf} = \varphi (\rho \beta)_p + (1 - \varphi) (\rho \beta)_f \]  

Based on Koo- Kleinstreuer (2004; 2005) equations, Viscosity and conductivity of nanofluid is formulated as summation of constant static part and variable Brownian part:

\[ \mu_{nf} = \mu_{Static} + \mu_{Brownian} \]  
\[ k_{nf} = k_{Static} + k_{Brownian} \]  

The Brinkman (1952) and Maxwell–Garnett’s (1904) model is utilized to expressing static viscosity and thermal conductivity respectively:

\[ \mu_{Static} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \]  

\[ \frac{k_{static}}{k_f} = \frac{(k_p + 2k_f) - 2\varphi(k_f - k_p)}{(k_p + 2k_f) + \varphi(k_f - k_p)} \]  

According to Koo- Kleinstreuer model (2004; 2005), \( \mu_{Brownian} \) and \( k_{Brownian} \) can be denoted as follows:

\[ \mu_{Brownian} = 5 \times 10^4 \lambda \varphi \rho_f \sqrt{\frac{BT}{2\rho_{np} R_{np}}} f(T, \Phi) \]  
\[ k_{Brownian} = 5 \times 10^4 \lambda \varphi \rho_f c_p f \sqrt{\frac{BT}{2\rho_{np} R_{np}}} f(T, \Phi) \]  

Where \( \rho_{np} \) and \( R_{np} \) are the nanoparticles’ characteristics as density and radius respectively and \( B \) denotes Boltzmann number. Equations (17) - (19) are approximated For the CuO-water nanofluid experimentally:
\[ \lambda = 0.0137(100\Phi)^{-0.8229} \text{ for } \Phi \leq 1\% \quad (17) \]

\[ \lambda = 0.0011(100\Phi)^{-0.7272} \text{ for } \Phi > 1\% \quad (18) \]

And

\[ f(T, \Phi) = (-6.04\Phi + 0.4705)T + (1722.3\Phi + 134.63) \quad (19) \]

\[ \text{for } 1\% \leq \Phi \leq 4\% \text{ and } 300K \leq T \leq 325K \]

128 The wall boundary conditions in dimensionless form are as follow:

\[ X = 0, \quad U = V = 0, \quad \theta = 1 \quad (20) \]

\[ X = 1, \quad U = V = 0, \quad \theta = 0 \quad (21) \]

\[ Y = 0, \quad Y = 1, \quad U = V = \frac{\partial \theta}{\partial Y} = 0 \quad (22) \]

129 Due to very slow velocity of nanofluid, it is considered that at boundaries of porous matrix the tangential stresses and velocity in the nanofluid and porous media approximately identical (Beckermann, 1987; Hadidi, 2016; Singh, 2011):

\[ T_{nf} = T_{PM}, \quad k_{nf} \frac{\partial T}{\partial n} = k_{eff} \frac{\partial T}{\partial n} \quad (23) \]

\[ U_{nf} = U_{PM}, \quad V_{nf} = V_{PM}, \quad P_{nf} = P_{PM} \quad (24) \]

\[ \mu_{nf} \frac{\partial U_{nf}}{\partial n} = \mu_{eff} \frac{\partial U_{PM}}{\partial n}, \quad \mu_{nf} \left( \frac{\partial V_{nf}}{\partial n} + \frac{\partial U_{nf}}{\partial t} \right) = \mu_{eff} \left( \frac{\partial V_{PM}}{\partial n} + \frac{\partial U_{PM}}{\partial t} \right) \quad (25) \]

132 The nanofluid’s Nusselt number is varied on the right and left boundaries according to equations below (Oztop, 2008):

\[ \text{Nu}_R = -\frac{k_{nf}}{k_f} \left( \frac{\partial \theta_{nf}}{\partial X} \right)_{X=1} \quad (26) \]

\[ \text{Nu}_L = -\frac{k_{nf}}{k_f} \left( \frac{\partial \theta_{nf}}{\partial X} \right)_{X=0} \quad (27) \]
The net dimensionless heat generation of porous matrix is expressed by appending the local Nusselt number over the right and left boundaries.

\[ \text{Nu} = \int_0^1 (\text{Nu}_R + \text{Nu}_L) \, dY \]  

(28)

For numerical solving and discretization of the governing equation above, a control volume technique was employed. A first order upwind method was used for the convective and diffusive terms. While SIMPLE procedure was employed for the velocity-pressure coupling (Patankar, 1980).

### 2.3 Grid independency and code validation

To assess independency of grid dimensions, different arrangement of grid dimensions were investigated at the condition of \( Ra=10^5 \), \( \epsilon=0.4 \), \( R_k=10 \) and \( q=1000 \). The average Nusselt number for different grid sizes is presented in Table 2. As shown in table 2, results in the grid size of 100×100 show less than 1\% variance with the results in the grid size of 120×120. Therefore, to keep a balanced trade-off between convergence time and solution accuracy, the adopted grid size in the computational domain was 100×100.

It must be noted all the computational calculations were carried out by computer with Processor: Intel-Core i3-3220 CPU @ 3.30GHZ, 4.00 GB Ram and 32-bit Operating System. Also Fig.2 illustrates the residuals of continuity, momentum, and energy equations’ solution. As can be seen from this figure, the convergence of the results is smooth. Therefore it can be said that, modeling this work in considered conditions and geometry is reliable.

Validation of the code was carried out in two steps. First, results obtained from the present work for an enclosure partly filled with porous material were compared to the values obtained from the Beckermann et al (1987). This comparison was conducted with conditions of \( Pr=1, R_k=1 \) and
C=0.55 and was shown in Fig. 3. It should be mentioned that the S reagent, the space which is not occupied with porous material along the x direction. Secondly, Nusselt number obtained from the present study for the cavity filled with CuO-water nanofluid were compared with the ones obtained from the Aminossadati and Ghasemi (2011). In Fig.4 it can be seen that the maximum difference between Nusselt number obtained in the present work and the data reported in Ref. (2011), in volume fraction of 0.02 and Rayleigh number of $10^6$, is about 0.9%.

3. Data processing

3.1. Determining the effective value of interstitial heat transfer coefficient

Figure 5 shows the variation of Brownian Nusselt number with conduction ratios for different porosities. Tests were conducted at the condition of $Ra=10^5$, $\phi=0.04$, $\varepsilon=0.2$, $q=1000$, and $H=10$. At low convection coefficients (e.g. 10), increasing the conduction ratio has no effect on temperature distribution and the internal heat generation and subsequently changes of this parameter has no effect on the Nusselt number. On the other side, because of lack of heat transferred between the porous matrix and nanofluid, and also due to the high temperatures of the matrix, the generated heat and the resulting Nusselt number would be extremely low. Thus, in order to clarify the effect of the porous matrix’s material on the Nusselt number tests were conducted at the higher convection coefficient and the value of $H$ has to be set to 100.

3.2. The variation of Brownian according to conduction

The contours of streamlines and isotherms in different porosities were conducted at the condition of $Ra=10^5$, $\phi=0.04$, $H=100$ and $q=1000$, with and without Brownian term. The change in Brownian Nusselt number in the various conduction and porosities is shown in Fig. 6. Due to Impermeability
of porous matrix in lower porosities, the only scheme for transfer of generated heat inside the porous matrix is conduction. Thus, variation of conduction ratio causes more heat transfer and a higher Nusselt number, consequently. Results show that the increment of Nusselt number is significantly proportional to increment of the conduction ratio. The further increment of porosity leads to further penetration of nanofluid inside the matrix. Therefore, the dominated regime changes from conduction to convection heat transfer. In this condition, the heat would be harvested from the porous matrix by convection of nanofluid. Hence Nusselt number will be high and approximately equal in all conduction ratios.

Figures 7 and 8 show the comparison of the nanofluid streamlines and isotherms for the condition with and without Brownian term. As can be mentioned earlier, the Brownian term increases the viscosity of nanofluid, so this term affects the velocity and smoothness of streamlines. The Fig. 7 shows that considering this term causes more smooth streamlines while ignoring this term results in secondary circulation even in lower porosities.

Figure 8 shows the isotherms for the condition with and without Brownian term and results were compared with each other. The higher values of gap among solid and dashed lines result in more Brownian effects. Presence of Brownian term enhances thermal conduction and temperature decrement of the porous matrix and bigger Nusselt number. The data reveals that in almost zero porosity (the exact value is 0.005) at the $R_k=0.1$, the Brownian term is low. Consequently the Brownian (solid) and non- Brownian (dashed) isotherms inside the matrix overlap eachother. By augmenting the conduction ratio, the difference between Brownian and non- Brownian isotherms increases which is indicative of bigger Brownian term effects.
Nusselt factor \((\text{Nu}_B/\text{Nu}_{WB})\) was specified as the Nusselt number in the presence of Brownian term \((\text{Nu}_B)\) to Nusselt number of the non-Brownian term \((\text{Nu}_{WB})\) ratio. Fig. 9 shows the variation of Nusselt factor with conduction ratio for various porosities.

According to this fact that at approximately zero porosities, porous matrix is almost impermeable, generated heat is transferred to outside the block purely by the heat conduction. Hence the generated heat in low thermal conduction ratios does not pass to nonporous field. Also, the Brownian term of nanofluid is not affected which in this case the value of Nusselt factor is low. By more increment of conduction ratio, the heat is released from porous matrix and influences viscosity and thermal conduction of nanofluid therefore heat transfer and Nusselt factor are enhanced. By increasing porosity in lower range of \(R_k\) values, the temperature of matrix also remains high that results in higher Brownian term and Nusselt factor. By approaching \(\varepsilon=1\) and the gradual removing of porous media, heat conduction is replaced by convection and as a result matrix temperature and consequently the Brownian term and Nusselt factor are decreased. The Nusselt factor for \(R_k=1\) surpasses corresponding value for \(R_k=10\) in middle range of porosity [0.2, 0.4] because of switching the dominated regime from conduction to convection. Finally it can be deduced that the mounting porous matrix into cavity full of nanofluid has significant effect on Brownian term value. This effect is more obvious in lower porosities specifically in a low conduction ratio. While in high margin value of porosity even neglecting the Brownian term can cause rational results.

### 3.3. The effect of Rayleigh number

Since Brownian term effect on conduction regime was significant, the effects of volume fraction of nanofluid and Rayleigh number were investigated in lower porosities at the condition of \(\varepsilon=0.4\).
R_k=1, q=1000 and H=100. Fig. 10 illustrates the Nusselt number variations versus volume fractions in different Rayleigh numbers. Results indicated that the increased Rayleigh number highly increased the Nusselt number in all volume fractions that can be attributed to the increased circulation velocity of the nanofluid inside the cavity and therefore more cooling of porous matrix. Increased volume fraction also resulted in higher Nusselt number.

Unlike the assumption of the homogenous nanofluid, in this state the increasing process of the Nusselt number would be reduced due to Brownian viscosity increase, especially in heat source region. For this reason, the growth of Nusselt number in Rayleigh number of $10^4$ is higher than those of $10^5$ and $10^6$, respectively, and similar results concerning behavior of nanofluid is reported by Aminossadati and Ghasemi (2011).

Variations of Nusslet factor related to different Rayleigh number and volume fractions were shown in Fig. 11. The highest Nusslet factor was seen to occur at Rayleigh number of $10^4$ which was due to the dominant effect of Brownian thermal conductivity on viscosity in lower Rayleigh numbers. In higher Rayleigh number, the circulation velocity of the nanofluid and the viscosity effect was higher in creating resistance which resulted in decreasing Nusselt factor compared to lower Rayleigh numbers. Instead, the increase of volume fraction of nanofluid caused to increased Nusselt factor and this increase was higher in lower Rayleigh numbers.

In lower Rayleigh numbers, conduction was the dominant regime in the cavity. Therefore, increasing volume fraction resulted in the higher thermal conduction of the nanofluid which plays a more significant role in Nusselt factor increase. Although, by increasing Rayleigh numbers viscosity increases so it had no significant effect on heat transfer due to lower circulation velocity of the fluid. In high Rayleigh numbers, by increasing volume fraction the effect of the viscosity
was higher and conduction effect was decreased that lead to the decreased Nusselt factor growth
compared to lower Rayleigh numbers.

4. Conclusion

In the present study, effect of mounting heat generating porous matrix in a close cavity on the
Brownian term of CuO-water nanofluid was studied numerically. Viscosity and Thermal
conductivity of nanofluid were assumed to be consisting of a static component and a Brownian
component that were functions of volume fraction of the nanofluid and temperature.

According to obtained results mounting the porous matrix caused to change the streamlines and
isotherms on nanofluid saturated cavity and enhance the value of Brownian term. The heat
conduction of porous matrix plays an important role on variation of Brownian term. Due to the
heat trapped in low heat conduction of porous matrix, its temperature and Brownian term were
raised. This fact resulted in the greatest value of Nusselt factor in this region. Also based on
decrement of conduction effect, the Brownian impact dropped where neglecting this term has an
insignificant influence. On the other hand the Brownian term must be considered in porous matrix
with lower porosities and conduction ratio.

It is obvious from the result in lower Rayleigh number the Brownian term impact has more
importance, but by increasing Rayleigh number and due to increment of nanofluid viscosity
influence the value of Brownian term was dropped severely. This impact caused to higher Nusselt
factor at lower Rayleigh numbers, and vice versa.

Increased volume fraction increased non-linear Nusselt number, i.e. the process of Nusselt number
increment was decreased gradually which could be due to the increased Brownian viscosity and it
has more significant effect when compared to the thermal conduction. Also, incorporation of
porous matrix resulted in increased contribution of Brownian term into Nusselt number at all volume fractions.

References


Table 1: nanoparticle and fluid properties (Aminossadati et al. 2011)

Table 2: Average Nusselt number for different grid dimensions.

Fig.1. Geometry of the problem

Fig.3. Comparison of the applied code with Beckermann (1987)

Fig.4. Comparison of the present code with Aminossadati and Ghasemi (2011)

Fig.5. Brownian Nusselt number for various $R_k$ and $\varepsilon$ at $H=10$

Fig.6. Nusselt number for various $R_k$ and $\varepsilon$ at $H=100$

Fig.7. Streamlines of Brownian (solid) and non-Brownian (dashed) cases

Fig.8. Isotherms of non-Brownian (dashed) and Brownian (solid) cases

Fig.9. Nusselt factor for various $R_k$ and $\varepsilon$

Fig.10. Total Brownian Nusselt number for different Ra and $\varphi$

Fig.11. Nusselt factor for different Ra and $\varphi$
Nomenclature

$c_p$  specific heat ($J \cdot kg^{-1} \cdot K^{-1}$)

$h$  convective heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$)

$K$  permeability of the porous medium ($m^2$)

$k$  thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$)

$Q_0$  constant coefficient of heat generation ($W \cdot K \cdot m^{-2}$)

Greek symbols

$\alpha$  thermal diffusivity ($m^2 \cdot s$)

$\beta$  thermal expansion coefficient, $1/K$

$\lambda$  modeling function Eq. (21-22)

$\phi$  volume fraction of nanoparticles

Subscripts

$f$  fluid

$nf$  nanofluid

$p$  nanoparticle

$PM$  Porous media

$s$  solid
Fig. 1. Geometry of the problem

219x224mm (96 x 96 DPI)
Fig. 2 residuals of continuity, momentum, and energy equations' solution
Fig. 3. Comparison of the applied code with Beckermann (1987)

303x301mm (96 x 96 DPI)
Fig. 4. Comparison of the present code with Aminossadati and Ghasemi (2011)

290x198mm (96 x 96 DPI)
Fig. 5. Brownian Nusselt number for various $R_k$ and $\varepsilon$ at $H=10$

316x206mm (96 x 96 DPI)
Fig. 6. Nusselt number for various Rk and ε at H=100

303x209mm (96 x 96 DPI)
Fig. 7. Streamlines of Brownian (solid) and non-Brownian (dashed) cases

652x841mm (101 x 101 DPI)
Fig. 8. Isotherms of non-Brownian (dashed) and Brownian (solid) cases

686x873mm (96 x 96 DPI)
Fig. 9. Nusselt factor for various Rk and ε

323x231mm (96 x 96 DPI)
Fig. 10. Total Brownian Nusselt number for different Ra and $\varphi$

317x216mm (96 x 96 DPI)
Fig. 11. Nusselt factor for different $Ra$ and $\phi$

$317\times219$ mm (96 x 96 DPI)
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Table 1: nanoparticle and fluid properties (Aminossadati et al. 2011)
Table 2: Average Nusselt number for different grid dimensions.

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322x97mm (96 x 96 DPI)