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Suppression of Spurious Responses in Microstrip Filters Using a Reduced Number of Resistors

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Abstract

The stop band of a microstrip filter was extended by suppressing its harmonics, using additional stubs together with a smaller number of resistors than in previous work. The possibility of suppressing the second to the fifth harmonics with one resistor per resonator is investigated in a simulation. Equations for initial estimates for losses and required coupling coefficients for the new stub design are given. In an example of a 4th order filter, a total of only two resistors were required to suppress the 2nd, 3rd and 4th harmonics to -35 dB; -29 dB was achieved in a fabricated filter. The pass band loss was increased by only 0.2 dB.

Indexing Terms: Absorption, harmonics, losses, microstrip microwave filters

1. Introduction

The upper stop band of microstrip filters is often limited by harmonic or other spurious responses. These can be suppressed by stubs or resonators which resonate at the spurious harmonic [1]-[3], microstrip indentations [4]-[7], or ground plane apertures [8]. An alternative is for neighbouring resonators to have the same fundamental, but differing higher order resonances [9],[10]. Resistive attenuation which is significant only in specific resonator modes has been used in tuneable oscillators [11],[12], and in filters [13]-[17]. In [17], several harmonics were suppressed, but each resonator required one resistor for the even order spurious bands, and a further resistor for each odd-order spurious peak, with the exception of the first and last resonators in which some suppression was already available from the input and output loading. In the present work, a simulation of a 4th order filter requires only one resistor within an additional stub, in each of two resonators to attenuate the 2nd, 3rd and 4th harmonics. Admittedly, the 4th harmonic in the measured filter was lower than expected and did not require attenuation. A second possible advantage over [17] is that the stubs, being directly coupled to the resonators, can have a larger coupling coefficient and therefore better suppression of some of the harmonics. Measured results with 29dB of attenuation are given.

A major consideration is the increase in pass band loss. Approximate equations are provided for feasibility studies, for a starting point for the iterations using an electromagnetic simulator such as SONNET®, and for a qualitative understanding. Various approximations in [17] will again be used to the limit of their validity, for the new configuration. In particular, the input and output points in a resonator with low Q-factor has a significant effect, exacerbated by a cubic dependence. The stubs also change the current distributions of the harmonics, which may change the coupling coefficients and the level of the harmonics; this has not been considered.
Fig. 1. Fundamental, second harmonic and third harmonic voltage waveforms in a microstrip resonator, assuming the twin-pronged stub is only a small perturbation.

2. Principle of operation

Fig. 1 shows the voltage waveforms of the fundamental and two harmonics in a straight microstrip resonator. A resistor is included in a forked stub which is placed slightly off-centre on the main line. The prongs are a quarter-wavelength long at the third and fifth harmonics respectively. At the fundamental, the input voltage to the stub is very small, being close to a voltage node, and furthermore the stub does not resonate; it therefore introduces very low loss, as required. At the third and fifth harmonics, the increased input voltage leads to more loss, which is proportional to the square of the voltage. But much more important, current is large because the stub resonates at these frequencies. At the even order harmonics, the stub is not resonant, but being near a voltage maximum, receives a large current. Thus several harmonics can potentially be attenuated using only one resistor. In the present work, the stub has only one prong so fewer harmonics are suppressed.

In fig. 2(a), input and output are via capacitive pads in a lower layer. Coupling coefficients increase with frequency because of the decreasing capacitive reactance, but in this case it is partially compensated by the decreasing reactance between the input pads and earth. The simulated response is shown in fig. 2(b). In this example, the odd order harmonics are split into two modes each, as well as being attenuated.

A possible problem is that the two prongs in the forked stub have a total length of \( \frac{1}{2} \frac{\lambda_1}{3.75} \), where \( \lambda_1 \) is the wavelength at fundamental frequency. They form a half-wave resonator at 3.75 times the fundamental. This reduces the attenuation at the fourth harmonic.

A single-pronged stub and the main line resonating at the third harmonic form a dual-mode resonator. In [18] and [19], the two modes are used in the pass band (or pass bands in a dual-band filter), but not for harmonic suppression. The off-centre tapping point is also remotely similar to the input and output taps of some microwave filters [20].
Fig. 2. Harmonic suppression in a hairpin variant of the microstrip resonator.
(a) Resonator layout. Dimensions in mm. Relative permittivity is 10.2, loss tangent is zero, dielectric thickness is 1.27 mm, and track width is 0.5 mm. Hatched region represents a 15 \( \Omega \) resistor. Input and output, shown dashed, comprise capacitor plates 0.4 mm above the ground plane.
(b) Response of the hairpin without (dashed) and with (solid line) the stub.

3. The relation between harmonic attenuation and pass band loss

A stepped impedance hairpin microstrip resonator with an additional stub is shown in fig. 3(a). The main line has sections of characteristic impedance \( Z_a \) and \( Z_b \), taken to be of equal length. The stub has a uniform characteristic impedance \( Z_c \). Short lengths of transmission line can be approximated by series inductors and parallel capacitors, but at odd-order resonances, the voltage minimum at the centre makes the capacitors unnecessary. Thus \( L \) and \(-L\) represent the offset of the stub from the centre of the main line. Straightforward circuit analysis gives

\[
\frac{V_{in}}{I_{out}} = R_L + jX_m + \frac{(\alpha L)^2 + (R_L + jX_m)^2}{2(R + jX_s)}
\]

or with no stub,

\[
\frac{V_{in}}{I_{out}} = R_L + jX_m
\]
3.1 Operation at the fundamental frequency

Without the stub, at frequency $f_1 + \Delta f$ near the resonance $f_1$, writing $Z_{m1}$, $X_{m1}$, $Z_{s1}$ and $X_{s1}$ for $Z_m$, $X_m$, $Z_s$ and $X_s$, and using $\tan \theta \approx \theta$, basic transmission line theory gives

$$Z_{m1} = jX_{m1} \approx 2 jZ_a \left( \tan^{-1} \left( \frac{Z_b}{Z_a} \right) \right) \frac{\Delta f}{f_1}$$

(3a)

$$\approx j \frac{\pi}{2} Z_a \frac{\Delta f}{f_1}$$

(3b)

where (3b) and subsequent equations labelled (b) apply to the special case where $Z_a = Z_b$. The external quality factor can be found from the 3 dB points in (2), yielding

$$R_L \approx Z_a \left( \tan^{-1} \left( \frac{Z_b}{Z_a} \right) \right) \frac{1}{Q_{lc}}$$

(4a)

$$\approx Z_a \frac{\pi}{4} \left( \frac{1}{Q_{lc}} \right)$$

(4b)

When the stub is added, this relation still holds approximately. The stub impedance is

$$Z_{s1} = jX_{s1} \approx -jZ_c \cot \left( \frac{\Delta f_1}{2f_3} \right)$$

(5a)
\[ \approx -j\sqrt{3}Z_c \]  

(5b)

Also, \( f_1 \), is still close to the resonance, and pass band loss will be estimated from this point. Assuming \( |R + jX_{s1}| \approx X_{s1}^2 \), (1) reduces to

\[ \frac{V_{in}}{R_L I_{out}} \approx 1 + \frac{(\omega L)^2 + R_L^2}{2R_L X_{s1}^2} (R - jX_{s1}) \]  

(6)

Even with \( R=0 \), the existence of the imaginary part implies that \( |V_{in}/R_L I_{out}|^{-1} < 1 \); a loss arises from resonator asymmetry. It need not occur if the filter as a whole is symmetrical, with resonators on each side of the centre line mirror images of each other. The imaginary part can be ignored when considering the loss introduced by the resistor.

\[ \frac{V_{in}}{R_L I_{out}} \approx 1 + \frac{(\omega L)^2 R}{2R_L X_{s1}^2} + \frac{R_L R}{2X_{s1}^2} \]  

(7)

The first term in (7) is related to the external Q-factor \( Q_{le} \), while \( Q_{la} \) and \( Q_{lb} \), respectively will be associated with the other two terms, which are based on resonator losses, and not on the pass band shape, which may not be exactly the same as for a single-pole resonator. Thus

\[ \frac{1}{Q_{la}} \approx \frac{(\omega L)^2 R}{2R_L X_{s1}^2} \]  

(8)

\[ \frac{1}{Q_{le}} \approx \frac{R_L R}{2X_{s1}^2} \]  

(9)

from which

\[ R = \frac{2X_{s1}^2}{(\omega L)^2} Z_a \left( \tan^{-1} \sqrt{\frac{Z_b}{Z_a}} \right) \frac{1}{Q_{la}} \]  

(10a)

\[ = \frac{\pi}{2} \frac{X_{s1}^2}{(\omega L)^2} Z_a \frac{1}{Q_{la}} \]  

(10b)

\[ R = \frac{2X_{s1}^2}{Z_a \left( \tan^{-1} \sqrt{\frac{Z_b}{Z_a}} \right) Q_{le}^2} \]  

(11a)

\[ \frac{8X_{s1}^2}{\pi Z_a Q_{le}^2} \]  

(11b)

The voltage at the node at the centre of the main resonator is not exactly zero \[17\], so pass band losses arise even when the stub is centrally placed, that is when \( \omega L = 0 \), \( R \neq 0 \), as given by the \( Q_{1\beta} \) term.
3.2 Suppression of the third harmonic, where it dominates the second harmonic

Addition of the stub may split the third order resonance into two modes, with neither at exactly three times the fundamental frequency. This paper loosely refers to both as the 3rd harmonic. The following discussion applies when the peaks are relatively large, so when they are suitably suppressed, the 2nd harmonic is also satisfactorily attenuated. At frequency $f_3 + \Delta f$ close to the third harmonic $f_3$ before the addition of the stub,

$$Z_{m3} = jX_{m3} \approx 2jZ_a \left( \pi - \tan^{-1} \left[ \frac{Z_a}{Z_m} \right] \frac{\Delta f}{f_3} \right)$$  \hspace{1cm} (12a)$$

$$\approx j \frac{3\pi}{2} Z_a \frac{\Delta f}{f_3}$$  \hspace{1cm} (12b)$$

$$Z_{s3} = jX_{s3} \approx j \frac{\pi}{2} Z_c \frac{\Delta f}{f_3}$$  \hspace{1cm} (13a)$$

$$\approx j \frac{\pi}{2} Z_c \frac{\Delta f}{f_3}$$  \hspace{1cm} (13b)$$

Since attenuation at third harmonic needs to be high, the damping due to $R$ is much larger than that due to $R_L$. $R_L$ is therefore omitted from (1). When $R$ is also negligible, the third harmonic splits into two peaks at frequencies $\pm \Delta f_p / f_3$, separated by $2\Delta f_p$, so the structure is equivalent to a pair of resonators with a coupling coefficient
\[ k_3 = \frac{2\Delta f_p}{f_3} \quad (14) \]

At the two peaks, \( \frac{V_m}{I_{out}} = 0 \), so from (1),

\[(\omega L)^2 = 2X_mX_{x3} + X_{m3}^2 \quad (15a)\]

\[= \left( \frac{2Z_c}{3Z_a} + 1 \right) \left( \frac{9\pi^2}{16} \right) Z_a k_3^2 \quad (15b)\]

Returning to non-zero \( R \), and first taking the case when the \( Q_{1a} \) term in the fundamental pass band is negligible compared with the \( Q_{1a} \) term, the third harmonic response for \( Z_a = Z_b \) and for a given level of fundamental pass band loss is found by substituting (10b), (15b) and \( R_L = 0 \) into (1),

\[
\frac{2Q_{1a}^{1/3}V_{in}}{\pi Z_a I_{out}} = 3jF + \frac{2.25AK^2 - 9F^2}{96Z_c^2/Z_a^2 A\pi^2K^2 + j2(Z_c/Z_a)F} \quad (16)
\]

where

\[ F = Q_{1a}^{1/3} \frac{\Delta f}{f_3} \quad (17) \]

\[ K = Q_{1a}^{1/3} k_3 \quad (18) \]

\[ A = \frac{2Z_c}{3Z_a} + 1 \quad (19) \]

For a constant pass band loss, that is, a constant \( Q_{1a} \) term, the resistance \( R \) varies as \( 96Z_c^2/Z_a^2 A\pi^2K^2 \) as given in the denominator.

For \( Z_a = Z_b = Z_c \), numerically-computed plots of normalised response \( \frac{2Q_{1a}^{1/3}V_{in}}{\pi Z_a I_{out}} \) against normalised frequency \( F \), for various values of normalised coupling coefficient \( K \), are shown by the family of dashed lines, which includes the thick black line with superimposed white dashes, in fig. 4. The peaks are minimised to a value of 0.517 when \( K \) has the value \( K_{opt} = 1.66 \quad (20) \)

Here, they are separated by an 8.1 dB trough, not very different from the 3 dB in [17], because choosing a 3 dB trough here would increase the peaks by less than 1 dB. An effective third-harmonic Q-factor \( Q_3 \) can be defined as the Q-factor of the original single mode resonator, with the stub replaced by a resistance inserted into the resonator itself, such that it has the same maximum response. For this equivalent resonator,

\[
\frac{2Q_{1a}^{1/3}V_{in}}{\pi Z_a I_{out}} = 3j \left( Q_{1a}^{1/3} \frac{\Delta f}{f_3} \right) + \frac{1}{0.517} \quad (21)
\]

\( Q_{1a} \) has no significance in the original resonator with no stub; it is an arbitrary constant with the same value as in (16), included to facilitate direct comparison. When \( \frac{\Delta f}{f_3} = \frac{1}{2Q_3} \), the real and imaginary parts of (21) are equal, resulting in
\[
\frac{1}{Q_{\alpha\alpha}} \approx 0.466 \left( \frac{1}{Q_3} \right)^3
\]  

(22)

From (18), (20) and (22), the required coupling coefficient is

\[
k_3 \approx 1.29 \left( \frac{1}{Q_3} \right)
\]  

(23)

For a given third harmonic attenuation, (22) estimates the pass band loss which has to be tolerated, while (23) gives the required value of \( k_3 \). The stub position could be varied in repeated simulations until this value is attained, and then and then a resistor can be introduced and varied until the dip between peaks is about 8 dB. Alternatively, basic transmission line theory shows that the shift of the stub position from the centre is \( \frac{\alpha_3 L \lambda_3}{2\pi Z_a} \) where \( \lambda_3 \) is the wavelength at the third harmonic, and \( \alpha_3 L \) can be found from (15b). \( R \) is determined by (10b).

The second case, when the \( Q_{\alpha\alpha} \) term is small, could be investigated by substituting (11b) instead of (10b) in (1), thereby keeping \( R \) and the \( Q_{1\beta} \) term constant, while \( K \) in the numerator is varied. Alternatively, the previous optimum value \( K_{\text{opt}}=1.66 \) can be inserted in the denominator of (16), which also keeps the \( Q_{1\beta} \) term constant. \( Q_{\alpha\alpha} \) is now an arbitrary constant, equal to its previous value when \( K=K_{\text{opt}} \). The value of \( K \) in the numerator is varied to see if a greater attenuation can be achieved in this second case. For \( Z_a = Z_b = Z_c \), the normalised response for various values of \( K \) is shown by the family of solid lines, which includes the thick line. The peaks are minimised as \( K \) tends to infinity. However, it can be shown that a large \( K \) results in a greater sensitivity to fabrication error. Furthermore, from (8) and (9), increasing \( K \) (and with it, \( \omega_3 L \)) too far invalidates the assumption that the \( Q_{1\beta} \) term is dominant. The previous value of \( K_{\text{opt}}=1.66 \) is therefore retained as the approximate optimum even though the peaks are 4.3 dB larger than for the actual optimum \( K \to \infty \). It will also be accepted as an approximate optimum when the \( Q_{\alpha\alpha} \) and \( Q_{1\beta} \) terms are comparable with each other.

Keeping \( Z_a = Z_b = Z_c \), the \( Q_{1\beta} \) term is found from (10b), (11b), (15b) and \( \alpha_3 = 3\alpha_1 \),

\[
\frac{1}{Q_{1\beta}} = \frac{1}{(Q_3 k_3)^2} A \frac{Q_3^2}{Q_{\alpha\alpha}} \frac{1}{Q_{1\beta}} \frac{Q_1}{Q_3}
\]  

(24)

and using (22) and (23),

\[
\frac{1}{Q_{1\beta}} \approx 0.169 \left( \frac{1}{Q_3} \right)^2 \left( \frac{1}{Q_{1\beta}} \right)^2
\]  

(25)

The total pass band loss, as described by \( Q \)-factors, is

\[
\frac{1}{Q_{\alpha\alpha}} + \frac{1}{Q_{1\beta}} = \alpha \left( \frac{1}{Q_3} \right)^3 + \beta \left( \frac{1}{Q_{1\beta}} \right)^2 \left( \frac{1}{Q_3} \right)
\]  

(26)

where

\[
\alpha = 0.466
\]  

(27)

\[
\beta = 0.169
\]  

(28)
These values are constant for the given values of microstrip characteristic impedance, and (26) gives the additional pass band loss which has to be tolerated for a given level of third harmonic attenuation. Because of the factor \((1/Q_{el})^2\), pass band loss may be larger for larger bandwidths, as explained in [17].

In a stepped impedance hairpin to be discussed, \(Z_a = 72 \ \Omega\), \(Z_b = 33 \ \Omega\) and \(Z_c = 45 \ \Omega\). Repeating the foregoing analysis, (29) gives the additional pass band loss which has to be tolerated for a given level of third harmonic attenuation. Because of the factor \(\frac{1}{Q_3}\), pass band loss may be larger for larger bandwidths, as explained in [17].

\[
\alpha = 0.202 \quad (29)
\]
\[
\beta = 0.092 \quad (30)
\]
\[
k_3 \approx 1.25 \left( \frac{1}{Q_3} \right) \quad (31)
\]

The depth of the trough between the peaks is 10.5 dB, and when the \(Q_{el}\) term is negligible, the peaks are 4.5 dB short of the true optimum. The improved \(\alpha\) and \(\beta\) arise from the larger value of \(\omega_3/\omega_1\), which is 4.28 instead of 3.0; this affects the first term in the denominator in (16), which depends on \(\omega_1\) and \(\omega_3\) in (10) and (15).

### 3.3 Second harmonic attenuation when dominated by the third harmonic

The second harmonic behaviour can be based on (1), except that the approximation \(\tan \theta \approx \theta\) is inappropriate. However, it is easier to simplify the circuit as shown in fig. 3(b). The off-centre tap position of the stub is modelled by the capacitors \(\pm C\) and inductors \(\pm L\). The capacitors cancel. The inductances also cancel approximately, because they are small perturbations, unlike the odd order resonances where they are large compared with the near-zero transmission line impedances. The two top ends of the resonator have the same polarity, so they are equivalent to a single transmission line with half the characteristic impedance. The source is placed in the stub, assuming that the excitation point does not affect the Q factor. The stub is properly taken to be a transmission line, which is an improvement over [17], where a series RC load is approximated by a parallel RC combination, and the capacitance is subsequently ignored. The input reactance is

\[
X_{in} = \frac{Z_a}{2(Z_a + Z_b)} \left( Z_a \tan \theta - Z_b \cot \theta \right) - Z_c \cot \left( \frac{f_2}{f_3} \theta \right) \quad (32)
\]

where

\[
\theta = \frac{\pi}{2} \frac{f}{f_2}, \quad (33)
\]

\(f_2\) is the second harmonic resonance before the addition of the stub, and \(\theta\) is the electrical length of one of the transmission line sections in the main line. Resonance occurs at \(f_{21}\), where \(X_{in} = 0\), while at \(f_{21} + \Delta f\),

\[
X_{in} \approx \frac{dX_{in}}{d\theta} \left. \frac{d\theta}{df} \right|_{\Delta f} \Delta f \quad (34)
\]

where, using (32),

\[
\frac{dX_{in}}{d\theta} = \frac{Z_a}{2(Z_a + Z_b)} \left( Z_a \sec^2 \theta + Z_b \csc^2 \theta \right) + Z_c \frac{f_2}{f_3} \csc^2 \left( \frac{f_2}{f_3} \theta \right) \quad (35)
\]

is evaluated at the resonance, where

\[
\theta = \frac{\pi}{2} \frac{f_{21}}{f_2} \quad (36)
\]
At the 3dB points, \(|X_{in}| = R\) and with the 3dB bandwidth equal to \(2\Delta f\), using (11a) or (11b) the quality factor \(Q_2\) is given by

\[
\frac{1}{Q_2} \approx \frac{8}{\pi} \frac{f_2}{f_{21}} \frac{d\theta}{X_{in}} Z_a \tan^{-1} \left( \frac{Z_b}{Z_b} \right) \frac{X_{a1}^2}{\beta_2} \left( \frac{1}{Q_3} \right)
\]

(37a)

and for \(Z_a = Z_b = Z_c\),

\[
\frac{1}{Q_2} \approx \frac{96}{\pi^2} \frac{f_2}{f_{21}} \frac{d\theta}{X_{in}} Z_c Z_a \beta_2 \left( \frac{1}{Q_3} \right)
\]

(37b)

This gives the available second harmonic suppression if \(1/Q_3\) has already been chosen.

When \(L = 0\) in (8) (deviating from the previously described optimum \(K\)), there is no third harmonic suppression under the given assumptions, but the second harmonic suppression is not affected. The \(1/Q_{1a}\) and \(1/Q_{1b}\) terms can therefore be associated respectively with the third and second harmonic suppression. However, when the optimum is sought, the terms are not independent.

The foregoing equations also apply for the 4th harmonic, with \(Q_4\) and \(f_{41}\) replacing \(Q_2\) and \(f_{21}\) and using a new value for \(dX_{in}/d\theta\). However, \(f_2\) is unchanged by virtue of (36), being the frequency where the phase shift of one transmission line section is \(\pi/2\). For \(Z_a = Z_b\), \(X_{in}\) is symmetric about \(f = 1.5f_2\), resulting in a less favourable, smaller \(1/Q_4\), but at least the absolute bandwidth of the 4th harmonic is equal to the 2nd harmonic.

### 3.4 Attenuation when the second harmonic dominates

If the second harmonic is the dominant feature which determines the stub properties, they can be evaluated by re-writing (26) as

\[
\frac{1}{Q_{1a}} + \frac{1}{Q_{1b}} \approx \left[ \alpha \left( \frac{Q_2}{Q_3} \right)^3 \right] \left( \frac{1}{Q_2} \right)^3 + \left[ \beta \left( \frac{Q_2}{Q_3} \right)^2 \right] \left( \frac{1}{Q_{1a}} \right)^2 \left( \frac{1}{Q_2} \right)^2
\]

(39)

where the quantities in the square brackets are constants for a given set of \(Z_a\), \(Z_b\) and \(Z_c\), and \(1/Q_2\) is a measure of the second harmonic suppression required. If there is surplus third harmonic suppression, the \(1/Q_{1a}\) term can be reduced by reducing \(L\), that is, deviating from the previously described optimum. Pass band loss is thereby reduced without compromising second harmonic attenuation. This may be considered in future work.
4. Filter Design and Measurements

Fig. 5 shows a fourth order, microstrip hairpin filter designed with a pass band centre frequency of 1000 MHz and a fractional bandwidth of 10%. Except for the different additions for harmonic suppression, and some fine adjustments, this filter is similar to one of the filters in [17], using a 1.27 mm thick RT Duroid 6010LM substrate and 18 microns of copper. The feed lines are also 1.25 mm wide for the usual 50 Ω impedance. Unlike [17], only two 3.2 mm chip resistors are used. Before adding the resistive stubs, some harmonic suppression was already achieved by increasing the capacitance between resonators with additional 4 mm lines. At the second harmonic, capacitive and inductive coupling have opposite signs but unequal magnitude in mixed dielectric (air + substrate) configurations. Equalising them using the additional capacitance therefore reduces the coupling at the second harmonic. The same additional capacitors can be used in parallel line quarter-wave couplers [21] to improve directivity. The second harmonic response does indeed become narrower, but at the cost of a wider third harmonic. This filter with a partially suppressed second harmonic is the basis of comparison with the final design.

The resistive stubs are near the middle of resonators 2 and 3. The tapping point, 3.5 mm from the centre of the resonator, and the 10 Ω resistance, were found from trial and error simulations, as the above theory had not yet been finalised. Further simulations were carried out to re-optimise the response, and the dimensions given in fig. 5 are the final values.

In the fabricated filter, the leftmost end of resonator 1 and the rightmost end of resonator 4, together with both resistive stubs, were shortened by up to 1 mm by manual cutting, to improve the response. The error may have resulted from the volume of the solder which was not considered in the simulations, or because the chip resistors were modelled too simply as resistive squares. Future devices might not need the adjustment. The measured result, shown in fig. 6a, shows 29 dB of attenuation up
to 3900 MHz, short of the intended 35 dB, but this is nevertheless a successful filter. Even with no resistors, the stubs introduce a little suppression, but the effect of the resistors is apparent, except for the fourth harmonic of the measured filter which was unexpectedly already very small so its suppression is not demonstrated here. Other data on 4th harmonic suppression is presented in the next section. The additional pass band attenuation, which is the difference between measurements with 10 ohm resistors and 0 ohm bridges, is 0.18 dB, compared with the simulation result of 0.23 dB in fig. 6(b). Based on the simulated filter third harmonic fractional band width of 0.15 at -35 dB, the equations estimate 0.27 dB. However, single-resonator simulations show a fractional band width of only 0.11, and the pass band loss estimate is then only 0.13 dB. This is an expected limitation in the simple use of $Q$ factor to estimate harmonic suppression. Nevertheless, a factor of two accuracy is useful, and the equations are very straightforward.

Fig. 6. Response of the hairpin filter with resistive stubs
(a) Upper stop band
Fig. 6(b). Pass band

Fig. 7. Stepped impedance hairpin filters used for additional loss measurements. Substrate and minimum track width as in fig. 5. Dimensions in mm. The basic resonator has dimensions 10 mm x 10 mm. Four separate devices 1(a), 1(b), 2(a) and 2(b), each resembling the illustration, were fabricated. 1(a) and 1(b) have connections in1 and out1, together with gap 1, with in2 and out2 omitted; stub offsets are given as (a) and (b). 2(a) and 2(b) are designated correspondingly; in2 and out2 require via pins. Resistors for the simulation are (a) 3.6 $\Omega$ or (b) 5.5 $\Omega$
5. Verification of equations for pass band effective $Q$-factor

The estimate for pass band loss in section 3.2 is now compared with simulations and measured results. In the previously described filter, $\frac{1}{Q_1} = \frac{1}{Q_{1a}} + \frac{1}{Q_{1b}}$ can be calculated from the 0.18 dB loss [17]. Alternatively, the additional loss in dB is [22]

$$L_{dB} = \frac{4.343\ f_1}{B}(g_2 + g_3)$$

(40)

where $B/f_1$ is the fractional bandwidth. Only the low pass prototype component values $g_2$ and $g_3$ are included since only resonators 2 and 3 contain lossy stubs. For a Chebyshev filter, $g_2$ and $g_3$ are not exactly equal, even though the corresponding band-pass resonators are a symmetrically placed, identical pair. However, working from first principles in the LC (inductance-capacitance) prototype circuit, it can be shown that the sum $g_2 + g_3$ in (40) gives the total loss of the two resonators to a reasonable approximation. With $L_{dB}$ measured, $B/f_1 = 0.1$, and taking prototype values for a filter with 0.1 dB ripple, the corresponding value of $Q_1$ was found.

Four second-order Butterworth filters, with the layout given in fig. 7, were also designed and measured. These employed 10 mm x 10 mm step impedance resonators, with characteristic impedances specified in section 3. $Q_1$ can be found from (40), with $g_2$ and $g_3$ replaced by $g_1$ and $g_2$. The stubs were off-centre by 1 mm or 2 mm, and the nearest available resistors were 4.7 $\Omega$ and 3.3 $\Omega$ respectively, sufficiently similar to the simulations considering the level of approximation required.

### Table 1. Comparison of Estimates, Simulations and Measured values of $Q_2$, $Q_3$ and $Q_4$.

* Asterisked values are based on four-resonator simulations or measurements.

<table>
<thead>
<tr>
<th>$Z_a$</th>
<th>$Z_b$</th>
<th>$Z_c$</th>
<th>$\frac{1}{Q_1}$</th>
<th>$\frac{Q_1}{Q_2}$</th>
<th>$\frac{Q_3}{Q_4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>$\Omega$</td>
<td>$\Omega$</td>
<td>Sim</td>
<td>Meas</td>
<td>Estimates</td>
</tr>
<tr>
<td>72</td>
<td>72</td>
<td>72</td>
<td>0.11</td>
<td>--</td>
<td>0.39</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>0.15*</td>
<td>0.17*</td>
<td>--</td>
</tr>
<tr>
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<td>33</td>
<td>45</td>
<td>0.02</td>
<td>--</td>
<td>0.43</td>
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<td></td>
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<td>0.05</td>
<td>0.07</td>
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<td></td>
<td></td>
<td>0.13</td>
<td>0.14</td>
<td>0.43</td>
</tr>
</tbody>
</table>

The simulations involved single resonators, similar to the plain hairpins and step impedance resonators shown in fig. 5 and 7, with external coupling either capacitive resembling fig. 2(a), or parallel line coupling. The additional pass band loss was evaluated by comparison between the simulated resistive and 0 $\Omega$ cases. The latter itself exhibits a small mismatch loss because the resonators are not perfectly symmetrical. The estimates of $Q_1$ are given in (26) - (30), using values of $Q_3$ and $Q_{1e}$ from simulations. All the results are shown in fig. 8. The variation between the estimate, the simulations and the measured results is less than a factor of about two, much smaller than the whole range of possible values, which is more than three orders of magnitude, so the estimate provides very useful information.

The calculated, simulated and (where available) measured values for the relationship between $Q_2$, $Q_3$ and $Q_4$ are given in table 1. The agreement is within a factor of 2.5. Possible reasons for the discrepancies are the large soldering pads which cause
additional capacitive loading, and the position of the hand-soldered resistor, which is not exactly as in the simulation, and the volume of the solder used in manual soldering.

Fig. 8. Additional pass band loss due to the resistive stubs, expressed as an effective Q-factor. The estimates from the equations in section 3.2, the simulations, and measured results are compared. For the plain hairpin, the stub is off-centre by 3.5 mm, and the stub resistor is 10 Ω. For the step-impedance resonators, the stub is off-centre by 2, 1, or 0.5 mm, and the stub resistors in the simulations are 5.5, 3.6 and 1.4 Ω respectively.
6. The effect on stub impedance on pass band loss

Loss constants $\alpha$ and $\beta$ are shown in fig. 9, for $Z_a=Z_b$ but with a variable $Z_c$. Simulations are based on a 35 $\Omega$, 0.625 mm line bent into a 20 mm x 19.875 mm U-shape on a 0.3175 mm thick substrate with $\varepsilon_r=10.2$. The stub widths are 0.125, 0.625, 1.125, and 1.625 mm, with characteristic impedances 74, 35, 24 and 18 $\Omega$ respectively. They are placed 2.875, 2, 1.625 and 1.375 mm from the centre of the main stub, and contain resistors 7, 3.6, 2.4 and 1.8 $\Omega$. There is reasonable agreement for the data presented, with an approximate best fit obtained by multiplying the simulated results by only 1.2. However, for $Z_a/Z_c=2.5$ (not shown), the simulated values are significantly smaller than the calculated estimates, probably because of a relatively large value of $R_L$ instead of the assumed $R_L=0$ for the third harmonic in (1). When a non-zero value of $R_L$ is inserted into the calculations, closer agreement is obtained. When $Z_a=72$ $\Omega$, $Z_b=33$ $\Omega$, $\alpha$ and $\beta$ are about one third of the values in fig. 9, while $Q_3/Q_2$ is approximately unchanged.

For a given level of third harmonic attenuation, pass band loss decreases as $Z_c$ increases, that is, towards the extreme left of the graph. However, when the second harmonic suppression is given, pass band loss increases when $Z_c$ is large, as shown by the inset which plots $\alpha(Q_2/Q_3)^3$ from (39). There is scope for choosing $Z_c$ depending on the relative second and third harmonic levels, but perhaps it would be reasonable simply to compromise with $Z_a=Z_c$, which is convenient for fabrication as it does not involve a wide range of characteristic impedances.

Fig. 9. Simulated (open symbols) and estimated (filled, black symbols) pass band attenuation constants $\alpha$ and $\beta$, together with the ratio $Q_3/Q_2$, for $Z_a=Z_b$, and variable $Z_c$. 

$\alpha = \frac{Z_a}{Z_c}$
7. Conclusion

Spurious resonances in microstrip filters have been suppressed using many fewer resistors than in similar previous work [17]. Estimates of the additional pass band loss were given, including a more accurate expression for the available second harmonic suppression for a given pass band loss. A measured filter shows excellent performance, and additional data confirm the design equations.

References