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Modal Combinatorialism is Consistent with S5.

Henry Taylor

Forthcoming in Thought. Please Cite Published Version

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Abstract

The combinatorial theory of modality has long been dogged by the supposed problem that it entails that S5 is not the correct logic for metaphysical modality. In this paper, I suggest a modification to combinatorialism, to eliminate this tension with S5. I argue that the resulting view is more in the spirit of combinatorialism than the original position.

1. Modal combinatorialism

Many have been attracted to the combinatorialist account of modality (Armstrong 1989, 1997; Skyrms 1981). But combinatorialism is incompatible with the thesis that S5 is the correct logic of metaphysical modality. Or so it has been claimed (Armstrong 1989, p.62; Lewis 1999, p.202). This is a problem, because S5 is the overwhelmingly dominant logical system for thinking about metaphysical modality.

Combinatorialism is a view of metaphysical (not epistemic, logical, or nomological) modality. It is based on an ontology of states of affairs. Each of these is a complex of a particular and a property, where properties are universals (Armstrong 1997). I use capitals for universals and lower case for particulars. So Fa is the state of affairs of a’s being F. The exception is the upper case ‘W’, which I reserve for worlds. The view concerns simple states of affairs: the most basic states of affairs upon which all others depend (Armstrong 1989).

The combinatorialist claims that the space of possible worlds is generated by recombining the different elements of actual states of affairs. Suppose that in the actual world, we have two states of affairs, Fa and Gb, and that nowhere in the actual world is there the state of affairs Ga or Fb. Nonetheless, we can ‘recombine’ the universals and particulars contained in Fa and Gb. This will give us a possibility, which is Ga and Fb (Armstrong 1989, ch.3; 1997, ch.10). Armstrong says:

‘The idea for possibility… is that all the combinations of simple particulars, properties and relations… constitute the possibilities for first-order states of affairs.’ (1997, p.160).
The focus of this paper is the supposed tension between combinatorialism and S5. I won’t discuss other arguments against combinatorialism. I first specify what this tension is, before showing how it should be resolved. I do not claim that my view is the only possible solution to the problem, just that it is an attractive one, which fits naturally with the combinatorialist view. I don’t have space to motivate S5 as the superior logic for metaphysical modality, though I note that it is the overwhelmingly dominant one (Williamson 2016; Lewis 1986, p.246). Armstrong himself claims that if combinatorialism can be made consistent with S5, that will be a ‘distinct advantage’ for the view (1997, p.166). Even for readers sceptical of S5, the fact that combinatorialism can be made consistent with it is an important result.

2. Strong actualism
We must highlight a commitment at the core of the combinatorialist project, as this will help us understand some of its central claims. Armstrong says:

‘The Combinatorialist… is an actual-world chauvinist. The actual world, and it alone, is genuinely a world. The actual universals set a limit, a limit given by the totality of their recombinations, to the possible universals. The possible is determined by the actual, and… cannot outrun the actual.’ (1989, p.56)

Because of the negative connotations of ‘chauvinist’, I shall refer to this view as ‘strong actualism’.

For the strong actualist, our modal metaphysics must be firmly grounded in the actual universals, and cannot outrun the resources afforded by them.¹ A full defence of strong actualism would require a discussion of the debate between actualism and possibilism, which I clearly do not have space for (e.g., Stalnaker 2012, Lewis 1986).² However, we can note that strong actualism is at least a reasonable thesis worth pursuing (Armstrong 1989, pp.7-13; cf. Lewis 1986, pp.108-115). Even for those unconvinced by this view, it is still an important and interesting question whether an adequate modal metaphysics can be developed using only the relatively austere resources that strong actualism affords. Further, even actualists who are not combinatorialists (Stalnaker 2012) will be interested in how the actualist position can be developed in an alternative manner.

¹ Armstrong was first a fictionalist (1989, pp.49-53) and then a deflationist (1997 pp.172-174) about other possible worlds.
² Possibilism: the view that there are non-actual entities. Actualism: the denial of possibilism.
3. Contraction and expansion
Armstrong introduces a principle of contraction, which states that we can exclude certain states of affairs from consideration (1989, p.62; 1997, pp.156-158). Suppose the actual world has Fa and Gb. The principle of contraction states that it is possible that Fa exist on its own, as it results from a contraction of the actual world: we exclude Gb to just leave Fa.

Could the world have been richer? Consider alien universals. A universal G is alien to a world W iff G does not exist in W, and is not constituted by any elements that exist in W (cf. Armstrong 1989, p.54, Lewis 1986, pp.91-92). Is it possible for there to be a universal that is alien to the actual world? Armstrong says no. We can see why by returning to strong actualism. In accounting for modal truths, we must look into the actual world. There is nothing in the actual world that can account for the truth of possibility claims about universals that are alien to the actual world (1989, pp.55-57). Notice that contracted worlds are not like that: the resources to account for contracted worlds can be found in the actual world (1989, p.62).

Strangely, Armstrong does allow that (at least some) possibility claims about alien particulars are true. His reasons are to do with quidditism and haecceitism (1989, pp.57-61). We can sidestep these issues here. I will proceed as if possibility claims about at least some alien particulars are true, but my solution will work even if we deny this (I show how in footnote 8).

4. Troubles with S5
Let Wa be the actual world, and for the sake of example, suppose it contains Fa and Gb. By the principle of contraction there is another possible world, which contains Fa, but not Gb. Call this W1. W1 is accessible from Wa because it is a contraction of Wa. Clearly, G is alien to W1. Let ‘∃x Gx’ just mean ‘there exists a G’. So ∃x Gx is true in Wa and false in W1.4

Consider the following principle, which I call ‘Universal Combinatorialism’ (UC).

(UC): For any world W, and any entity ϕ, ϕ is possible in W iff ϕ is obtainable by

3 For further discussion of the combinatorialist rejection of alien universals, see Melia (2003, pp.137-141). Melia does not discuss the friction with S5. This is my own focus (rather than the plausibility of the combinatorialist rejection of alien universals itself).
4 For a proposition to be true in a world just means that in that world, that proposition is true. The notion of ‘true of a world’ will not be used in this paper (Stalnaker 2012, pp.46-48).
recombination of the universals and particulars in the simple states of affairs in W.  

As we have seen, in determining what is possible for the actual world, we look inward at the stock of actual particulars and universals in the actual states of affairs. (UC) effectively says that other worlds must do the same: each world must look at its own stock of universals and particulars, and by recombining them, we get the possibilities for that world.

If we accept (UC) then since G does not exist in W₁, and isn’t the result of recombining the universals and particulars in W₁, it follows that ¬◊∃xGx is true in W₁. But if we accept that ¬◊∃xGx is true in W₁, then it follows that ¬□◊∃xGx is true in W₄. To see that this follows, suppose that □◊∃xGx were true in W₄, then it would follow by the elimination rules for necessity (and the accessibility of W₁ to W₄) that ◊∃xGx is true in W₁, which creates a contradiction.

So if we accept that ¬◊∃xGx is true in W₁, we must accept that ∃xGx and ¬□◊∃xGx are both true in W₄. Obviously, this violates the distinctive axiom of B:

(B): P → □◊P

Of course, (B) is an axiom of S₅, as well as of B (S₅ is stronger than B). So both the systems B and S₅ must be rejected. This is why Armstrong recommends accepting S₄, which does not include (B) as an axiom (1989, p.62). Rejecting (B) also involves rejecting the symmetry of accessibility. In our above example, W₁ would be accessible to W₄ but not vice versa. Armstrong summarises the situation by saying: ‘symmetry of accessibility then fails... one must content oneself with an S₄ modal logic’ (1989, p.62).

The problem is the reasoning that takes us to the claim that ¬◊∃xGx is true in W₁, because only with this claim can the problems arise. The principle underlying this problematic reasoning is (UC), because that is what takes us from the claim that G is alien to W₁ to the claim that ¬◊∃xGx is true in W₁.

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5 'Entity' is intended to apply to universals, particulars, relations and states of affairs.

6 An inconsistency with the distinctive axiom of S₅ (◊P → □◊P) can be derived in a similar way. From the truth of ∃xGx in W₄ it follows that ◊∃xGx is true in W₄ (by P → ◊P, which is a theorem of T, and therefore also a theorem of S₄, B and S₅). From the truth of ◊∃xGx in W₄ it follows that □◊∃xGx is true in W₄ (by the characteristic S₅ axiom). But by the reasoning given above, a contracted world like W₁ (which is accessible to W₄) and (UC) implies the truth of ¬□◊∃xGx in W₄. Contradiction.
5. Two other proposals
Before giving my own suggestion, I briefly consider two other ways of blocking the inference to the claim that \( \neg \Box \exists x Gx \) is true in \( W_1 \). I do not try to refute these suggestions, but I do point out some of the negative consequences of them, which my own solution avoids.

5.1 Necessitism
We could reject the principle of contraction. Then, all worlds would have the same stock of universals. We would not have a case where universals are alien to one world but not to another, and it would no longer follow that there are possibility claims about these universals that are true in some worlds and false in others. This would entail that every universal is a necessary existent, a version of necessitism (Williamson 2010, 2013). Necessitism is a highly controversial metaphysical commitment. Embracing necessitism is likely to have more controversial consequences down the line. Williamson’s necessitism crucially depends on accepting the existence of non-concreta (2010, p.665). If we can avoid commitment to necessitism and non-concreta, that will make the resultant position more powerful, as it can be accepted by more parties. Interestingly, Armstrong himself explicitly rejects both of these commitments, as he believes that all universals are contingent (1997, pp.156-158) and that only concreta exist (1997, p.8).

5.2 Alien properties again
We could adopt an approach that Armstrong himself toys with: accepting the possibility of alien universals after all. He argues that the universals in the actual world have relations of difference to each other. He suggests that from this, we can construct the idea of a universal that is again different from all of these (1997, p.167), which would be a universal that is alien to the actual world (this line of thought has some similarities with Stalnaker 2012). By accepting the possibility of alien universals, it might be thought that we can sidestep the problems outlined above.

This approach can only account for the truth of a general existential claim: that there could have been more universals than there are. But we need our theory to block the reasoning to the claim that \( \neg \Box \exists x Gx \) is true in \( W_1 \). That is a claim about the specific universal G. The claim that there could have been more universals than there actually are is not in tension with this claim, so it does not help. Armstrong himself was no fan of his own proposed solution. He says that it ‘does leave me uneasy’ because ‘some violence is done to combinatorial intuitions’ (1997, p.167). He was not even convinced that it could preserve S5,
saying ‘it may be argued that some important sort of inaccessibility still remains’ (1997, p.170), though he does not elaborate on these claims. In any case, I will discuss this idea no more.

6. **The solution**

Combinatorialism and strong actualism do not themselves entail (UC). When we reject (UC), a solution emerges that fits naturally with combinatorialism. Take the set of all of the *actual* world’s universals. Call this (ALL). We should reject (UC) in favour of the claim that for any possible world, the set of universals that that world could contain is not fixed by the universals in that world, but rather is fixed by the universals in (ALL). That is, (contra (UC)) when we are considering what is possible in a world, it is now permissible to include universals that are alien to that world, so long as we only pick from members of (ALL) (so long as we only pick from actual universals).7

How does this solve the problem? $W_a$ contains $Fa$ and $Gb$, and $W_1$ is a contraction of $W_a$, containing only $Fa$. $W_1$ is accessible from $W_a$. In this example, the set (ALL) contains $F$ and $G$. The problem was that (given (UC)) we could derive the claim that $\neg \diamond \exists x Gx$ is true in $W_1$. But this no longer follows, because we have rejected (UC). We have replaced it with the claim that what is possible for a world is fixed by the universals in the actual world. Since $G$ is in the actual world (and is thus a member of (ALL)), $\diamond \exists x Gx$ is now true in $W_1$. The reasoning that led us to the claim that $\neg \diamond \exists x Gx$ is true in $W_1$ is blocked. Without this claim, we can no longer generate any contradiction with (B) or with the characteristic S5 axiom. Further, there is no longer a failure of the symmetry of accessibility ($W_a$ and $W_1$ are now accessible from each other).

More informally, the solution works by rejecting the claim that worlds should look inwards at their own pools of universals and particulars to determine what is possible for them. Rather, they all look to

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7 Divers briefly suggests a solution that is in some ways similar, saying that what is accessible at all worlds should be determined by what can be generated by the actual world, where ‘actual’ is read rigidly (2002, p.216). However, his discussion is very brief. He does not outline the clash with (B) and the characteristic S5 axiom in the formal detail that I have done. Indeed, he discusses a version of the clash between combinatorialism and S5 (2002, pp.207-208) but does not apply the solution to that version, so the clash with S5 remains (I address this version of the clash with S5 in section 8). Divers also does not explain the rationale behind this solution in detail, or discuss the relation of the view to necessitism, alien properties, counterfactuals, or the modal status of combinatorialism itself. None of this is intended as a criticism of Divers. It is merely to point out that this paper and his work have different concerns, and that without the work provided here, the defence of S5 is of limited force.
the actual world, which fixes the central pool of resources. Notice that when we accept S5, we need not mention accessibility at all. Since all worlds can access all other worlds, we can define possibility (in any world at all) as ‘true in at least one possible world’. With S5, accessibility can safely drop away from our definition of possibility. 

7. Actual privilege

Someone could object: why should we index the pool of available universals to the ones in the actual world? Doesn’t this privilege the actual world in some way? The answer is yes! It does privilege the actual world, and it should. Combinatorialism embraces strong actualism, which entails that ‘[t]he actual universals set a limit, a limit given by the totality of their recombinations, to the possible universals.’ (Armstrong 1989, p.56. My emphasis). It is thus entirely in the spirit of the view that we should privilege the stock of universals possessed by the actual world over those of other worlds.

In fact, in light of the view that the limits of modal space should be fixed by the actual universals, commitment to the troublesome principle (UC) looks rather peculiar. When we consider the truth of modal propositions at other possible worlds, it is strange to confine our attention to just that world and to cut off the resources of the actual world from consideration. On the view I propose, we always look to the actual world when considering the truth of any modal proposition. In this way, the view is much more in the spirit of strong actualism than the version Armstrong himself puts forward.

I am now allowing that possibility claims about G can be true in W₁, even though G is alien to W₁. In section 3 I mentioned that we should reject the possibility of universals that are alien to the actual world, because by outstripping the resources of the actual world, we clash with strong actualism. The same problem does not recur here, because we only allow actual universals to be part of the central pool of universals. We need not worry that our modality is outstripping actuality, because G can be found in the actual world.

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8 It may be suggested that the problem recurs in the case of possibility claims about particulars (rather than universals) that are alien to W₁. This is not a problem because (as mentioned above) Armstrong allows possibility claims about alien particulars to be true, so the problem never arises for particulars. Even if we do not allow that possibility claims about alien particulars can be true, we can just tweak the strategy proposed so that (ALL) includes the actual universals and actual particulars. The rest of the solution applies in the same way.
To highlight the difference between myself and Armstrong, it is helpful to distinguish between the principles that determine how the space of possible worlds is constructed, and the principles that determine how modal claims are to be assessed at possible worlds. We can accept that Armstrong was right that the space of possible worlds should be constructed using ‘the simple particulars and simple universals of our world’ (1997, p.161). However, assessing the truth of modal claims in a world is a further and very different question. We must address this latter question for our modal metaphysics to be complete. When Armstrong addresses this, he assumes a principle like (UC): that all worlds must look inwards. This is what leads to our troubles, and must be replaced. The view proposed here gives us a new way of assessing the truth of modal claims at possible worlds, which is strongly principled, given the commitment to strong actualism, and which avoids the problems with Armstrong’s view. On this view, all worlds look at the actual world in assessing their modal claims.

8. Troublesome counterfactuals?
Consider the following counterfactual:

(ψ) If G had not existed, it would have been impossible for there to exist a G.

To recap: in our example, Wᵢ (the actual world) does in fact contain G. (ψ) is asking counterfactually what would have been the case if it had not. On one hand, there is a pull toward saying that (ψ) is true. For combinatorialism, the possibilities are generated by what universals and particulars there are, so it seems logical that if there had been fewer universals and particular, then there would have been fewer possibilities.

But there are excellent reasons to reject the truth of (ψ). Assuming the possibility of (ψ)’s antecedent, if we accepted the truth of (ψ), then it would follow that ‘it is possible for there to be a G’ is false in some possible world other than Wᵢ. We would then have to accept that ‘it is possible for there to be a G’ is only *contingently* true in Wᵢ, even though G exists in Wᵢ. This would violate (B) which states that if something

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9 Thanks to an anonymous referee for helping me see this.
10 Strictly speaking, this only follows if we assume that the world in question is accessible to Wᵢ. On S5, all worlds can access all other worlds, so we can assume this in the context of demonstrating a clash with S5.
exists then it is necessarily possible. Since (B) is an axiom of S5, we would then reintroduce a clash with S5.

Fortunately, my view does not imply that \( \langle \psi \rangle \) is true. \( W_a \) does in fact contain \( G \), so \( G \) is a member of (ALL): the set of all actual universals. On the standard analysis, a counterfactual is true just in case the consequent is true at the closest world where the antecedent is true (Lewis 1973). So, to analyse \( \langle \psi \rangle \), we must look to the world that is closest to \( W_a \) which does not contain \( G \). Say that is \( W_i \). At \( W_i \), the claim that it is impossible for there to exist a \( G \) is false, because \( G \) is a member of (ALL), and so \( \Diamond \exists x \neg Gx \) is true at all worlds. Thus, the consequent of the counterfactual is false at the closest world where the antecedent is true, and \( \langle \psi \rangle \) is itself false. This is as it should be. Recall that, for the combinatorialist, the limits of modal space are fixed by the universals and particulars that actually do exist. They have privilege and they alone. Modal space is not fixed by what would have been the case if the world had been poorer, as this counterfactual supposes. It is affected by what actually is the case.

This reasoning allows us to sidestep another way that the clash with S5 is sometimes expressed. It is sometimes said that since possibility claims are dependent on actual universals (for the combinatorialist), then if there had been fewer universals, then fewer possibility claims would have been true, indicating that possibility claims are contingent (Lewis 1999, p.202; Divers 2002, p.207). But rejecting \( \langle \psi \rangle \) allows us to reject the claim that if there had been fewer universals, then fewer possibility claims would have been true. So this way of expressing the clash with S5 is blocked.

Why can \( \langle \psi \rangle \) appear plausible? Because it is easy to forget how much combinatorialism is grounded in the actual universals. We understand the main principles of combinatorialism: that it takes the actual universals and particulars, and by recombining them, it gives us modal space. So, we can easily imagine the same principles operating on a restricted set of universals to generate a different modal space. However, to believe that this represents a genuine possibility is to assume precisely what the combinatorialist denies: it is to assume that modal space is affected by what the world would have contained if (counterfactually) things had been different, rather than what the world actually does contain.

The combinatorialist can of course accept that a scenario where modal space is restricted is perfectly coherent and conceivable, whilst denying that it represents a genuine metaphysical possibility. The
combinatorialist believes that what is possible is fixed by what actually is, not by what we can conceive of, so she already accepts that there are some scenarios that are conceivable, but not metaphysically possible (Armstrong 1989, pp.71-76).

9. *Combinatorialism: Necessary or Contingent?*

On my proposal, does combinatorialism end up contingent or necessary? This depends on how one characterises the view. If we take combinatorialism to be the view that something is possible iff it is obtainable by recombination of the simple universals and particulars that exist at the actual world, then combinatorialism will be necessarily true. However, if we take it to be the view that something is possible iff it is obtainable by recombination of the simple universals and particulars that exist, then combinatorialism will be true in the actual world, but false in contracted worlds like W₁. This is because (on the proposed view) in W₁ there are some things that are possible, but are unobtainable by recombination of the resources in W₁. Combinatorialism, so stated, is contingent.

Is this a problem? There is a vast literature on such ‘advanced modalizing’ issues (Divers 1999, Jago 2016) and I cannot engage with it all here. But three points are relevant. First, the contingency in question stems from the fact that the actual world is special: its resources (and not those of other worlds) are recombined to produce the possibilities. As we have seen, this special treatment is not problematic because it is already at the core of the actual-world chauvinist view that motivates combinatorialism. Second, there is nothing itself problematic about our modal theory being contingent. The modal status of our modal metaphysics is a highly theoretical and abstract issue. We are better off letting the theory that is the best independent account of modality decide this question, rather than assuming an answer in advance. Third, even if we were convinced that our theory of modality should be necessarily true, this would only show that we should not state combinatorialism in the way that makes it turn out contingent. If we do have reason to prefer a modal theory that is necessary, then we have reason to prefer the statement of the view (given above) on which it is necessary.

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11 Provided, of course, that ‘actual’ is read rigidly, as designating the world that is in fact actual.
12 Provided that ‘exists’ is restricted in scope: applying only to those entities in the world in question.
10. Conclusion
Given the overwhelming dominance of S5 modal logic, it is perhaps unsurprising that combinatorialism has not been extensively discussed as part of the current movement for integrating modal metaphysics and modal logic (Williamson 2013, Stalnaker 2012). If this paper is correct, then this neglect is unjustified. Combinatorialism deserves discussion as a modal theory that is actualist in the extreme, and also metaphysically and logically robust.
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