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DOI:
10.1088/1742-6596/381/1/012009

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2012 J. Phys.: Conf. Ser. 381 012009

(http://iopscience.iop.org/1742-6596/381/1/012009)

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Abstract. The emergence of clustering in light nuclear systems is explored using the deformed harmonic oscillator as a starting point. The experimental evidence for various geometrical arrangements of clusters in carbon-12 is discussed.

1. Introduction

The emergence of organised behavior in complex systems coordinated through local, nearest neighbor, interactions is well documented in systems ranging from biological, physical through to sociological [1]. Such self-organization, which arises from a series of well defined, sometimes complex, interactions, can produce remarkable behavior. Perhaps one of the preeminent examples is the behavior of slime-molds. These organisms are amoeboid in character but are capable of extraordinary behavior. Physarum polycephalum, for example, has been used in a variety of laboratory based experiments which explore how such a colony develops an optimal strategy for harvesting distributed food sources. A rather striking example is the recent work in which the local geography of the Tokyo area is replicated and a slime mold is used to find the optimal connectivity between the satellite towns and cities [2]. The comparison between the slime-mold and human solution (rail networks) is then rather interesting.

In a system which is closer to nuclei, clusters have been observed in water [3,4]. Here quite complex cluster structures appear whose existence is due to a delicate balance between van de Waals type interactions and hydrogen bonds. Understanding the details of these molecular structures also indicates the influence of many-body interactions. The appearance of clusters is a minimization of the energy of the molecular system. The nucleus has variously been

considered to be a nucleon fluid. The nucleon-nucleon interaction is inherently complex and also contains many-body components and hence analogues of water clusters might be expected. One obvious difference between water and nuclei is size. On the scale of the water molecule the system size is effectively infinite, whereas with nuclei the boundary/surface has an important role. The nucleons are located in orbitals whose properties are constrained by the condition that at the boundary the wave-functions must tend to zero. In water the clusterisation is driven by the nature of the molecule-molecule interaction only. The overlap of nucleons in identical orbitals gives rise to enhanced interaction/binding energy.

For a system such as the alpha-particle where all four nucleons occupy the same, lowest energy, orbital the binding is maximal and such a configuration is energetically favored. In nuclei such as ^{8}Be, ^{12}C, ^{16}O evidence for correlations persists (see Fig. 1). From Fig. 1 it is clear that the binding energy per nucleon is maximal for a given element when N=Z and in particular when A=$4n$ (n=1, 2, 3 ….). This observation naturally led to the conclusion that this special class of nuclei could be described in terms of geometric arrangements of alpha-particles [5]. Apparent confirmation is found when plotting the binding energy of such systems as a function of the number of alpha-particle interactions, or bonds [5] – Fig. 2. These systematics reveal a linear dependence indicative of a relatively constant alpha-alpha interaction energy.

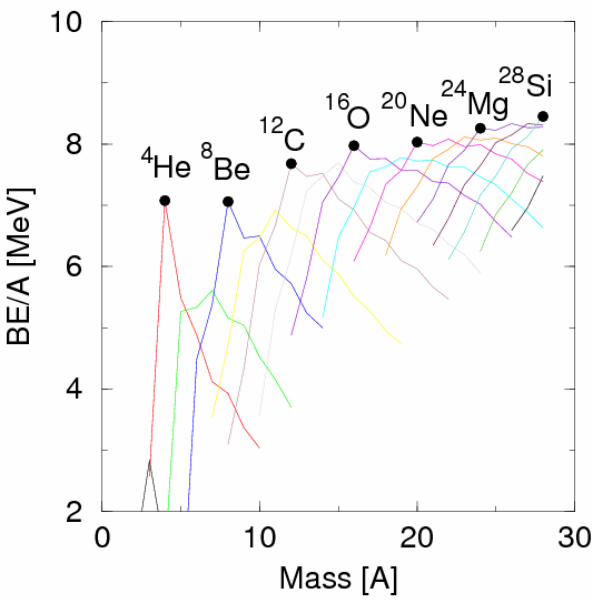

Figure 1. The binding energies per nucleon for the light elements. The lines show common elements. The maxima for a given isotope chain coincides with nuclei with even (and equal) numbers of protons and neutrons.

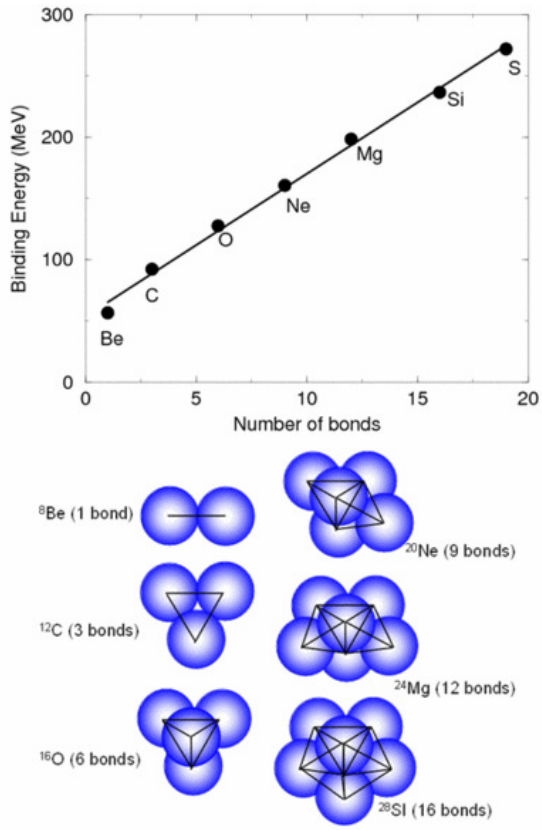

Figure 2. (top) Binding energies of light alpha-particle nuclei as a function of bonds between the alpha-particles. (bottom) Proposed arrangements of the alpha-particles.

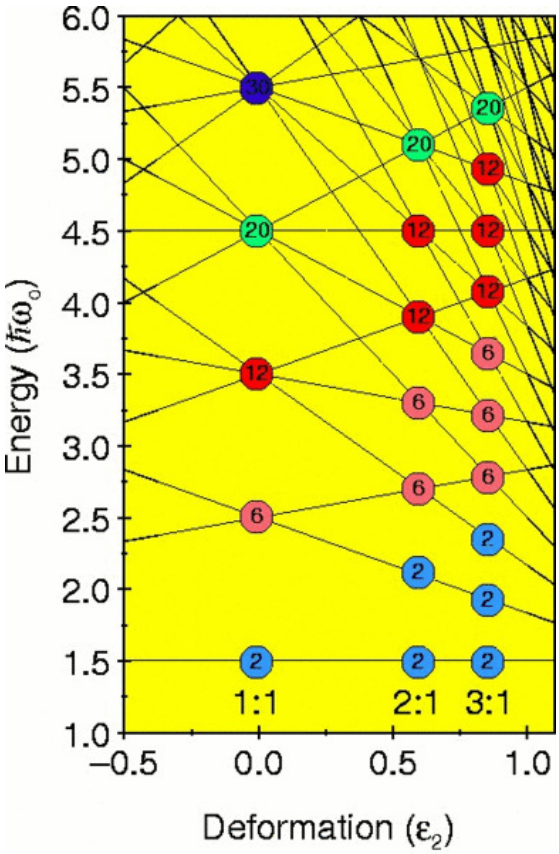

Figure 3 The energy levels of the deformed harmonic oscillator (DHO).

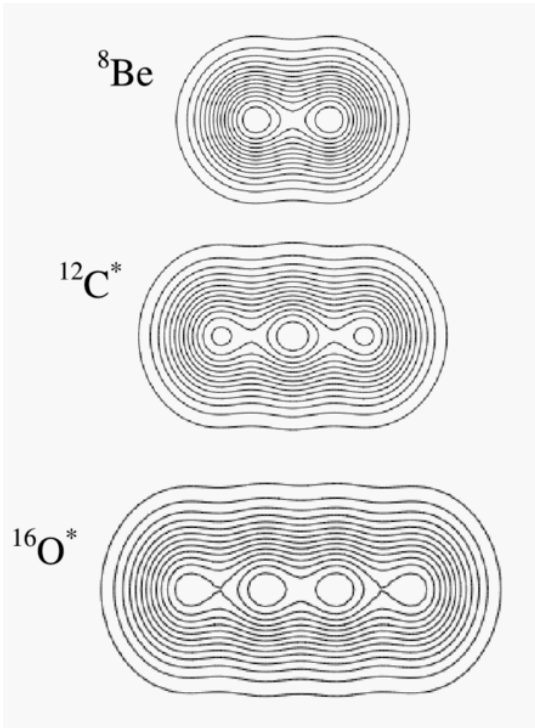

Figure 4. Densities calculated in the harmonic oscillator corresponding to the ground state of ^{8}Be and excited states in ^{12}C and ^{16}O.

This rather simplistic description does not provide the complete picture of the ground-state structure of such nuclei, but interestingly it does form the basis for understanding their symmetries (see below). Nevertheless, the high binding energy associated with alpha-particle correlations does mean that such particles have a very strong influence on the structure of light nuclei.

What has been discussed so far is how the nature of the strong force produces alpha-clusters and the potential arrangement of those clusters. This is completely analogous to the water molecule case. The role of the boundary conditions can be explored using the deformed harmonic oscillator (DHO). The energy levels of the 3D-DHO are shown in Fig. 3. As the potential is deformed in the prolate direction, $\varepsilon>0$, then the energy levels rearrange themselves. Those associated with oscillations along the deformation axis are lowered whilst those with oscillations perpendicular are raised in energy. As is well documented (see Ref. [6] and references therein), this gives rise to shell structure associated with deformed nuclei and hence enhanced stability. A second feature that emerges is the symmetries at these new shell gaps. This is indicated by the numbers shown in Fig. 3. At a deformation of the potential of 2:1 the degeneracies associated with the spherical oscillator, 2, 6, 12, 20…, are repeated twice and at 3:1 they are repeated three times. It is tempting to associate these two and three-fold symmetries with cluster structures. For example, at 2:1 2+2 is equivalent to alpha+alpha and

2+6 is associated with ^{16}O+alpha and similarly at 3:1 that 2+2+2 can be linked to alpha+alpha+alpha etc…

This interpretation is reinforced through the calculated densities. Fig. 4 shows the densities corresponding to the 2:1, 3:1 and 4:1 structures linked with the 2+2, 2+2+2 and 2+2+2+2 symmetries, respectively. These densities reveal the same symmetries that appear in the magic numbers. This similarity whereby the densities reinforce the symmetries observed in the degeneracy is interesting. This coincidence can be understood in terms of an underlying SU(3) symmetry; Nazarewicz and Dobaczewski [7] showed that at a deformation of 2:1 that the level scheme could be described by two coupled SU(3) groups and at 3:1 three groups, etc…. The appearance of cluster-like structures in ab-initio type calculations strongly reinforces the assertion that correlations and clustering dominates in light nuclear systems [8]. The nature of these structures is related to the boundary conditions giving rise to the characteristic standing wave patterns. These symmetries which are found in the deformed harmonic oscillator, both in the magic numbers and densities, reinforce the formation of cluster structures from alpha-particles borne from the robustness of the ^{4}He nucleus.

As noted above, these symmetries permeate many nuclear models. For example, the patterns of the harmonic oscillator may be found in the densities predicted by the Hartree-Fock mean-field approach and even calculations which deal with the nuclear force from first principles [8]. However, the acid test of any model is the overlap with experimental observations.

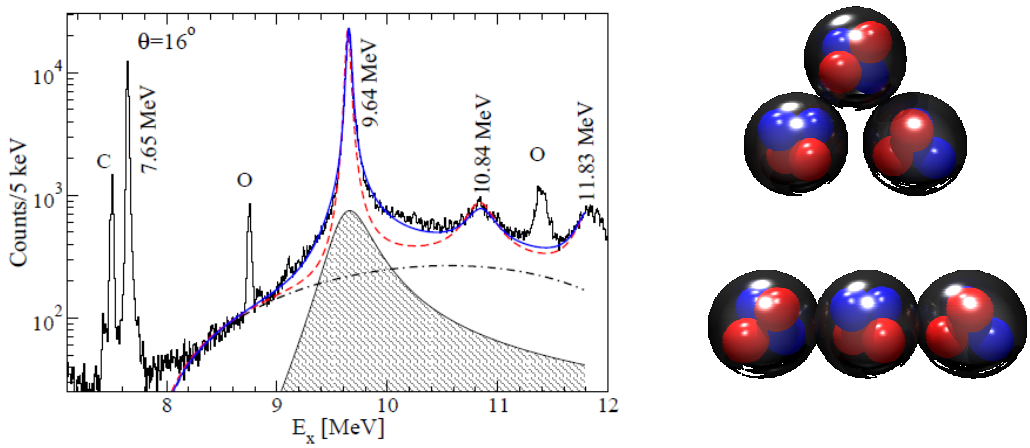

Figure 5. Left-hand-side. ^{12}C excitation energy spectrum showing the broad 2^{+} state required to reproduce the shape of the experimental spectrum, from Ref. 8. Right-hand-side. Possible arrangements of the alpha-clusters in either a triangle (top) or chain (bottom). The experimental data indicate that the triangular structure is preferred.

2. Structure of ^{12}C

After ^{8}Be, one of the most important cluster structures to define experimentally is that associated with three alpha-particles in ^{12}C. In a system of two clusters the arrangement is clear, but there are a number of competing possibilities for a three component system (see. Fig. 5). The determination of the dominant component/arrangement is both an experimental and theoretical challenge. The complexity of the Greens Function Monte Carlo (GFMC) approach prohibits its extension currently to nuclei with A>12, and ^{12}C currently the focus of the calculations. In this nucleus it is excited states, as opposed to the ground-state, which is the focus in terms of alpha-cluster calculations. In fact, the state of interest is the 7.65 MeV, 0^+, Hoyle-state. The Hoyle-state because its existence was proposed by Sir Fred Hoyle some 50 years ago [9]. This state is the gateway through which carbon is synthesized via what is known as the triple-alpha process. Without this state there would be no organic life. Calculations of the properties of the Hoyle-state using the GFMC approach are still on-going. However, calculations using the Antisymmetrized Molecular Dynamics (AMD) [10] and its cousin the FMD approach (Fermionic Nuclear Dynamics) [11] reveal a highly clustered state. Its' very dilute nature, together with the 3alpha cluster structure has been interpreted in terms of an alpha-condensate by Schuck and co-workers [12], this would be a new bosonic state of nuclear matter. Very recent calculations using the framework of chiral effective field theory have been able to closely reproduce the energy of the Hoyle-state [13]. Although they offer little detailed structural information, they do suggest a larger charge radius than the ground-state, as observed experimentally. As with the ^{8}Be ground-state, the width of the 7.65 MeV state indicates a significant cluster structure and, in agreement with theoretical predictions, a large charge radius (see Ref. [11]) indicating a volume which is 3-4 times that of the ^{12}C ground state.

Revealing the collective excitations of such a state could have significant implications for understanding its structure. In the alpha-condensate model this would correspond to a 2^+ state which is associated with the conversion of one of the bosons from s-wave to d-wave. In a more traditional cluster model it would correspond to a collective rotation and the moment of inertia would then reveal the details of the cluster arrangement.

Recent studies of the ^{12}C(α,α') [14] and ^{12}C(*p*,*p*') [15,16] (see Fig. 5) reactions indicate the presence of a 2^+ state close to 9.6-9.7 MeV with a width of 0.5 to 1 MeV. The state is only weakly populated in these reactions, presumably due to its underlying cluster structure, and is broad. Consequently, its distinction from other broad-states and dominant collective excitations (e.g. the 9.6 MeV, 3^-) makes its unambiguous identification challenging. Further evidence for such an excitation comes from measurements of the ^{12}C(γ,3α) reaction performed at the HIGS facility, TUNL [19]. Here a measurable cross section for this process was observed in the region of 9-10 MeV which cannot be attributed to known states in this region. Furthermore, the angular distributions of the alpha-particles are consistent with an L=2 pattern, indicating a dominant 2^+ component. Based on a rather simple description of this state in terms of three alpha-particles with radii given by the experimental charge radius, it is possible to use the 2 MeV separation between the Hoyle-state and the proposed 2^+ excitation to draw some conclusions as to the arrangements of the clusters (see Fig. 5). This would indicate that rather than a linear arrangement of the three clusters that a more appropriate description would be a loose arrangement of the alpha-particles in something approaching a triangular.

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doi:10.1088/1742-6596/381/1/012009

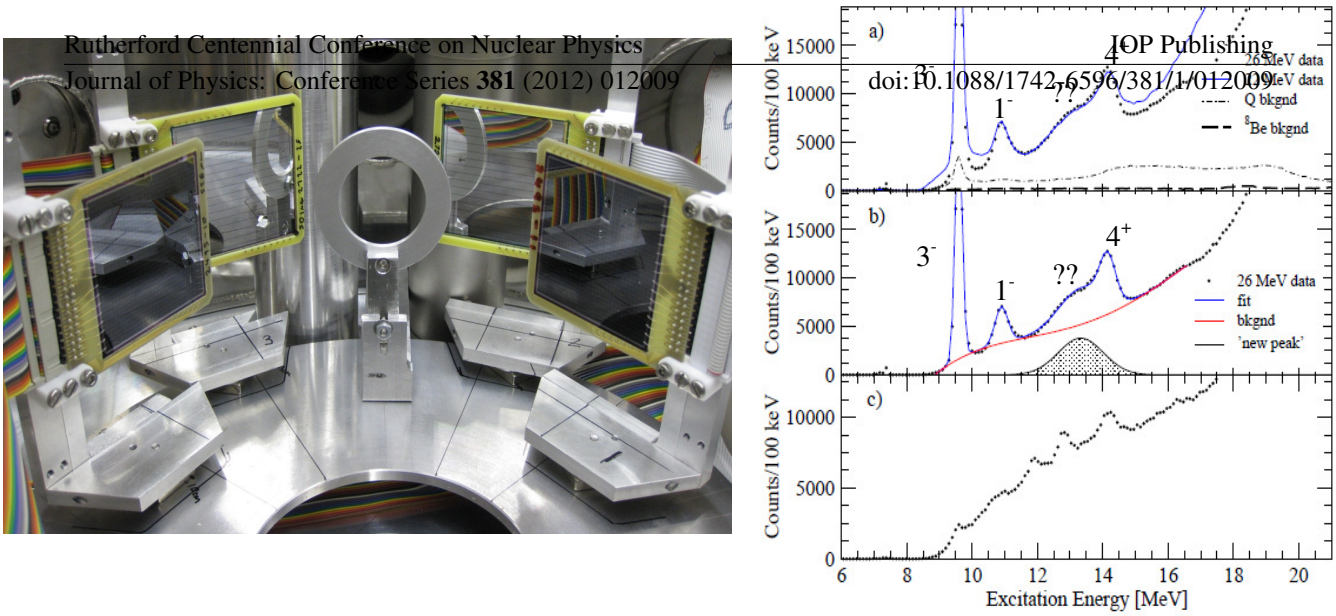

Figure 6. Left-hand-side. Arrangement of the 4 silicon strip detectors used in the measurements in Ref. [11]. The beam passes from bottom to top in this picture. Right-hand-side. Carbon-12 excitation energy spectra. a) The blue line shows the measurement at 22 MeV. The spectrum corresponding to measurements at 26 MeV is shown by the dots. The backgrounds obtained by gating above the ^{8}Be peak (bold dashed line) and both above and below the Q-value peak (dot-dashed line) are both illustrated. b) Fit to the 26 MeV data is given by the blue solid line. The polynomial background (red-line) and line-shape for the new peak (shaded area) is shown. c) Excitation energy spectrum for events not proceeding via the decay to the ^{8}Be ground-state. The proposed new state is indicated by "??"

A natural extension of such a conclusion is that there should also be a collective 4^+ state. Using the simple $j(j + 1)$ scaling, a 4^+ excitation close to $E_x(^{12}C) = 14$ MeV would be expected. We have performed recent measurements of the two reactions ^{9}Be(α,3α)n and ^{12}C(α,3α)^{4}He [18]. In these measurements three alpha-particles were detected in an array of four silicon strip detectors (shown in Fig 6). The analysis required that two of the three alpha-particles came from the decay of the ground-state of ^{8}Be. For the decay of ^{12}C to ^{8}Be+α this ensures that the decay process can proceed through only natural parity states (i.e. 0^+,1^-, 2^+ ….). This restricts the complexity of the excitation energy spectrum. The measurements for both the ^{9}Be and ^{12}C targets reveal the known 3^-, 1^- and 4^+ states at 9.64, 10.84 and 14.08 MeV, respectively. However, there is an additional component to the spectrum close to 13.3 MeV with a width estimated to be 1.7 MeV (Fig. 6). It is believed that this is not a contaminant and is observed with similar properties in all spectra. Angular correlation measurements made using the ^{12}C target are not definitive, but indicate a 4^+ assignment.

Such measurements are challenging and it is difficult to be absolutely certain that the feature does not correspond to an experimental artifact. Nevertheless, the appearance of a 4^+ state close to the energy predicted based on the extension of the Hoyle-state and proposed 2^+ excitation. Such a feature is also observed in measurements of the neutron energy produced in the ^{9}Be(α,3α)n reaction [19]. However, it is clear that further measurements are required to confirm, or otherwise, this new state in ^{12}C. Moreover, analysis of beta-decay measurements populating alpha-decay states in ^{12}C indicate that there may be broad 2^+ strength in ^{12}C but at somewhat higher energies than suggested in the proton and alpha inelastic scattering measurements [20]. This contradiction between the two types of measurements remains to be reconciled and hence a firm conclusion regarding the excitation structure of ^{12}C above the alpha-decay threshold remains to be established.

It is possible that a better understanding of the nature of the 7.65 MeV state is emerging, but certainly there is great scope for further measurements, confirming or otherwise the above conclusions. The question then arises as to the structure of the ground-state. In the case of ^{12}C, the system can be constructed from a variety of geometric arrangements of three alpha-particles. It might be expected that the compact equilateral-triangle arrangement might be the lowest energy configuration. Such an arrangement possesses a D_{3h} point group symmetry. The corresponding rotational and vibrational spectrum is described by a form [21]

$$E = E_0 + Av_1 + Bv_2 + CL(L + 1) + D(K \pm 2l)^2 \quad (1)$$

where $v_{1,2}$ are vibrational quantum numbers, and v_2 is doubly degenerate; $l = v_2, v_2 - 2, \ldots.1$ or 0, L is angular momentum, M would be its projection on a laboratory fixed axis and K a body-fixed axis [21]. A, B, C and D are adjustable parameters. The spectrum of states predicted by the choice $A = 7.0$, $B = 9.0$, $C = 0.8$ and $D = 0.0$ MeV is shown in Fig. 7 [21].

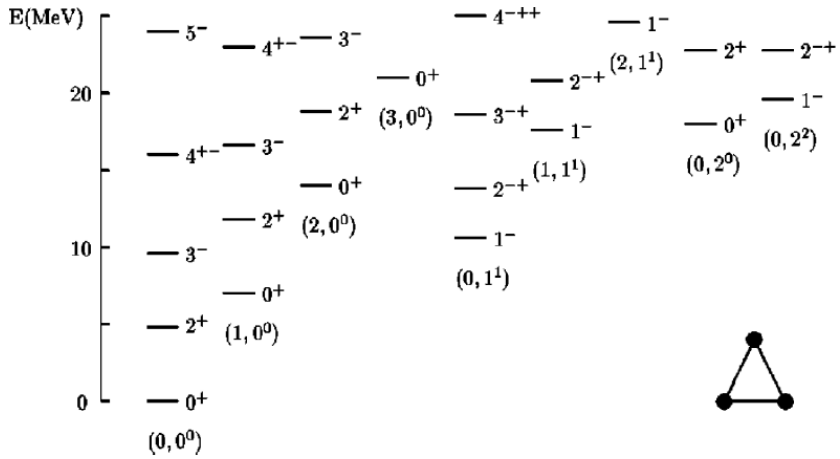

Figure 7. Spectrum of the energy levels of an equilateral triangle configuration. The bands are labeled by $(v_1;v^l_2)$. See Ref. [21] for further details.

The ground state band, $(v_1;v^l_2) = (0,0^0)$, contains no vibrational modes and coincides well with the observed experimental spectrum. Here the states correspond to different values of K ($K = 3n$, $n = 0,1,2...$) and L. For K=0, L=0, 2, 4 etc....., which is a rotation of the plane of the triangle about a line of symmetry, whereas for $K \neq 0$ $L = K$, K +1, K +2..... In the present case K=0 or 3 is plotted with the parity being given by $(-1)^K$. The K=0 states coincide well with the well-known 0^+ (ground-state), 2^+ (4.4 MeV) and 4^+ (14.1 MeV) states. The K=3 states correspond to a rotation about an axis which passes through the centre of the triangle, the first of which has spin and parity 3^- and coincides with the 9.6 MeV, 3^-, excited state. A prediction of this model is that there should be a 4^- state almost degenerate with the 4^+ state. A recent measurement involving studies of the alpha-decay correlations indicated that the 13.35 MeV unnatural-parity state possessed J^π=4^- [3]. The close degeneracy with the 14.1 MeV 4^+ state would appear to confirm the D_{3h} symmetry. In this picture the 0^+ state at 7.65 MeV would correspond to a vibrational mode (v_1=1). The coupling of rotational modes would then produce a corresponding 2^+ state at 4.4 MeV above 7.65 MeV, i.e. 12.05 MeV; there is no known 2^+ state at this energy, pointing to the more complex structure of this state.

3. Conclusions

Universally, both very simple and complex calculations reveal that symmetries are extremely important in driving the structure of light nuclei. This coupled with the well-bound, inert, nature of the alpha-particle means that alpha-clustering plays an extremely important role. The characterization of such clustering is extremely challenging, but there is a broad spectrum of evidence for this behavior. Providing a detailed characterization of the cluster states has been made for a number of systems, in particular ^{20}Ne. However, even for nuclei as simple as ^{12}C there remain unanswered questions. We have shown here that we are moving towards a more detailed understanding, but have not finally arrived.

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