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Experimental investigation on bubble departure diameter in pool boiling under sub-atmospheric pressure

Wenzhong Gao¹, Jiaye Qi¹,Xuan Yang¹, Jiahao Zhang¹, Dawei Wu²

(¹. Merchant Marine College, Shanghai Maritime University, Shanghai, 201306, China; ². School of Marine Science and Technology, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom; College of energy engineering, ZheJiang University, Hangzhou, 310058, China)

Abstract

The objective of this research is to compare the nucleate boiling characteristic of the calcium chloride aqueous solution with distilled water at sub-atmospheric pressure by analyzing the growing bubble dynamics in order to provide some data for optimizing the design of the dehumidification system. The experiments were carried out with water and calcium chloride solution on the upper surface of a polished stainless steel plate, under sub-atmospheric pressure ranging from 3.6kPa to 22.0kPa. An artificial nucleation site was created in the center of the plate to generate the isolated bubbles. A high-speed camera was used to capture the images of dynamic bubbles, and the relevant parameters on bubble dynamics were measured and calculated by frame-by-frame image treatment.

Experimental results show that bubble diameter tends to increase with the pressure decrease which means the lower vapor density and stronger surface tension force at sub-atmospheric boiling. The influence of superheat and sub-cooling degree were also analyzed. Additionally, a complex boiling regime of calcium chloride solution with irregular bubble dynamic parameters was observed. Finally, bubble growth dynamics under sub-atmospheric were analyzed and the force balance equation were established. It is shown that the dynamic effect especially the inertial force dominated the growth stage under sub-atmospheric boiling. A new bubble departure diameter correlation within ±20% deviation was proposed.
**Keywords:** nucleate pool boiling; sub-atmospheric pressure; bubble departure diameter; calcium chloride aqueous solution

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_v$</td>
<td>the vapor pressure (Pa)</td>
</tr>
<tr>
<td>AAD</td>
<td>the average absolute deviation (%)</td>
</tr>
<tr>
<td>AD</td>
<td>the average deviation (%)</td>
</tr>
<tr>
<td>$q$</td>
<td>heat flux (kW m$^{-2}$)</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $Pr = \nu/\alpha$</td>
</tr>
<tr>
<td>$Ar$</td>
<td>Archimedes number, $Ar =</td>
</tr>
<tr>
<td>$a$</td>
<td>the length of the semi-minor axis (m)</td>
</tr>
<tr>
<td>$b$</td>
<td>the length of the semi-major axis (m)</td>
</tr>
<tr>
<td>$T_{sat}$</td>
<td>the saturation temperature(℃)</td>
</tr>
<tr>
<td>$T_w$</td>
<td>the heated wall temperature(℃)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>the saturation temperature(℃)</td>
</tr>
<tr>
<td>$C$</td>
<td>a parameter in fritz correlation</td>
</tr>
<tr>
<td>$t_g$</td>
<td>bubble growth period (s)</td>
</tr>
<tr>
<td>$t_w$</td>
<td>bubble waiting period (s)</td>
</tr>
<tr>
<td>$D_d$</td>
<td>bubble departure diameter (mm)</td>
</tr>
<tr>
<td>$D_{eq}$</td>
<td>the equivalent bubble diameter (mm)</td>
</tr>
<tr>
<td>$P_{\infty}$</td>
<td>ambient pressure (Pa)</td>
</tr>
<tr>
<td>$P_{wall}$</td>
<td>the heated wall pressure (Pa)</td>
</tr>
<tr>
<td>$F_B$</td>
<td>buoyancy force (N)</td>
</tr>
<tr>
<td>$F_I$</td>
<td>inertial force (N)</td>
</tr>
<tr>
<td>$F_M$</td>
<td>Maragoni force (N)</td>
</tr>
<tr>
<td>$F_p$</td>
<td>pressure force (N)</td>
</tr>
<tr>
<td>$F_s$</td>
<td>surface tension force (N)</td>
</tr>
<tr>
<td>$f$</td>
<td>the bubble departure frequency (Hz)</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration (ms$^{-2}$)</td>
</tr>
<tr>
<td>$h_l$</td>
<td>the liquid height (cm)</td>
</tr>
<tr>
<td>$g_c$</td>
<td>gravitational acceleration correction factor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity (m$^2$s$^{-1}$)</td>
</tr>
<tr>
<td>$h$</td>
<td>the heat transfer coefficient (kW m$^{-1}$K$^{-1}$)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>contact angle, deg</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity (Pa s)</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>liquid density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>liquid density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$h_{lv}$</td>
<td>latent heat (J/kg)</td>
</tr>
<tr>
<td>$Ja$</td>
<td>Jacob number, $Ja = \rho_l c_p \Delta T/\rho_v h_{lv}$</td>
</tr>
</tbody>
</table>
In recent years, due to new environmental restrictions and ever-increasing demand of energy efficiency, absorption refrigerating plants and liquid desiccant dehumidification systems have become more and more popular. In such systems, nucleate boiling is a widely used method of regeneration due to its high heat transfer coefficient between solid and liquid. Bubble growth and detachment processes have a major impact on nucleate boiling heat transfer. A bubble growth cycle includes:

1. Nucleation period, a bubble nucleates from a single site on the heating surface;
2. Growing period, the bubble grows larger over time, during this period, the convection between liquid and wall is enhanced because of the hot capillary action in vapor-liquid interface;
3. Departure period, when the bubble grows to a certain size, it will depart from the surface, at the same time, liquid with lower temperature will flow into the area that the bubble used to be, so that the wall will be cooled instantly and a lot of heat will be exchanged. It is believed that the nucleation boiling heat transfer is directly influenced by the bubble detachment, so both of the bubble departure diameter and the bubble departure frequency are important parameters that need to be conveyed.

Compared with the boiling heat transfer at atmospheric pressure, it has been found that the boiling heat transfer is deteriorated with lower heat transfer coefficient [1] and larger temperature fluctuations [2-3] at sub-atmospheric pressure. However, only a few studies [2,4-8] on pool boiling at sub-atmospheric pressures have directly characterized bubble formation, growth and dynamic behavior, and developed the correlations for bubble dynamics. The lack of work is partly due to the complexity of the boiling environment at sub-atmospheric pressure. In sub-atmospheric boiling, bubble behavior is quite different from that at the atmospheric conditions. Large non-spherical bubbles, long waiting period, the non-homogeneous boiling environment [5] indicate that traditional models do not apply in the sub-atmospheric conditions. It’s also difficult to isolate an individual influencing factor, since the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>$K_1$</td>
<td>a parameter defined by Jensen and Memmel [20], $K_1 = (Ja/Pr)^2(AR)^{-1}$</td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>vapor density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>surface tension coefficient (N m$^{-1}$)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>the Maragoni correlation factor</td>
</tr>
<tr>
<td>$L$</td>
<td>the calibrated length of the ruler (m)</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure (Pa)</td>
</tr>
</tbody>
</table>
different phenomenon in sub-atmospheric pool boiling is a result of the combined
effects of various factors.

One of the early sub-atmospheric pool boiling experiments was conducted by
Yagov et al. [4]. The results of boiling of water, ethyl alcohol and NaCl solution
under low pressures were presented. Extremely long period in vapor formation,
millimeters-sized bubbles and severe perturbation of the liquid were observed during
the experiment. Van Stralen et al. [2] discovered the bubble growth rates of water and
organic component at sub-atmospheric pressures. They found that both the bubble
growth time and departure radius increase with decreasing pressure. “Rayleigh”
bubbles and a high-velocity liquid jet were observed under the low pressure, which
was attributed to the occurrence of dry area according to their theory. Giraud et al. [5]
pointed out that it was the non-homogeneity environment that results in the different
phenomena in the sub-atmospheric pool boiling. The instantaneous boiling curves
were presented and a particular “cyclic boiling regime” was investigated in which
wall temperature fluctuations can reach 20K. It was also highlighted that the liquid
height was an important parameter. Although some typical phenomena were presented
and analyzed, those studies remain qualitative. It is still unclear how the theoretical
correlations of bubble dynamics obtained under atmospheric condition change under
sub-atmospheric conditions. Zajaczkowski et al. [6] analyzed several correlations for
heat transfer coefficient in sub-atmospheric conditions, and found the most accurate
correlation, but no bubble dynamic parameters were involved. Non-dimensional
characteristic radius and time scale parameters were used by Kim et al. [7] to compare
the difference between bubble growth behavior at atmospheric and sub-atmospheric
pressure. However, the comparison may not be valid because the liquid height was
unknown. Michaie et al. [8] used image processing software to calculate some
quantitative parameters in sub-atmospheric boiling, but no further analysis was
presented. In atmospheric or higher pressure, bubble dynamic parameters and their
correlations with influencing factors are studied by numerous researchers. Bubble
departure diameter and bubble departure frequency are the crucial parameters
referring to bubble dynamics. The correlation proposed by Fritz [9] as shown Eq. (1)
has withstood the test of time and were modified by various researchers. The equation
was based on pure liquid and liquid mixture, where \( \theta \) is the contact angle.

\[
D_d = 0.0146\theta \sqrt{\frac{2\sigma}{g(\rho_l-\rho_v)}} \tag{1}
\]
Cole and Rohsenow [10] have modified the Fritz correlation by involving heat flux appearing in Jacob number.

\[ D_d = C\alpha^2 \left[ \frac{2\sigma g_c}{g(\rho_1 - \rho_v)} \right]^{1/2} \]  

(2)

Other mostly used correlations in previous studies are presented in Table 1. All of the correlations were obtained from Mohanty et al. [11]. Those correlations worked well in their experiments, but it is still unknown whether these correlations can predict well under sub-atmospheric boiling conditions.

**Table 1**

<table>
<thead>
<tr>
<th>Bubble departure diameter and frequency correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ D_d = 0.0208 \left[ \frac{\sigma g_c}{g(\rho_1 - \rho_v)} \right]^{1/2} \left[ 1 + 0.0025 \left( \frac{dD}{dt} \right)^2 \right] ]</td>
</tr>
<tr>
<td>[ D_d = 0.0454 \left[ \frac{2\sigma}{g(\rho_1 - \rho_v)} \right]^{1/2} ]</td>
</tr>
<tr>
<td>[ D_d = 0.25(1 + 10^5 K_1)^{1/2} \left[ \frac{\sigma g_c}{g(\rho_1 - \rho_v)} \right]^{1/2} ]</td>
</tr>
<tr>
<td>For ( K_1 &lt; 0.06 ), where ( K_1 = \left( \frac{\ln a}{Pr} \right)^2 (Ar)^{-1} )</td>
</tr>
<tr>
<td>[ D_d = 0.19(1.8 + 10^5 K_1)^{2/3} \left[ \frac{\sigma}{g(\rho_1 - \rho_v)} \right]^{1/2} ]</td>
</tr>
<tr>
<td>[ D_d = 0.1649 \alpha^{0.7} \left[ \frac{\sigma g_c}{g(\rho_1 - \rho_v)} \right]^{1/2} ]</td>
</tr>
</tbody>
</table>

Though so many researches have been done to investigate bubble departure mechanisms, study about binary mixtures and correlations to predict bubble departure diameter still arises wide interest. In this work, calcium chloride aqueous solution is used as the working fluid to improve our research on dehumidification system regeneration and also to make a thermo-physical properties comparison with distilled water under the influence of sub-atmospheric pressure. The experiment was conducted on the upper surface of a smooth stainless-steel plate heated by a controlled oil bath. High speed visualization technology was used to monitor the boiling surface, and image processing software was used to extract data. The influence of pressure, superheat, and subcooling degree were presented. Meanwhile, bubble dynamics behavior and force balance were analyzed. Since most of the empirical correlations used in the system design are based on data at atmospheric pressure, by analyzing experimental phenomena, experimental data, and by force analysis, a new correlation suitable for predicting the bubble departure diameter of water and binary mixtures under sub-atmospheric temperature is proposed.
2. Experimental

2.1 Setup

The schematic experimental apparatus of pool boiling are shown in Fig.1. The boiling vessel was constituted by two stainless steel blind plates (diameter 160mm) and a toughened glass cylinder with 90mm inner diameter and 200mm height. They were held together by four bolts and nuts. O-rings seals were placed between the two blind plates and the glass cylinders with the aim of pledging gas-tightness. The vessel was evacuated through a valve connected to a vacuum pump at the top of the upper blind plate. The vapor was condensed through another valve connected to a condenser in order to recycle the evaporated working liquid. Due to the need of frequent assembly and disassembly as well as high gas-tightness, all components of the experimental facilities were made in vacuum technology (ISO-KF).

A stainless steel ruler with the length of 150mm was placed inside the glass cylinder as the reference length to measure the liquid height and bubble size. Two K-type thermocouples were embedded in the upper blind plate and sealed by sealant. They were used to measure the vapor and the liquid temperatures. Another four K-type thermocouples were embedded into the lower blind plate close to the heating surface to evaluate the heat flux with the help of Fourier’s law. The calculation of the heat flux and its uncertainty was based on the work of Gong et al. [17]. A fast response thermocouple was placed on the upper surface to measure the instantaneous wall temperature. A pressure transducer with range of 0–20 kPa was used to measure the vapor pressure. Agilent data logger was used to record temperatures, pressures and heat flux. The uncertainties of the measurement instruments are summarized in Table 2.

The experiments were carried out on the upper surface of the lower blind plate. The surface of the stainless steel blind plate was polished with the measured roughness of 0.4μm. An artificial nucleation site with the diameter of 150μm and depth of 70μm [18] was created at the center of the blind plate. The boiling vessel was heated by a controlled oil bath containing silicone oil. During the experiment the oil temperature was set to a constant value in order to maintain a constant given temperature.

A high speed camera was placed near the glass cylinder to capture the pictures of bubbles growth, detachment and breakup. The camera highest acquisition frequency is
1000fps and the resolution is 800×600 pixels. A halogen backlight device was placed in the opposite position to illuminate the boiling area. The 150W halogen lamp can produce intense and homogeneous light to make a decent contrast for clear analysis. The images were analyzed by AOS Imaging Studio software to get the bubble departure diameters. From the captured images, a physical dimension of 90 mm corresponds to 354 pixels, leading to 0.254 mm per pixel. An image processing software was used to measure the contact angle through analyzing the images, as shown in Fig. 2. The contact angle values of water and calcium chloride solution are 35.36 degrees and 32.29 degrees, respectively. As each photo have its own time-stamp, bubble growth period and bubble waiting period can be calculated by finding the creation and detachment frame during a frame-by-frame treatment.

### 2.2 Determination of the bubble equivalent diameter

Under sub-atmospheric pool boiling conditions, the bubble shape is no longer sphere. In our experiment, a flattened spheroid shaped bubble was observed as shown in Fig. 3a, which is in agreement with Ref [4, 8]. The geometry of a flattened spheroid is shown in Fig. 3b.

As shown in Fig. 3a, the spheroid shape is almost axis-symmetric and a bit non-symmetric in the vertical direction. Therefore, neither of the equatorial diameters 3a nor 3b shown in Fig. 3b can be a good sample of the equivalent diameter. In most previous studies, researchers used the equivalent diameter of a sphere with the same volume. So the bubble equivalent diameter can be calculated as in Eq. (3),

\[
D_{eq} = 2\sqrt{ab}
\]  

### 2.3 Experimental uncertainty

As mentioned above, the geometry length of bubbles was measured by counting pixels in a captured image. The error of the measurement is of ±2 pixels at the beginning of the growth and ±1 pixels at the end of the growth and the uncertainty of the bubble diameter is ±2.6%. The error of the measurement of contact angle is ±1 degree.
Due to the vacuum environment and good insulation measures, an adiabatic condition can be obtained. According to the Fourier Law, heat flux can be calculated by Eq. (4):

\[ q_i = \lambda_i \left( T_{i+1} - T_{i} \right) / \left( x_i - x_{i-1} \right) \]  

(4)

Where \( x_i \) is the distance between corresponding measurement point and the heating surface, \( i=1,2,3,4 \), as shown in Fig. 4; \( T_i \) is the temperature, \( \lambda_i \) is the average thermal conductivity of stainless steel between the \( i \)-1 point and \( i \) point.

The average heat flux is:

\[ q = \frac{1}{3} \sum_{i=1}^{3} \left[ \lambda_{i+1} \left( T_{i+1} - T_i \right) \left( x_{i+1} - x_i \right) \right] \]  

(5)

According to the Taylor experimental error analysis formula, synthetic standard uncertainty (U) is calculated by standard uncertainty (\( u_i \)). Since all temperatures are measured by thermocouples whose precision is \( \pm 0.1 \) K, the temperature uncertainty is:

\[ u(T) = \sqrt{ \left( \frac{\partial(T)}{\partial x_1} u(x_1) \right)^2 + \left( \frac{\partial(T)}{\partial x_2} u(x_2) \right)^2 + \left( \frac{\partial(T)}{\partial x_3} u(x_3) \right)^2 } \]  

(6)

For the length measurement, the standard deviation of ten times length measurements is obtained as:

\[ u(\bar{x}) = \sqrt{ \frac{1}{n(n-1)} \sum_{i=1}^{n} (l_i - \bar{L})^2 } \]  

(7)

So the uncertainty of heat flux can be expressed as:

\[ u(q) = \sqrt{ \left( \frac{\partial f(q)}{\partial T_1} u(T_1) \right)^2 + \left( \frac{\partial f(q)}{\partial T_2} u(T_2) \right)^2 + \left( \frac{\partial f(q)}{\partial T_3} u(T_3) \right)^2 + \left( \frac{\partial f(q)}{\partial T_4} u(T_4) \right)^2 + \left( \frac{\partial f(q)}{\partial l_1} u(l_1) \right)^2 + \left( \frac{\partial f(q)}{\partial l_2} u(l_2) \right)^2 + \left( \frac{\partial f(q)}{\partial l_3} u(l_3) \right)^2 + \left( \frac{\partial f(q)}{\partial l_4} u(l_4) \right)^2 } \]  

(8)

### 2.4 Procedure and operating conditions

Since the experiments need to be implemented under sub-atmospheric pressure, a high level of gas-tightness is required. The entire system, including the condenser and the vacuum pump were cleaned and tested before the experiments. After the test solution was prepared and pipetted into the boiling vessel through a valve, the vacuum
pump was then turned on. When the pressure of the system was stabilized and the dissolved gases were released from the solution, the controlled oil bath was switched on to heat the boiling vessel allowing the saturation temperature of the solution to be reached. Meanwhile, the condenser was turned on to keep the system pressure from rising by condensing the vapor. After the system reached a steady state, the high-speed camera and the halogen lamp were turned on. The visual information as well as the experimental parameters including the surface temperature($P_{wall}$), the liquid temperature($\rho gh$) and the system pressure($P_v$) were collected and recorded in a PC-based data acquisition system.

The experiments were conducted with distilled water and calcium chloride aqueous solution. Reagents and distilled water were used to prepare the experimental solution analytically. Distilled water was used in the first set of experiment for eight values of vapor pressure $P_v$ starting from 3.6 kPa up to 22.0 kPa. The liquid height was set as a constant value of $H_l = 15.0$ cm for all sets of experiments. The controlled oil bath was set with different oil temperature in each set of experiment to discover the relationship between wall temperature and bubble parameters. Calcium chloride aqueous solution was conducted in the second set of experiments with the same liquid height. Moreover, in order to provide some basic data for future research on promoting the application of low-grade waste heat in solution dehumidification system, five values of vapor pressure $P_v$ starting from 4.2 kPa up to 20.0 kPa were adopted. The full operating conditions and selected physical properties of the experimental solution are presented in Table 3.

3. Results and discussion

3.1 Bubble growth cycle at different sub-atmospheric pressure

3.1.1 Boiling environment

Since the experiment was conducted at pressure ranges from 3.6kPa to 22kPa and the working fluids including water and calcium chloride solution, the boiling environment is non-homogeneous mainly due to the following reasons. First, the pressure generated by the liquid height may be in the same order of magnitude as the fluid saturation pressure, which means different liquid height leads to different pressure, as a result, the pressure and saturation temperature of the working fluid can
be highly non-homogeneous[2]; Secondly, the different properties of the components
in the binary system result in different boiling characteristics, which leads to
concentration gradient and temperature gradient starting from the heated surface, the
concentration gradient and temperature gradient therefore, strengthen inhomogeneity.
Generally speaking, the pressure gradient, concentration gradient and temperature
gradient leads to the special boiling environment at sub-atmospheric pressure, as well
as the non-homogeneity on bubble growth and shape.

3.1.2 Boiling phenomena analysis

In sub-atmospheric pressure, both the bubble size and shape are quite different from
those under the atmospheric conditions. The bubble life cycle of distilled water in 4.2
kPa and 22.0kPa is shown in Fig. 5a. As shown in this figure, at \( P_v = 4.2 \) kPa a
bubble nucleates in the artificial nucleation site on the boiling surface and grows with
a hemi-sphere shape during the initial 10ms. Then, the bubble is prolonged into a
centimeter-sized spheroid shape before detachment. A liquid jet was observed
penetrating the lower side of the bubble before the bubble collapses. At \( P_v = 20 \) kPa,
a bubble of the similar shape with the lower pressure was observed, but it was much
smaller in size. When the bubble is about to depart, the secondary bubble penetrated
the first one and a mushroom shape was observed. The phenomena were similar with
those observed by previous studies [2,4,5,8]. According to Van Stralen [2], the
high-velocity liquid jet in 4.2kPa is owing to the liquid depression after the primary
bubble departs and the formation of the mushroom shape is due to the dry area
beneath the center of the first bubble. The author compared this behavior to film
boiling because of the low value thermal-conductivity of the vapor.

The bubble growth curves of distilled water at different pressure are shown in Fig.
6a. Take \( P_v = 4.2 \) kPa for example, the initial 40ms can be recognized as the rapid
growth period in which the liquid inertia and surface tension force governs. From
40ms to the time that bubble departs, the thermal diffusion dominates the growing
process during which bubble grows at slow speed. Comparing the curves at different
pressure, it’s obvious that with the increase of the pressure level, both the bubble
equivalent diameter and bubble growth time decreases. According to Michaie [8], this
is because the vapor density \( \rho_v \) grows rapidly with the decrease of pressure while the
liquid density \( \rho_l \) is basically unchanged. Hence, vaporizing certain mass of liquid
require a much larger vapor volume in lower pressure than higher pressure. Surface tension force also increases with the decrease of pressure resulting in the difficulty in the bubble detachment and induces a longer bubble growth time. In general, the analysis mentioned above shows that our experimental method and procedure was verified to be appropriate.

The typical bubble shape of calcium chloride aqueous solution in different pressure is shown in Fig. 5b. Analogously, bubbles of spheroid shape were observed. In lower pressure, large bubbles and high liquid jet were also observed. Owing to the relatively high saturation temperature, there is absence of mushroom bubble in 20kPa which leads to the diminution of the dry area and liquid depression.

The boiling phenomenon of aqueous salt solution can be quite different from the boiling of distilled water based on the following reasons. First, the thermo-physical properties of the salt solution are different from distilled water. The change in saturation pressure, contact angle and surface tension force leads to different boiling characteristics. Secondly, the salt solution belongs to the binary systems, which is different from the pure liquid. During the boiling transfer, the light component in the solution near the heated surface is boiled at first. As the boiling continues, the concentration of the solution near the heated surface is increased, which leads to the change of physical properties near the heated surface.

3.2 Bubble departure diameter and frequency at different sub-atmospheric pressure

3.2.1 Bubble departure diameter and frequency of distilled water

Fig. 6b shows the relationship between bubble departure diameter and pressure of distilled water. It’s clear that the bubble departure diameter increases as pressure decreases. At the lowest pressure ($P_v = 4.2 \text{kPa}$), bubble departure diameter can reach 85mm while at higher pressure ($P_v = 20 \text{kPa}$) the bubble size is much smaller. As mentioned above, vapor density and surface tension force is the main reason behind this phenomenon. In fact, in most bubble departure diameter correlations, the bubble diameter is proportional to the number. As the pressure decreases, vapor density $\rho_v$ increases, which results in the larger size bubble at lower pressure.

The bubble waiting time and frequency at different pressure of distilled water are shown in Fig. 6c. Bubble waiting time is defined as the time interval between the first departing bubbles to the initiation of the successive bubble from the same nucleating
cavity. In the range from 3.6kPa to 22.0kPa, the bubble waiting time decreases from 8s to 0.2s. As a result, bubble departure frequency, defined as the reciprocal of the sum of the bubble waiting time and growth time, increases from 0.1Hz to 4Hz. The long waiting time in the lower pressure conditions is owing to the large size bubbles. The detachment of a bubble of that size leads to strong movement in the surrounding fluid. The liquid near the heating surface is then cooled to approximately the saturation temperature, which need a lot of time to be re-heated to reach the nucleation temperature. As for the mushroom shape (the secondary bubble penetrated the first one) appeared at 20kPa, as shown in Fig. 6e, t1 is the waiting time between the first bubble and the second one, t2 is the waiting time between the first bubble and next first bubble, and t3 is the waiting time between the mushroom and the next mushroom. Considering t1 is small and the secondary bubble is unstable because it is triggered by the jet flow, so we usually use t2 as the waiting time, as shown in Fig. 6c.

A comparison between experimental data and literature data was made, as shown in Fig. 7 [3]. Considering the difference of working condition and methods, the error of departure diameter at the same pressure is almost within 20%, which means the results of this experiment are accurate.

### 3.2.2 Comparison with bubble departure diameter and frequency of distilled water and calcium chloride solution

The comparison of the bubble departure diameter and frequency is shown in Fig. 8a. It can be found that the boiling of calcium chloride solution at sub-atmospheric pressure is particularly irregular especially in higher pressure conditions. The irregular departure diameter and frequency reflect the complexity of the boiling conditions of salt solution at sub-atmospheric pressure. In sub-atmospheric pressure, the temperature gradient, pressure gradient and the concentration gradient create a volatile and complex boiling condition. Moreover, the surface tension gradient can be formed by the concentration gradient, which leads to Marangoni flow. The liquid motions caused by preceding bubbles and the size of the previous bubble also have effect on bubble dynamic parameters. This multi-factor combined effect leads to the irregular departure diameter and frequency. Therefore, it is reasonable to compare the overall trend rather than single bubble parameters. Overall speaking, comparing water in the same pressure, the bubble departure diameter of the calcium chloride aqueous solution decreased and the departure frequency increased. Although the increase of the surface
tension force make the bubble harder to detach from the heating surface resulting in longer waiting time and larger bubble diameter. The saturation temperature of the calcium chloride aqueous solution is higher than water in the same pressure, which leads to smaller vapor density. As a result, the bubble departure diameter decreases and the bubble departure frequency increases.

Fig. 8b shows the bubble departure diameter and frequency in different concentration. As shown in Fig. 8b, the irregular distributions still occur in higher concentration. It can be concluded that with the increase of concentration, the bubble departure diameter decreases and the bubble departure frequency increases. The smaller vapor density and stronger Marangoni flow is the reason behind these changes.

3.3 The effect of wall superheat and subcooling degree at sub-atmospheric boiling

Large wall temperature fluctuation was observed during bubbles grew and detach process, which is in agreement with Ref [3, 4, 5]. Therefore, the compared wall superheat is defined in Eq. (5) as

\[ ΔT = T_w(t) - T_{sat}^* \]

\[ T_{sat}^* = 17.834\ln P_{wall} - 116.62 \]

\[ P_{wall} = P_{sat} + \rho gh \]

where \( T_w(t) \) is the wall temperature when the nucleation begins and \( T_{sat}^* \) is the saturation temperature near the surface. Fig. 9a shows the bubble growth curves in different wall superheat. As the wall superheat rises, both the bubble equivalent diameter and bubble growth time increases, owing to the thicker thermal boundary layer in higher wall superheat [19].

The typical bubble growth curve of saturated boiling and subcooled boiling is shown in Fig. 9b. As shown in the diagram, with the increase of subcooling degree, bubble size decreased which is in agreement with the experimental data of Qiu et al. [20] and the simulation results of Dhir et al. [21]. In saturated boiling, liquid has no need for absorbing heat from bubbles. While in sub-cooled boiling, as shown in Fig. 10, the vapor forming the bubbles re-condenses and release heat. With the increase of the bubble size, the surface area of condensation becomes larger, which results in the
increase of condensation rate. However, due to the limitation of the heat flux, the condensation rate is constrained as well as the bubble size. Therefore, subcooling degree is a constraint on the bubble departure size. Although Marangoni flow that occurs in sub-cooling conditions tends to hinder the departure of the bubbles, considering that the saturated bubble diameter is much larger than that under sub-cooling conditions, the bubble growth period under saturated boiling is still longer than that under sub-cooling conditions at sub-atmosphere.

3.4 A new bubble departure diameters correlation

3.4.1 Comparisons of several correlations for bubble departure diameter

Although the typical bubble shape was observed and the influence with some experimental parameters was summarized, the effects of thermal diffusion, dynamic and non-homogeneous environment in sub-atmospheric pressures still need to be determined. The experimental data was compared with different bubble growth theories and correlations to examine the relative importance of these effects.

One of the foremost bubble growth correlations were proposed by Plesset and Zwick [22] as in Eq. (10). By neglecting the surface tension force and dynamic effects, the bubble growth is only limited by the thermal diffusion process. Accordingly, the solution predicts the instantaneous bubble radius for thermal diffusion dominated growth. The thermal diffusion controlled growth solutions proposed by Plesset and Zwick [22] as well as Forster and Zuber [23] and others are in satisfactory agreement with the experimental data in atmospheric and higher pressures. However, according to Cole and Shulman [12] the heat diffusion limited theory failed to predict the situation under sub-atmospheric conditions. On the other hand, the well-known extended Rayleigh [24] equation as shown in Eq. (11) is based on an equilibrium balance from the dynamic point of view.

\[ D = 3.908 \alpha \sqrt{a t} \]  

(10)

\[ \frac{p_s(T_c) - p_s}{\rho_l} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{2\sigma}{\rho_l} \]  

(11)

As is illustrated in Fig. 11, when the pressure was about 20kPa and the Jakob number approximately equaled to 267, the theoretical curves lay above the experimental data. With the pressure being reduced to 7.3kPa and the Jakob number...
equaled to 662, the deviation of the Plesset and Zwick correlation became larger. In
the lowest pressure 4.2kPa and the highest Jacob number 1786, the experimental data
in the early growing stage coincided with the Rayleigh curves, while the Plesset and
Zwick correlation didn’t predict well in this situation. These comparisons indicate that
the early growth stage of bubble growth process under sub-atmospheric pressures is a
significant inertia controlled region.

3.4.2 The force balance of the bubble under sub-atmospheric pressure

In order to discover the dynamic behavior and obtain a useful bubble growth model
under sub-atmospheric pressure, it’s important to identify which of the acting forces
are dominant. The force balance of the isolated bubble is presented in this chapter to
further analyze the influences of each force under sub-atmospheric pressure. As is
mentioned above, the flattened spheroid shape is the typical shape of an isolated
bubble under sub-atmospheric boiling conditions. The geometric parameters and force
analysis of an isolated bubble is shown in Fig. 12a.

Generally speaking, buoyancy force $F_b$ and pressure force $F_p$ are the positive
forces are those tend to push the vapor bubble on to the heating surface. While the
drag force $F_d$, surface tension force $F_s$, the Marangoni force $F_M$ and inertial force $F_i$
are the negative forces that prevent bubbles from departing from the heating surface.
As a result, the expression of force balance is presented in Eq.(12):

$$F_b + F_p = F_M + F_s + F_i + F_d$$  \hspace{1cm} (12)

The expressions of each force are:

1. Buoyancy force $F_b$ is caused by the movement of the fluid due the non-uniform
density.

$$F_b = \frac{4}{3} \pi R_{eq}^3 g (\rho_l - \rho_v)$$  \hspace{1cm} (13)

2. Pressure force $F_p$ is the contact capillary pressure force caused by the surface
tension in the solid-liquid interface.

$$F_p = \pi R_c^2 \frac{2\sigma}{R_{eq}}$$  \hspace{1cm} (14)
3. The inertial force $F_i$ is deduced by the velocity of the bubble and the volume expansion.

$$F_i = \frac{d}{dt}(m_v)$$

(15)

4. The Maragoni force $F_M$ is caused by surface tension gradient in the vapor-liquid interface. In atmospheric conditions the surface tension gradient is mainly caused by temperature gradient. However, in sub-atmospheric conditions the pressure gradient should not be neglected.

$$F_M = \xi_t \frac{d\sigma}{dt} \Delta T + \xi_p \frac{d\sigma}{dt} \Delta p$$

(16)

5. The surface tension force $F_s$ acts on the vapor-liquid interface is shown in Eq.15

$$F_s = 2\pi R_c \sigma \sin \theta$$

(17)

The relationship between the forces acting on the isolated bubble and the bubble radius was presented in Fig. 12b and Fig 12c. As can be seen from the curves, in sub-atmospheric boiling conditions, the buoyancy force $F_b$ and inertial force $F_i$ are obviously the dominant forces. Due to the low vapor density in lower pressure, the large volume of bubbles leads to the exponential growth of the buoyancy and inertial force compared to the situation in atmospheric pressures. On the other hand, the influence of the other three forces is relatively small and can be neglected in the sub-atmospheric pressure. The curves also concurs with the comparison result in the previous chapter that the inertial forces are the major factor that hinders the bubble detachment.

The results also indicate that the bubble departure diameter correlations such as Fritz correlations based on the static force balance of the buoyancy and surface tension force may not be valid under sub-atmospheric pressure. Considering the significant inertia controlled region, an attempt should be made to find a new correlation in the sub-atmospheric pool boiling conditions.

### 3.4.3 New correlation

As is mentioned above, in sub-atmospheric pressures, the initial force is the dominant negative force. As a result, the force balance equation can be written below,
\[
g(\rho_l - \rho_v)V(t_g) = \frac{d}{dt_g}(m,v) \quad (18)
\]
where \( V(t_g) \) is the volume of the bubble.

Assume that the equivalent bubble diameter \( D_{eq} \propto t_g^n \), \( \rho_v \ll \rho_l \) and \( v = \frac{dh}{dt_g} \), the Eq.18 can be rewritten as:

\[
\frac{d}{dt_g}\left(t_g^{3n}\frac{dh}{dt_g}\right) = gt_g^{3n} \quad (19)
\]

Therefore, the \( h(t_g) \) can be written as:

\[
h(t_g) = \frac{gt_g^2}{6n+2} \quad (20)
\]

At the moment of a bubble’s detachment, as is shown in Fig. 13, the bubble is a flattened spheroid shaped and the \( h(\tau_g) \) can be written in this form:

\[
h(\tau_g) = b \quad (21)
\]

By analyzing the experimental data, it can be found that the ratio of minor axis and the major axis is approximately a constant value \( b/a = 0.3 \). Therefore, Eq.(21) can be rewritten as:

\[
h(\tau_g) = \frac{\sqrt{30}}{10} R_{eq} \quad (22)
\]

Kim et al[11] pointed out that the bubble radius is proportional to the power of 2/3-1/2 of growth time. According to our experimental data, the parameter \( n \) is equal to 1/2. So the bubble departure diameter is:

\[
D_d = \frac{2\sqrt{30}}{15} gt_g^2 \quad (23)
\]

### 3.4.4 Comparison of bubble departure diameters correlations with experimental data

The six most commonly used bubble departure diameters correlations were chosen to compare with the experimental data. As defined in Eq. 24 and 25, the average deviation (AD) and the absolute average deviation (AAD) are used to evaluate those correlations. The results are presented in Table 4 and the comparisons between the experimental and predicted bubble diameters are shown in Fig. 14.
As is presented in Table 4, Fritz [9] and Cole and Shulman [12] didn’t predict well in sub-atmospheric pressure, with deviations more than 95% in all of the conditions. However, due to lack of Jacob number in their correlations, the effect of the system pressure is neglected. As a result, their correlations are limited in sub-atmospheric conditions. Cole [13] modified Fritz correlation by taking a constant number 0.04 instead of the contact angle and involving the Jacob number. The data show the trends and work reasonably in 20kPa, but in lower pressure large deviations are observed.

By integrating three dimensionless numbers $Ja$, $Pr$ and $Ar$ into $K_1$, the correlations proposed by Kuyayeladze and Gogonin [14] and Jensen and Memmel [15] were quite similar. The correlation developed by Jensen and Memmel [15] showed big deviations in all of the conditions except for 12.3kPa. Kuyayeladze and Gogonin [14] agreed well within ±30% at three conditions (20kPa water, 7.2kPa water and 7.2kPa CaCl₂ solution). However, in lower pressure (5.6kPa), their correlation was not applicable because the dimensionless number $K_1$ got out of range ($K_1 > 0.06$). The correlations proposed by Kim and Kim [16] showed the best results with deviations with ±30% in all of the conditions. Dimensional analysis involving the characteristic time scale and characteristic bubble radius scale were used in their studies.

The comparison between the experimental and Eq.19 is illustrated in Fig. 15a. As can be seen from the picture, the new correlation predicted well in all of the experimental conditions. The predicted departure diameters of the new correlation were within ±20% deviation, which showed great improvement to predict in sub-atmospheric pool boiling conditions.

**3.4.5 Comparison of new bubble departure diameter correlation with different literature data**
In this section, a comparison between literature data [2][25][26] and prediction data that calculated by new correlation was made. Stralen and Cole [2] took water as working fluid, the working pressure ranges from 4.08kPa to 20.28kPa. While Stralen, Sluyter and Cole [25] took water, water-ethanol and water-1-butanol as working fluid, the pressure ranges from 3.6kPa to 6.6kPa. The working fluid of Cole and Shulman’s [26] experiment was water, the pressure was between 6.7kPa and 13kPa. As shown in Fig. 15b, the new correlation predicts well in most of the experimental conditions, especially for water(4.08kPa)[2,26] and binary mixtures (3.6kPa-6.6kPa)[26].

4. Conclusions

Nucleate pool boiling experiments of water and calcium chloride solution were carried out at sub-atmospheric pressure in a range of 3.6kPa to 22.0kPa. A high-speed camera was used to capture the bubble images. The bubble dynamic parameters were measured and calculated, the bubble growth curves in different boiling conditions were obtained. The main conclusions are listed as follows.

For distilled water, due to the low vapor density and high surface tension force, the bubble diameter tends to increase as pressure decreases. Additionally, as the wall superheat rises, the thermal boundary layer become thicker and results in larger bubble departure diameters. In sub-cooling conditions, the condensation effect and the Marangoni flow hinder the bubble growth, resulting in a smaller bubble size. The experiments of water verified the methods and procedure successfully, and provide basic data for the new correlation.

For calcium chloride solution, a complex boiling regime with irregular bubble dynamic parameters was observed. Comparing to water under the same pressure, the effect of smaller vapor density outweighs that of the surface tension force, which results in the decrease in departure frequency and increase in departure diameter. Moreover, with the increase of concentration, the bubble departure diameter tends to decrease and frequency tends to increase respectively owing to the smaller vapor density and stronger Marangoni flow.

Finally, based on bubble force balance analysis, the relationship of buoyancy force, pressure force, the inertial force, the Marangoni force and the surface tension force was analyzed. Considering sub-atmospheric pressure environment, the inertia force is the dominant negative force, so after simplification of force balance equation, a new
correlation was proposed. The developed correlation can predict the whole data of distilled water and calcium chloride solution within ±20% deviation. For the literature data, the developed correlation has a mean deviation of 17.2% and 6.3% for water and binary mixtures, respectively.

Acknowledgments

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Reference


Figure 1. Schematic of the experimental apparatuses. (A) Boiling Vessel; (B) Condenser; (C) Power Supply; (D) Valve; (E) Vacuum Pressure Transducer; (F) Vacuum Pump; (G) Agilent Data Logger; (H) Controlled Oil Bath; (I) High Speed Camera; (J) Halogen Backlight Device; (K) Stainless Steel Ruler

Fig. 2. Processed bubble images of distilled water(a) and calcium chloride aqueous solution(b)

\[ P_v = 20\text{kPa}, H_i = 15\text{cm}, T_i = 60^\circ\text{C} \]
Figure 3. A flattened spheroid shaped bubble (a) and the geometry of a flattened spheroid (b).

Fig. 4. Corresponding measurement points schematic diagram.
Figure 5. Typical Bubble shape under different pressure (a) (distilled Water, $H_1 = 15\text{cm}, \Delta T = 20\text{K}$), (b) (calcium chloride aqueous solution, $H_1 = 15\text{cm}, \Delta T = 20\text{K}$, wt = 30%)
**Graph a:**
- Bubble equivalent diameter $D_{eq}$ (mm) vs. time $t$ (ms)
- Data points for different wall pressures: 4.2 kPa (black square), 5.6 kPa (red circle), 7.4 kPa (blue triangle), 20 kPa (green triangle).

**Graph b:**
- Bubble departure diameter vs. wall pressure ($P_{wall}$) (kPa)
- Data points for different pressures.
(Distilled Water, $H_1 = 15\text{cm}, \Delta T = 40\text{K}, \Delta T_{\text{sub}} = 0\text{K})$

Figure 6. Bubble growth curves(a), bubble departure diameter(b), bubble waiting time(c) and departure frequency(d) at different pressure and bubble waiting time of mushroom shape bubble at 20kPa(e).

Figure 7. Experimental bubble departure diameter in comparison with literature data[2]
Figure 8. The comparison between distilled water and calcium chloride aqueous solution of bubble departure diameter and frequency in different pressure (A: Average value) 

$H_l = 15\text{cm}$, $\Delta T = 20\text{K}$, $\text{wt} = 30\%$ (a) and the bubble departure diameter and frequency in different concentration (A: Average value) (calcium chloride aqueous solution, $H_l = 15\text{cm}$, $\Delta T = 20\text{K}$) (b)
Figure 9. The bubble growth curves under different values of superheat temperature (Distilled Water, $P = 5.6\text{kPa}, H = 15\text{cm}$) \text{(a)} and the bubble growth curves in saturated boiling and subcooled boiling (Distilled Water, $P = 20\text{kPa}, H = 15\text{cm}, \Delta T = 30\text{K}$) \text{(b)}
Figure 10. The subcooled boiling mechanism
Figure 11. Comparison of the growth rate correlations with experimental data

(Distilled Water, $h_i = 15$cm)
Figure 12. The geometric parameters and force acting on an isolated bubble (a) and the relationship between the forces acting on the isolated bubble and the bubble radius 4.2Kpa (b)

101 kPa (c)

Figure 13. The growth and departure behavior of an isolated bubble under subatmospheric pressure
Predicted Data Dd (mm) vs Experimental Data Dd (mm)

**a. 20kPa Water**

- Fritz [23]
- Cole [18]
- Kuyayeladeze and Gogonin [19]
- Cole and Shulman [17]
- Kim and Kim [21]
- Jensen and Memmel [20]

**b. 12.3kPa Water**

- Fritz [23]
- Cole [18]
- Kuyayeladeze and Gogonin [19]
- Cole and Shulman [17]
- Kim and Kim [21]
- Jensen and Memmel [20]
Figure 14. Comparisons between the experimental and predicted bubble diameters with different correlations in different operating conditions.
Figure 15. Comparisons between the experimental data and the new bubble departure diameter correlation in different operating conditions (a) and Comparisons of literature data and new bubble departure diameter correlation prediction data (b)
Table 1

Bubble departure diameter and frequency correlations

\[ D_d = 0.0208 \left[ \frac{\sigma \rho_c}{g(\rho_l - \rho_v)} \right]^{1/2} \left[ 1 + 0.0025 \left( \frac{dD_d}{dt} \right)^{3/2} \right] \]
Cole and Shulman [12]

\[ D_d = 0.04J_a \left[ \frac{2\sigma}{g(\rho_l - \rho_v)} \right]^{1/2} \]
Cole [13]

\[ D_d = 0.25(1 + 10^5 K_1)^{1/2} \left[ \frac{\sigma \rho_c}{g(\rho_l - \rho_v)} \right]^{1/2} \]
Kutateladeze and Gogonon [14]

For \( K_1 < 0.06 \), where \( K_1 = \left( \frac{1 + 10^5 T}{\rho_l T} \right) (\text{Ar})^{-1} \)

\[ D_d = 0.19(1.8 + 10^5 K_1)^{2/3} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2} \]
Jensen and Memmel [15]

\[ D_d = 0.1649 J_a^{0.7} \left[ \frac{\sigma \rho_c}{g(\rho_l - \rho_v)} \right]^{1/2} \]
Kim and Kim [16]

---

Table 2

Uncertainties of the measurement instruments and calculation

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<th>Instrument</th>
<th>Uncertainty</th>
</tr>
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### Table 3
Operating parameter and physical properties

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<table>
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### Table 4
The statistical analysis of the bubble departure diameter at subatmospheric pressures

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