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An alternative Z-score measure for downside bank insolvency risk

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Abstract

We derive a Z-score measure reflecting downside bank insolvency risk, drawing on a Chebyshev inequality in terms of the lower semivariance. As then illustrated empirically for US banks, this may provide a useful alternative, or robustness check, to the more commonly used Z-score measure based on the standard Chebyshev inequality.
Keywords: bank; insolvency risk; Z-score; downside risk; semivariance
JEL classification: G21; G28

1. Introduction

A widely used risk measure in the empirical banking literature to reflect a bank's probability of insolvency is the Z-score; it is both relatively simple and can be calculated entirely from accounting information.[1] The Z-score draws on the well-known Chebyshev inequality, and is generally attributed to Boyd and Graham (1986), Hannan and Hanweck (1988) and Boyd et al. (1993).

In particular, defining bank insolvency as a state where $(EQ + ROA) \leq 0$, with EQ the bank's capital-asset ratio and ROA its return on assets, Hannan and Hanweck (1988) and Boyd et al. (1993) showed that if ROA is a random variable with finite mean μ_{ROA} and variance σ^2_{ROA}, the standard Chebyshev inequality[2] suggests an upper bound of the probability of insolvency P as

$$P(ROA \leq -EQ) \leq Z^{-2} \equiv P, \tag{1}$$

where the Z-score is defined as

$$Z \equiv (EQ + \mu_{ROA})/\sigma_{ROA} > 0. \tag{2}$$

Our paper presents a related approach to the derivation of an alternative Z-score measure to reflect a bank's probability of insolvency, which might be better at capturing downside risk. To this end, instead of drawing on the standard Chebyshev inequality, we exploit the lesser-known Chebyshev inequality in terms of the lower semivariance (see Berck and Hihn 1982). An empirical illustration using quarterly call report data for US banks demonstrates the potentially complementary information contained in this "downside risk" Z-score. As a consequence, this alternative measure may prove of practical use to applied researchers in the areas of banking and financial stability, at a minimum as a straightforward robustness check for results obtained with the traditional measure, and also potentially serve financial regulators and market participants more generally.

Section 2 now introduces our alternative "downside risk" Z-score measure; Section 3 empirically illustrates how it compares to the traditional measure when taken to US data; and Section 4 concludes the paper.

[1] For some recent papers using this methodology, see e.g. Pino and Sharma (2019), Caiazza et al. (2018), Berger et al. (2016), Doumpos et al. (2015), Vazquez and Federico (2015), Hakenes et al. (2014), Berger et al. (2014), Delis et al. (2014), Fang et al. (2014), Fu et al. (2014).

[2] The standard Chebyshev inequality states that for a random variable X with finite mean μ and variance σ^2, it holds for any $k > 0$ that $P\{|X - \mu| \geq k\} \leq \sigma^2/k^2$ (see e.g. Ross, 1997, p. 396).

2. Bank insolvency and downside risk: an alternative Z-score measure

The importance of downside risk when return distributions are skewed has been emphasized as early as Roy (1952); in our context, potential down movements in bank income and/or capital matter more in relation to bank distress than potential up movements. The lower semivariance σ^{2-}, which is defined as $\sigma^{2-} = (n-1)^{-1}\sum_n (\min(x-\mu_x, 0))^2$, could arguably be better at capturing this asymmetric impact than the common variance, which obscures it.

We can allow for this by considering an alternative "downside risk" Z-score, given as $Z_d \equiv (EQ + \mu_{ROA})/\sigma^-_{ROA}$. Defining bank insolvency as a state where $(EQ + ROA) \leq 0$, with EQ the bank's capital-asset ratio and ROA its return on assets. we can show that Z_d relates to a corresponding probability of insolvency as follows

Proposition. *If* ROA *is a random variable with finite mean* μ_{ROA} *and lower semivariance* $\sigma^{2-}_{\mathrm{ROA}}$*, where* $\sigma^{2-}_{\mathrm{ROA}} = (n-1)^{-1}\sum_n (\min(\mathrm{ROA} - \mu_{\mathrm{ROA}}, 0))^2$*, an upper bound of the bank's probability of insolvency* P *is given by*

$$\mathrm{P}(\mathrm{ROA} \leq -\mathrm{EQ}) \leq (Z_d)^{-2} \equiv P_d, \tag{3}$$

where the Z-score measuring downside insolvency risk is defined as

$$Z_d \equiv (\mathrm{EQ} + \mu_{\mathrm{ROA}})/\sigma^-_{\mathrm{ROA}} > 0. \tag{4}$$

Proof. This is an application of the Chebyshev inequality in terms of the lower semivariance (see Berck and Hihn 1982): it states that for a random variable X with finite mean μ and lower semivariance σ^{2-}, it holds for any $m > 0$ that $\Pr\{X \leq \mu - m\sigma^-\} \leq \frac{1}{m^2}$. Setting $X = ROA$ and $m = (EQ + \mu_{ROA})/\sigma^-_{ROA}$, we obtain our result.

Corollary. *The "downside risk" Z-score* Z_d *developed in the Proposition relates to bounds on the corresponding probability of insolvency that are tighter, by a factor of* $\sigma^{2-}_{\mathrm{ROA}}/\sigma^2_{\mathrm{ROA}}$*, than those given for the traditional Z-score* Z *by the standard Chebyshev inequality.*

Proof. This follows straightforwardly from the fact that $Z_d = \sigma_{ROA} Z/\sigma^-_{ROA}$ and $\sigma^2_{ROA} = \sigma^{2-} + \sigma^{2+}$, with σ^{2+} the corresponding upper semivariance, as shown in Berck and Hihn (1982).

This relevant corollary is, of course, not entirely surprising as the Chebyshev inequality in terms of the lower semivariance incorporates additional information about the degree of asymmetry of the underlying distribution of ROA which is not captured by the more general Chebyshev inequality.[3]

3. Empirical illustration

We now briefly illustrate how the Z-score measures defined by Equations (2) and (4) compare when taken to the data. To this end, we examine a dataset based on quarterly call reports for US banks, extracted from WRDS. We clean for obvious outliers/erroneous data, and calculate time-varying Z and Z_d measures[4] using moving mean and standard deviation/lower semi-standard deviation estimates for ROA (with window width eight quarters), combined with current period values of EQ.[5] In order to capture both pre-crisis and crisis periods, we then retain all banks for which we are able to construct a complete set of time-varying Z and Z_d measures for the period 2001q1-2010q4 (resulting in data for 5305 banks in total).

Table 1 presents descriptive statistics for the differences in means and standard deviations of Z and Z_d measures, calculated per bank, for the full sample 2001q1-2010q4, as well as the pre-crisis/crisis samples 2001q1-2005q4 and 2006q1-2010q4. Analogous statistics are computed for the differences in means and standard deviations of the corresponding probability bounds P and P_d, given by Equations (1) and (3), as well as for the correlation coefficients between Z-score measures Z and Z_d and the corresponding probability bounds P and P_d, respectively.

We observe that while Z_d measures are on average larger than Z ones, across all three samples, the differences in corresponding probability bounds P and P_d are on average fairly small. This notwithstanding, the maximum differences in corresponding probability bounds P and P_d can be substantial, and even more so in the separate pre-crisis/crisis samples. Analogous relationships hold for the variability of Z-score measures Z and Z_d and their corresponding probability bounds P and P_d, similarly across all three samples. We further note that correlations between Z-score measures Z and Z_d, as well as their corresponding probability bounds P and P_d, are very high on average, and slightly more so in the crisis period than the pre-crisis one. Interestingly, the minimum correlations between Z-score measures Z and Z_d, as well as their corresponding probability bounds P and P_d, are significantly lower in the

[3] See also the Appendix for an analogous comparison of corresponding probability bounds when using an alternative interpretation of the traditional Z-score Z that draws on the one-sided Chebyshev inequality instead (see Lepetit and Strobel 2015).

[4] Stata code for the calculation of these measures is available from the authors on request.

[5] For a discussion of different approaches to the construction of time-varying Z-score measures see Lepetit and Strobel (2013).

separate pre-crisis/crisis samples than for the full sample.

Figures 1 and 2 show scatter plots of Z vs Z_d measures and the corresponding probability bounds P vs P_d, respectively; these illustrate the close but nevertheless nontrivial relationship between these alternative measures (computed averaged per bank and for the full sample). In addition, Figures 3 and 4 present scatter plots of pre-crisis vs crisis correlations between Z-score measures Z and Z_d, as well as their corresponding probability bounds P and P_d. Both displays further demonstrate the potentially complementary information contained in the "downside risk" Z-score Z_d developed in this paper, suggesting it as a potential alternative to the more commonly used Z-score measure Z, serving at a minimum as a straightforward robustness check for results obtained with the traditional measure.

4. Conclusion

We derived a Z-score measure reflecting downside bank insolvency risk, drawing on a Chebyshev inequality in terms of the lower semivariance. An empirical illustration using quarterly call report data for US banks suggests that this may provide a useful alternative to the more commonly used Z-score measure based on the standard Chebyshev inequality. It may therefore be of practical use to applied researchers in the domains of banking and financial stability, at a minimum as a straightforward robustness check for results obtained with the traditional measure, but also potentially serve financial regulators and market participants more generally.

Appendix

An alternative interpretation of the traditional bank insolvency risk measure Z, drawing on the one-sided Chebyshev inequality, was more recently given by Lepetit and Strobel (2015): if the bank's return on assets ROA is a random variable with finite mean μ_{ROA} and variance σ^2_{ROA}, the one-sided Chebyshev inequality[6] suggests a refined upper bound of the probability of insolvency as

$$P(ROA \leq -EQ) \leq \left(1 + Z^2\right)^{-1} \equiv P_r,$$

[6] The one-sided Chebyshev inequality states that for a random variable X with finite mean μ and variance σ^2, it holds for any $a > 0$ that $P\{X \leq \mu - a\} \leq \frac{\sigma^2}{\sigma^2+a^2}$ (see Ross, 1997, p. 414, or previously, Feller, 1971, p. 152).

Table 1: Comparison of Z and Z_d measures, and their corresponding probability bounds P and P_d, for sample of US banks (per bank, N=5305)

Full Sample: 2001q1-2010q4

	mean	sd	min	max
diff_mean_z	-19.9437	12.6109	-111.8239	-0.9297
diff_mean_p	0.0027	0.0067	0.0000	0.1172
diff_sd_z	-12.3363	9.2669	-91.0827	1.5368
diff_sd_p	0.0038	0.0105	-0.0085	0.1531
corr_z	0.9691	0.0283	0.5900	0.9998
corr_p	0.9641	0.0577	0.3346	1.0000

Pre-crisis sample: 2001q1-2005q4

	mean	sd	min	max
diff_mean1_z	-21.2099	15.0786	-151.0849	-0.8699
diff_mean1_p	0.0017	0.0075	0.0000	0.2269
diff_sd1_z	-10.0271	9.5263	-101.5699	2.8820
diff_sd1_p	0.0016	0.0070	-0.0084	0.1820
corr1_z	0.9550	0.0622	0.1024	0.9999
corr1_p	0.9449	0.0807	0.0327	1.0000

Crisis sample: 2006q1-2010q4

	mean	sd	min	max
diff_mean2_z	-18.6775	13.6711	-165.7303	-0.6457
diff_mean2_p	0.0037	0.0101	0.0000	0.2025
diff_sd2_z	-11.1279	9.8793	-119.6034	6.4833
diff_sd2_p	0.0039	0.0118	-0.1212	0.1737
corr2_z	0.9655	0.0526	-0.0269	1.0000
corr2_p	0.9598	0.0717	0.2168	1.0000

with EQ the bank's capital-asset ratio and the Z-score Z again defined as $Z \equiv (EQ + \mu_{ROA}) / \sigma_{ROA} > 0$.

In the context of our "downside risk" Z-score Z_d, it is then straightforward to show that

$$P_d < P_r \text{ for } \sigma^{-}_{ROA} / \sigma_{ROA} < \sqrt{Z^2 / (1 + Z^2)},$$

with $P_d \geq P_r$ otherwise; this would suggest $\min(P_d, P_r)$ as a best bound on the probability of insolvency overall.

As illustrated in Figure 5, which plots the resulting best bounds for values $\kappa = \sigma^{-}_{ROA} / \sigma_{ROA} \in \{0.3, 0.5, 0.7\}$, apart from for very low Z-score levels, the "downside risk" Z-score Z_d developed above relates to bounds on the corresponding probability of insolvency that are generally, and potentially substantially, tighter than those given for the traditional Z-score Z by the one-sided Chebyshev inequality. Again, this is a natural consequence of the fact that it incorporates additional information about the degree of asymmetry of the underlying distribution of ROA.

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Figure 1: Scatter plot of Z vs Z_d measures (average per bank, full sample)

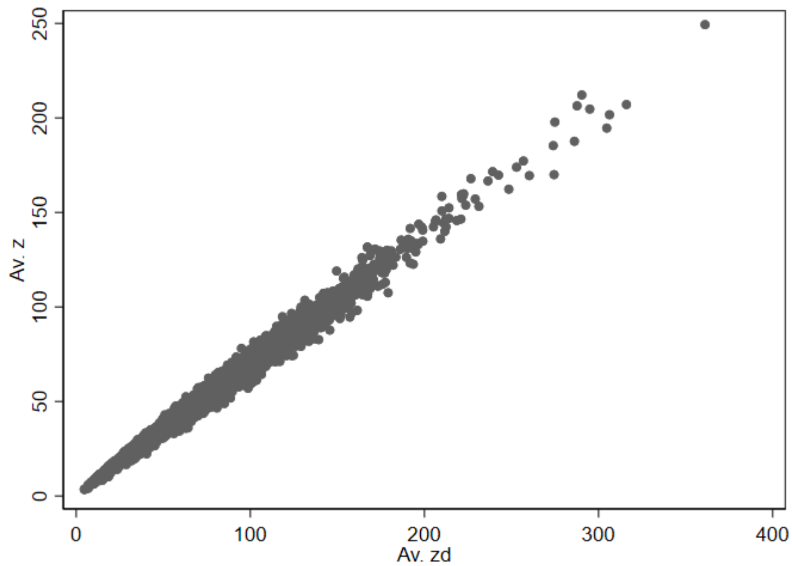

Figure 2: Scatter plot of corresponding probability bounds P vs P_d (average per bank, full sample)

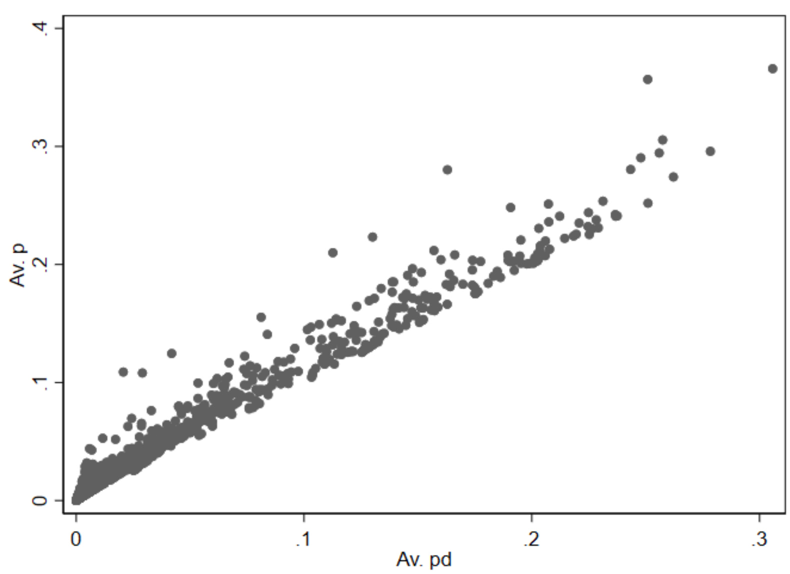

Figure 3: Scatter plot of correlations between Z and Z_d measures (per bank, pre-crisis vs crisis sample)

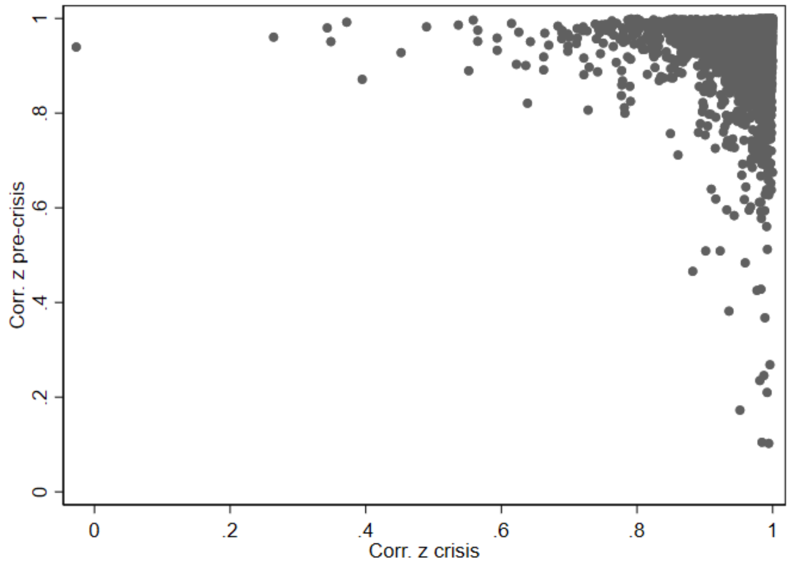

Figure 4: Scatter plot of correlations between corresponding probability bounds P and P_d (per bank, pre-crisis vs crisis sample)

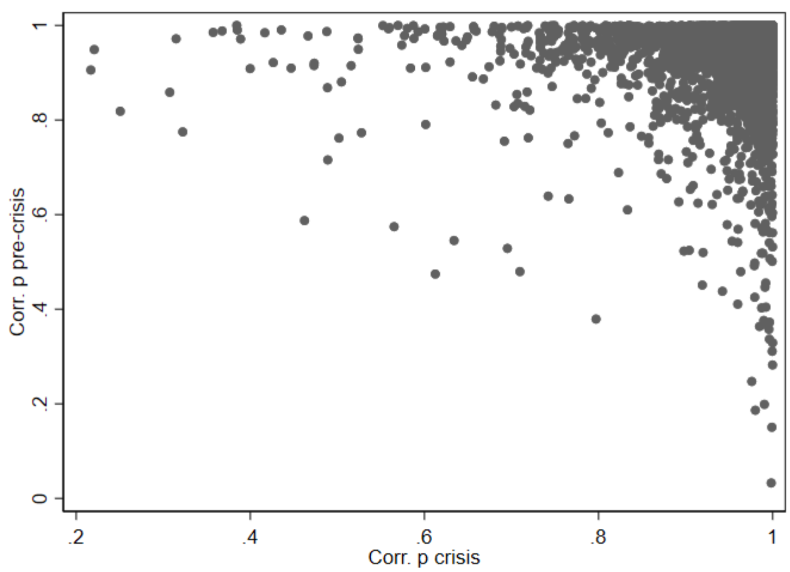

Figure 5: Best probability bounds for $\kappa = \sigma^{-}_{ROA}/\sigma_{ROA} \in \{0.3, 0.5, 0.7\}$, given $Z_d = Z/\kappa$