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Abstract

In this paper a novel approach based on evolutionary polynomial regression (EPR) is presented to develop constitutive models of materials. Stress-strain data are used to train EPR and develop EPR-based material models. It is shown that it is possible to construct the material stiffness matrix (Jacobian) using partial derivatives of the developed EPR models. The EPR-based Jacobian matrix is implemented in finite element (FE) model and two boundary value problems are used to verify the proposed EPR-based FE methodology. It was shown that the EPR-FE can successfully be employed to analyse structural problems with both linear and non-linear material behaviour.

Keywords

Constitutive model; evolutionary computing; finite element method; stiffness matrix; EPR

1. Introduction

Constitutive models are relationships between two or more physical quantities that represent different aspects of material behaviour and predict the response of that material to applied loads, displacements, etc. Several constitutive models have been developed to predict the behaviour of different materials. Among these models there are simple elastic models (Hooke's law) or elasto-plastic models (e.g., combination of Tresca or von Mises yield criterion with hardening) which are usually used to describe the behaviour of metals [1]. Another example is soil where various models have been developed to predict its behaviour such as elasto-plastic models (e.g., [2]) or models based on critical state theory [3]. In conventional constitutive modelling of materials, an appropriate mathematical model is initially selected and the parameters of the model (material parameters) are identified from appropriate physical tests on representative samples to capture the material behaviour. When these constitutive models are used in finite element analysis, the accuracy with which the selected material model represents the various aspects of the actual material behaviour and also the accuracy of the identified material parameters affect the accuracy of the finite element predictions.

In recent years, by rapid and effective developments in computational software and hardware, alternative computer aided pattern recognition approaches have been introduced to constitutive modelling of materials. The main idea behind pattern recognition systems such as neural network, fuzzy logic or genetic programming is that they learn adaptively from experience and extract various discriminants, each appropriate for its purpose. Artificial neural networks (ANNs) are the most widely used pattern recognition procedures that have been employed for constitutive modelling of materials. Recently other data mining techniques such as evolutionary methods have been utilised to modelling the behaviour of materials.

In this paper a new approach based on evolutionary algorithms is presented to develop material constitutive models. Furthermore, the implementation of the developed material models in finite element (FE) method is described and different examples are provided to illustrate the potential of the proposed approach in analysing boundary value problems in engineering.

2. Historical Background

The application of ANN for constitutive modelling was first proposed by Ghaboussi et al. [4] for modelling the concrete behaviour. The use of ANN was continued by Ellis et al. [5] and Ghaboussi et al. [6], who applied this technique to constitutive modelling of geomaterials. These works indicated that neural network constitutive models (NNCM) can capture nonlinear material behaviour and complex interaction between variables of the system without having to assume the form of the relationship between input and output variables.

The incorporation of NNCM in finite element analysis has been discussed by a number of researchers. Shin and Pande [7] presented an approach to compute the tangential stiffness matrix of the material using partial derivatives of the NNCM. The potentials of the presented approach were examined by analysing a rock specimen under uniaxial cylindrical compression with fixed ends. Shin and Pande [8] used the self-learning FE code (the finite element code with embedded NNCM) to identify elastic constants of an orthotropic materials from a structural test. They proposed a two-step methodology in which in the first step, the monitored data from a structure test are used to train a NN based constitutive model implemented in a finite element code. In the second step, the first order partial derivatives of developed NNCM are used to form the stiffness matrix and compute the material elastic parameters. Hashash et al. [9] described some of the issues related to the numerical implementation of a NNCM in finite element analysis. They derived a formula to compute consistent Jacobian matrix (stiffness matrix) for NN material models. For validation, the derived formula was implemented in the commercial finite element code "ABAQUS" through its user defined material subroutine (UMAT) to analyse a number of numerical examples including analyses of a beam bending and a deep excavation problem. UMAT and VUMAT are both user subroutines to implement non-standard constitutive equations in ABAQUS. UMAT is used to define material models in ABAQUS/Standard while VUMAT is used for ABAQUS/Explicit. UMAT must update the stresses and solution-dependant variables to their values at the end of the increment for which it is called. It also must provide the material Jacobian matrix for the mechanical constitutive model. On the other hand the material Jacobian matrix is not required in VUMAT [10]. Furukawa and Hoffman [11] proposed an approach to material modelling using neural networks which could describe monotonic and cyclic plastic deformation and its implementation in a FE model. They developed two neural networks, each of which was used separately to learn the back stress and the drag stress. The back stress represented kinematic hardening, and the drag stress represented isotropic hardening. After training and validation stages of NNs, the NNCM was implemented in a commercial FE package, MARC, using its user subroutine for material models. The implementation involved the determination of the stiffness matrix which was given by the sum of the elastic stiffness matrix and the plastic stiffness matrix. The elastic matrix was derived from Young’s modulus and Poisson’s ratio and only the plastic matrix was updated using developed neural networks. Kessler et al. [12] showed the incorporation of a neural network (NN) material model in ABAQUS, through its user subroutine VUMAT.

They developed a NN model for 6061 Aluminium under compression and different temperatures. The developed NN model was incorporated in ABAQUS via VUMAT to carry out analysis and the results were compared to the two other conventional build-in models of ABAQUS (power law model and tabular data). The results revealed that the NN-based finite element model can provide a better prediction in comparison to the other two. Haj-Ali and Kim [13] presented a neural network constitutive model for fibre reinforced polymeric (FRP) composites. Four different combinations of NN models were considered in this study. Data for training the NN models were gathered from off-axis compression and tension tests performed with coupons cut from a monolithic composite plate manufactured by pultrusion process. The results of predictions provided by NN models were compared to the experimental data and good agreement was observed. Furthermore a notched composite plate with an open hole was tested and the results were used to examine its FE model using the developed NNs model. For this purpose the developed NNs were implemented in ABAQUS material user subroutine. The results of finite element model were compared to the experimental results in an unknown point where the response of structure was linear. It was shown that the model was capable of predicting the general linear behaviour of the composite at this point but showed a little diversion as strain increased. No comparison was made between the results of the FE model with NN constitutive model and the experimental data around the hole, where the behaviour was more nonlinear.

The presented approach in this paper overcomes the drawbacks of NNCMs. The proposed EPR approach combines numerical and symbolic regression to find concise mathematical expressions that describe the system being study. Polynomial structures of equations produced by EPR allow users to take advantage of the mathematical properties of the developed models. In what follows a brief description of EPR is presented. It will also be shown that how EPR is employed to construct material models. Moreover implementation of the developed EPR models in FE is presented.

3. Evolutionary Polynomial Regression

Evolutionary polynomial regression (EPR) is a data-driven technique based on evolutionary algorithms, designed to search for mathematical expressions in form of polynomial structures representing a system. EPR is a two-step technique in which in the first step it searches for symbolic structures using an ad hoc but simple genetic algorithm (GA) and in the second step EPR estimates constant values by solving a linear Least Square (LS) problem. A typical formulation of the EPR expression is given as [14]:

$$y = \sum_{j=1}^{m} F(\mathbf{X}, f(\mathbf{X}), a_j) + a_0 \quad (1)$$

where y is the estimated output of the system; a_j is a constant value; F is a function constructed by the process; $\mathbf{X}$ is the matrix of input variables; f is a function defined by the user; and m is the number of terms of the expression excluding bias a_0. The general functional structure represented by $F(\mathbf{X}, f(\mathbf{X}), a_j)$ is constructed from elementary functions by EPR using a GA strategy. The GA is employed to select the useful input vectors from $\mathbf{X}$ to be combined together. The building blocks (elements), of the structure of F, are defined by the user based on understanding of the physical process. While the selection of feasible structures to be combined is done through an evolutionary process; the parameters a_j are estimated by the least square method.

3.1 EPR Objective Functions

In order to get the best symbolic model(s) of the system being studied, EPR is provided with different objective functions to optimise. EPR introduces a set of multidimensional strategies for model selection, based on a comprehensive analysis of complexity (including number of terms, number of inputs) and fitness of models. EPR can work both in single as well as multi-objective configurations. In single-objective EPR, an objective function is used to control the fitness of the models without allowing unnecessary complexities enter in the models. In the case of multi-objective strategy two or three objective functions are introduced in which one of them will control the fitness of the models, while at least one objective function controls the complexity of the models. The multi-objective strategy returns a trade-off surface (or line) of complexity vs. fitness which allows the user to achieve a lot of purposes of the modelling approach to the phenomenon studied [15]. Detailed explanation of the EPR can be found in [14-16].

4. EPR-based Material Modelling

In material modelling using EPR, the raw experimental or in-situ test data are directly used for training the EPR model. In this approach as the EPR learns the constitutive relationships directly from the raw data, it is the shortest route from experimental data to numerical modelling. In addition in this approach there are no material parameters to be identified and as more data become available, the material model can be improved by re-training of the EPR using additional data.

The source of data, the adopted training approach and the way the trained EPR model is to be used have significant effects on the choice of input and output parameters. An EPR model formulated in the form of total stress-strain relationships might be used for modelling of materials that are not strongly path-dependent. A similar approach has been employed by some researchers such as Ghaboussi et al. [17], and Shin and Pande [8] for training neural network based material models. In this approach in 2D problems strain variables (i.e. $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \gamma_{12}$) which represent the strain components in a 2D continuum, can be considered as inputs, and the corresponding stress variables ($\sigma_{11}, \sigma_{22}, \sigma_{33}, \tau_{12}$) as outputs. Due to the nature of EPR which represents the model as a mathematical equation; for each of the output parameters an equation needs to be developed. This is in contrast with artificial neural network (ANN) where you can have more than one parameter in output.

Data from material tests can be used to train EPR models. Usually a single test on a sample of a material provides a set of stress-strain relationships for a single stress path. However generally all the material tests that involve loading along the principal axes results in the shear components (shear strains, shear stresses) being zero. As a result, an EPR model trained in this way would not be suitable to be incorporated in a finite element (FE) analysis, since all the components of the stress and strain tensors must be taken into account during analysis. Therefore to obtain an EPR model with the potential to be incorporated in FE framework, the EPR-based model should be trained along global axes with non-zero shear components. To overcome this issue, a strategy is employed here to generate additional data from an ordinary material test. This strategy can be applied when the material being studied is isotropic or isotropy can be assumed. This strategy was first proposed by [18]. The strategy is described here for 2D problems but it can be easily extended to 3D. In this strategy, the additional data will be generated in two steps. In the first step it is assumed that the material is isotropic. The assumption of isotropy enables us to exchange the normal components, thereby to double up the data. Then transformation of each of the exchanged stress-strain pairs is carried out by rotating the datum axes (X-Y) from the original axes (1-2) where the material tests have been carried out. By rotating the axes, non-zero shear stresses and strains and their corresponding normal components can be obtained as a function of the rotation angle θ. The number of expanded stress-strain relationships is subject to the number of different rotation steps. This strategy can result in a large amount of training data (depends on the size of the original data and the number of rotational steps) which mean additional training time will be required. To avoid any unnecessary training run any duplicated stress-strain pairs in the expanded data must be removed. Depending on the stress path used in the test, one of the normal components can be zero or some normal components can have same value. In this case some parts of the data become duplicated and have to be eliminated.

5. EPR-based Jacobian Matrix

Any material model that is intended to be incorporated in finite element method should provide material stiffness matrix, also called Jacobian matrix, and can be defined as the following equation.

$$\mathbf{J} = \frac{\partial(d\boldsymbol{\sigma})}{\partial(d\boldsymbol{\varepsilon})} \qquad (2)$$

where, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the vectors of stresses and strains respectively. This matrix is defined explicitly for different material models. For instance the stiffness matrix for a linear elastic material model obeying the Hook's law in a plane stress geometrical condition is defined as follow [19]:

$$\mathbf{D} = \frac{E}{(1-\nu)^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix} \qquad (3)$$

where E represents elastic modulus and ν is the Poisson's ratio.

Hashash et al. [9] recommended to use consistent Jacobian (Equation 4) and proposed a method to estimate partial derivation of NNCM to create the Jacobian matrix.

$$\mathbf{J} = \frac{\partial \Delta \boldsymbol{\sigma}^{\mathbf{i+1}}}{\partial \Delta \boldsymbol{\varepsilon}^{\mathbf{i+1}}} \qquad (4)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the vectors of stresses and strains respectively and i+1 denotes the next state of stresses and strains. Clearly this formulation of Jacobian matrix needs the constitutive model to be constructed in an incremental form.

On the other hand Shin and Pande [7] and Shin and Pande [8] used direct derivatives of NNCM (Equation 5) and proposed a procedure to calculate the first order partial derivatives of NNCM.

$$\mathbf{D}_{\mathrm{NN}} = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}} \qquad (5)$$

In this paper direct derivative of EPR-based material models is used to construct the Jacobian matrix for materials.

6. Implementation of EPR material models in FEM

The developed EPR-based material models can be implemented in a widely used general-purpose finite element code ABAQUS through its user defined material module (UMAT). UMAT updates the stresses and provide the material Jacobian matrix for every increment in every integration point [10].

The material Jacobian matrix can be derived using the developed EPR models using the following equation:

$$J_{EPR} = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\varepsilon}} \quad (6)$$

Equation 6 can be applied in both elastic and inelastic regions because an EPR constitutive model (EPRCM) does not require the definition of a transition between elastic and inelastic regions (i.e. yield points). The Jacobian matrix resulting from the EPRCM can be directly incorporated in a conventional FE code instead of the conventional elasto-plastic constitutive matrix. The way, in which EPRCM is incorporated in finite elements, is shown in Figure 1.

7. Numerical Examples

To illustrate the developed computational methodology described in the previous sections, two examples of application of the developed EPR-based finite element method to boundary value problems are presented. In the first example, the application of the methodology to a case of linear elastic behaviour is examined. In the second example, the method is applied to a material with non-linear behaviour.

7.1 Example 1

A hypothetical test is conducted in this example in order to generate the required data to train EPR. The data is obtained from a finite element simulation of this hypothetical test. Figure 2 shows a sample of a material being tested under a tensile load T along axis 2 together with its deformed shape. The test is carried out under plane stress conditions. The original shape of the sample is drawn in dashed line. The size of the sample is 10 cm ×5 cm. The sample is made of a linear elastic material with a Young's modulus of $E = 500$ Pa and a Poisson's ratio of $\nu = 0.3$. Although the sample is only loaded along the axis 2, the deformations are measured along both axes 1 and 2 (stress in direction 1 is zero). The sample is loaded up to 20 Pa. The stress-strain data along axis 2 obtained from this test together with the strains measured in axis 1 are employed to extend the data using the strategy described in section 4. The stresses and strains in directions 1 and 2 obtained from the hypothetical test (which contain zero shear components) were exchanged assuming that the sample in the test is isotropic. This doubled the data; however the shear components of this data are still zero. For that reason, transformation of each stress-strain pair was carried out by an angular step $\Delta\theta = 3°$. This allowed the generation of all the possible combinations of stresses and strains with non-zero shear components.

Since the model studied here represents a two dimensional plane stress case, only three components of stresses and three components of strains (out of plane strains are also zero) exist in the model. These are $(\sigma_{11}, \sigma_{22}, \tau_{12})$ for stresses and $(\varepsilon_{11}, \varepsilon_{22}, \gamma_{12})$ for strains. Three EPR models were developed each corresponding to one of the stress components. In other word the inputs and outputs of the three models were:

Model 1 $\quad input: \varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$ $\quad output: \sigma_{11}$

Model 2 $\quad input: \varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$ $\quad output: \sigma_{22}$

Model 3 $\quad input: \varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$ $\quad output: \tau_{12}$

The target is to find a constitutive relationship in the general form of equation 1, where the matrix of inputs, $\mathbf{X}$, for each model is:

$$\mathbf{X} = \begin{bmatrix} \varepsilon_{11}^1 & \varepsilon_{22}^1 & \gamma_{12}^1 \\ \varepsilon_{11}^2 & \varepsilon_{22}^2 & \gamma_{12}^2 \\ \varepsilon_{11}^3 & \varepsilon_{22}^3 & \gamma_{12}^3 \\ \cdots & \cdots & \cdots \\ \varepsilon_{11}^i & \varepsilon_{22}^i & \gamma_{12}^i \end{bmatrix} \tag{7}$$

where superscript i represents the i^{th} row of the data. It should be noted that unlike ANN-based constitutive models [9, 20], in EPR the values of inputs and outputs do not need any normalisation or calibration before or after training and therefore these values can be used as they are. Before training the EPR, the data were randomly shuffled in order to make sure that the obtained models have no bias on a particular part of the data.

The database was divided into two independent sets. One set was used for training to obtain the models and the other one for validation to verify the performance of the obtained constitutive models. Although some researchers have studied the extrapolation capabilities of models developed by EPR [21, 22]; however like any other data mining technique, EPR does not demonstrate a good performance for data beyond the training range (i.e. extrapolation). It was therefore decided to choose the verification data in the range of the training data to avoid extrapolation as much as possible. Usually around 80% of data is used for training the model and the other 20% is used for validation.

Before starting the EPR process, some of the EPR parameters must be adjusted to control the obtained models. These parameters can control the optimisation techniques (i.e. single-objective or multi-objective), number of terms of the mathematical expressions, range of exponents, EPR structures and the type of the functions used to construct the EPR models. For this example the range of exponents were limited to [0 1], number of terms to 8, and “No Function” option was chosen. Also multi-objective strategy was used to develop the EPR functions. Applying the EPR procedure, the evolutionary constitutive material modelling starts from a constant mean of output value. By increasing the number of evolutions it gradually picks up the different participating parameters in order to form equations representing the constitutive relationship. Each proposed model is trained using the training data and tested using the testing data. The level of accuracy at each stage is evaluated based on the coefficient of determination (CoD). If the CoD for a model is not acceptable or the other termination criteria (e.g., maximum number of generations, maximum number of terms) are not satisfied, the current model should go through another evolution in order to obtain a new model. The results of EPR including the obtained equations, and CoD values for training and validation sets are presented in the equations 8-15 and Table 1.

Model 1

$$\sigma_{11} = 5.081 \tag{8}$$

$$\sigma_{11} = 425.0854\varepsilon_{11} + 2.0669 \tag{9}$$

$$\sigma_{11} = 549.4505\varepsilon_{11} + 164.8352\varepsilon_{22} - 5.1338 \times 10^{-12} \tag{10}$$

Model 2

$$\sigma_{22} = 5.119 \tag{11}$$

$$\sigma_{22} = 425.5103\varepsilon_{22} + 2.0598 \tag{12}$$

$$\sigma_{22} = 164.8352\varepsilon_{11} + 549.4505\varepsilon_{22} - 1.2134 \times 10^{-11} \tag{13}$$

Model 3

$$\tau_{12} = -0.0009978 \quad (14)$$

$$\tau_{12} = 192.3077\gamma_{12} + 3.2319 \times 10^{-12} \quad (15)$$

It can be seen from the obtained equations and Table 1 that for each model an equation with 100% accuracy (i.e. CoD = 100%) is achieved. It is also seen that despite the fact that we have fed EPR with three inputs (i.e. $\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$) in all three models, EPR has only picked up the inputs that have greater effects on the models. This is more interesting when we compare them with the equations that we get from classic theory of elasticity.

Equations (16), (17), and (18) describe the relationship between the strains and stresses for an elastic material [23].

$$\varepsilon_{11} = \frac{1}{E}\left[\sigma_{11} - \nu\left(\sigma_y + \sigma_{33}\right)\right] \quad (16)$$

$$\varepsilon_{22} = \frac{1}{E}[\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})] \quad (17)$$

$$\gamma_{12} = \frac{1}{G}\tau_{12} \quad (18)$$

In these equations E represents elastic modulus, ν is the Poisson's ratio and G is shear modulus which is related to elastic modulus and Poisson's ratio through following equation [23]:

$$G = \frac{E}{2(1+\nu)} \quad (19)$$

If we substitute the values of E and ν from the hypothetical test in equations (16) to (19) and re-arrange them, then the following equations are obtained:

$$\sigma_{11} = 549.45\varepsilon_{11} + 164.83\varepsilon_{22} \tag{20}$$

$$\sigma_{22} = 164.83\varepsilon_{11} + 549.45\varepsilon_{22} \tag{21}$$

$$\tau_{12} = 192.31\gamma_{12} \tag{22}$$

These equations are in an excellent agreement with those obtained from EPR (i.e. equations 10, 13, and 15 respectively). This shows that the EPR models have captured the relationships between stresses and strains with a superior accuracy.

The developed EPR models are used in FE analysis of a plane stress plate with a circular hole in its centre. Figure 3 shows the geometry, boundary conditions and loading of the plate. Due to the symmetry in the geometry of the plate only a quarter of the plate is modelled and therefore appropriate boundary conditions are provided on the bottom and left sides of the model. The model is made of 100 isoparametric 8-node elements and is stretched along Y direction. Using equations (6) and (10, (13), (15) the EPR-based material Jacobian matrix for this example is computed and presented in the following equation.

$$\mathbf{J}_{\mathrm{EPR}} = \begin{bmatrix} \frac{\partial \sigma_{11}}{\partial \varepsilon_{11}} & \frac{\partial \sigma_{11}}{\partial \varepsilon_{22}} & \frac{\partial \sigma_{11}}{\partial \gamma_{12}} \\ \frac{\partial \sigma_{22}}{\partial \varepsilon_{11}} & \frac{\partial \sigma_{22}}{\partial \varepsilon_{22}} & \frac{\partial \sigma_{22}}{\partial \gamma_{12}} \\ \frac{\partial \tau_{12}}{\partial \varepsilon_{11}} & \frac{\partial \tau_{12}}{\partial \varepsilon_{22}} & \frac{\partial \tau_{12}}{\partial \gamma_{12}} \end{bmatrix} = \begin{bmatrix} 549.45 & 164.83 & 0.00 \\ 164.83 & 549.45 & 0.00 \\ 0.00 & 0.00 & 192.31 \end{bmatrix} \tag{23}$$

On the other hand the conventional stiffness matrix for an isotropic, elastic material for plane stress case in terms of Young's modulus and Poisson ratio is given in equation 3. If we substitute the values of $E = 500$ Pa and $\nu = 0.3$ into equation 3 then we get the following matrix:

$$\mathbf{D} = \begin{bmatrix} 549.45 & 164.83 & 0 \\ 164.83 & 549.45 & 0 \\ 0 & 0 & 192.31 \end{bmatrix} \tag{24}$$

By comparing the equation (23) and (24) it can be seen that the Jacobian matrix obtained from EPR models is in an excellent agreement with the conventional elastic plane stress stiffness matrix. The EPR Jacobian matrix is implemented in UMAT and the above structure is analysed under the given loading and boundary condition. The problem is also analysed using the elastic material model provided by ABAQUS which requires elastic modulus and Poisson's ratio. The vertical displacement of the crown of the hole (node 44) versus the applied tension is compared between standard finite element analysis and EPR-based finite element method and the results are depicted in Figure 4. An excellent agreement can be seen from this figure between the displacement of node 44 using standard FE and EPR-based FE analyses. This shows that the EPR-based material Jacobian matrix is successfully implemented in the finite element analysis and the methodology can be used to predict the behaviour of a linear elastic material.

7.2 Example 2

A sample of a material with a non-linear behaviour is utilised in this example to perform another hypothetical test in finite element. The test is conducted in plane stress condition and it is assumed that the testing sample is made of an isotropic material. The data gathered from this numerical simulation is expanded using the same strategy as described before and the data fed into EPR for training. In this example the range of exponents were limited to [0 1 2], number of terms to 10, “No Function” option was chosen, and multi-objective strategy was used to develop the EPR functions. A set of EPR equations was obtained (10 equations for each model) after the training and testing procedure is finished. For each model the complexity of the equations is growing as the CoD is increasing. Each equation belongs to an evolutionary step of EPR. A summary of CoD values for training and testing of each equation for all three models is presented in Table 2.

In order to choose the best EPR model representing the material behaviour, the performance of EPR models from each evolutionary step is examined by its incorporation in a finite element model of the same hypothetical test. Using the evolved equations, the Jacobian matrix corresponds to each evolutionary step (10 in total) is constructed. Then the calculated Jacobian matrix for each evolutionary step is implemented in finite element to analyse the model of the material test. After analysis, the stress-strain curve of each model is recorded to compare the stress-strain behaviour of the different evolutionary steps provided by EPR with the results of simulated test. These stress-strain curves are presented in Figure 5. Since the EPR model corresponds to the first evolutionary step was consisted of constant values, its Jacobian matrix becomes zero and therefore it is not possible to perform an analysis for the 1st step. From figure 5 it can be seen that as the EPR steps increase, the predictions provided by EPR models become closer to the test data. Once the EPR model from 8th step is incorporated in the FE, it is seen that the results are in a very good agreement with the test data; this is also valid for the 9th step. However it can be seen that the 10th model (red dashed line) has provided less accurate results in compared to the 8th and 9th steps. One reason for this could be the over-fitting problem in the EPR equations of 10th model. From the results of this example it can be concluded that the best EPR model representing the material being tested is Equation 2-8. This model will be used to analyse a boundary value problem of a plate under biaxial tension.

A 2D plane stress panel subjected to an in-plane biaxial tension is set up to evaluate the potential of the proposed EPR-based FE method. The model of the panel with applied load and its surrounding boundary conditions is shown in Figure 6. It is assumed that the plate is made of the same non-linear material as tested above. The FE analysis of the panel is first carried out using a standard FE model with 270 isoparametric elements. For standard FE analysis elasto-plastic material model with Young's modulus and Poisson's ratio for elastic region and tabulated stress-strain data for plastic region are used. For the EPR-FEM, Equation 8-2 is used to form the Jacobian matrix and analysis is carried out under the identical loading and boundary conditions to the standard FE. The results of the two different analyses are compared in terms of maximum stresses and maximum strains (Figures 7 and 8). The results reveal that the EPR-FEM provides a very close (less than 2% difference for max principal stress and less than 0.7% for max principal strain) prediction to those of the standard FEM.

8. Conclusions

In this paper a novel approach for material modelling using evolutionary polynomial regression (EPR) is presented. EPR is a hybrid data mining technique in which it searches for symbolic structures using genetic algorithm and estimates constant values by least square. Stress-strain data from synthetic experiments are employed to train EPR and develop EPR-based material models. A methodology was introduced to incorporate the developed EPR models in FE. It was shown that it is possible to construct the material stiffness matrix (also known as Jacobian) using partial derivatives of the developed EPR models. The EPR-based

Jacobian matrix was implemented in FE model and a number of boundary value problems were used to verify the methodology. The results of the EPR-FEM are compared with standard FE where conventional constitutive models are used to model the behaviour of materials. These results showed that EPR-FEM can successfully be employed to analyse different structural engineering problems.

The main benefit of using EPR-based material models are that it provides a unified approach to constitutive modelling of all materials (i.e., all aspects of material behaviour can be implemented within a unified environment of an EPR model); it does not require any arbitrary choice of the constitutive (mathematical) models. In addition in EPR-based material models there are no material parameters to be identified and the model is trained directly from experimental data. EPR is capable of learning the material behaviour directly from raw experimental data; therefore, EPR-based material models are the shortest route from experimental research (data) to numerical modelling. Another advantage of EPR based constitutive model is that as more experimental data become available, the quality of the EPR prediction can be improved by learning from the additional data, and therefore, the EPR model can become more effective and robust.

A trained EPR-based model can be incorporated in a FE code in the same way as a conventional constitutive model. It can be incorporated either as incremental or total stress-strain strategies. An EPR-based FE method can be used for solving boundary value problems in the same way as a conventional FE. However, the incorporation of an EPR-based constitutive model in FE procedure avoids the need for complex yielding/plastic potential/failure functions, flow rules, etc.; there is no need to check yielding, to compute the gradients of the plastic potential curve or to update the yield surface.

It should be noted that, for practical problems, the data used for training EPR, should cover the range of stresses and strains that are likely to be encountered in practice. This is due to the fact that EPR models are good at interpolation but not so good at extrapolation. Therefore, any attempt to use EPR-based FE method for loading conditions that may lead to stresses or strains outside the range of the stresses and strains used in training of the EPR may lead to unacceptable errors. The proposed EPR-FEM methodology is generic and it is not limited to any particular material or boundary value problem.

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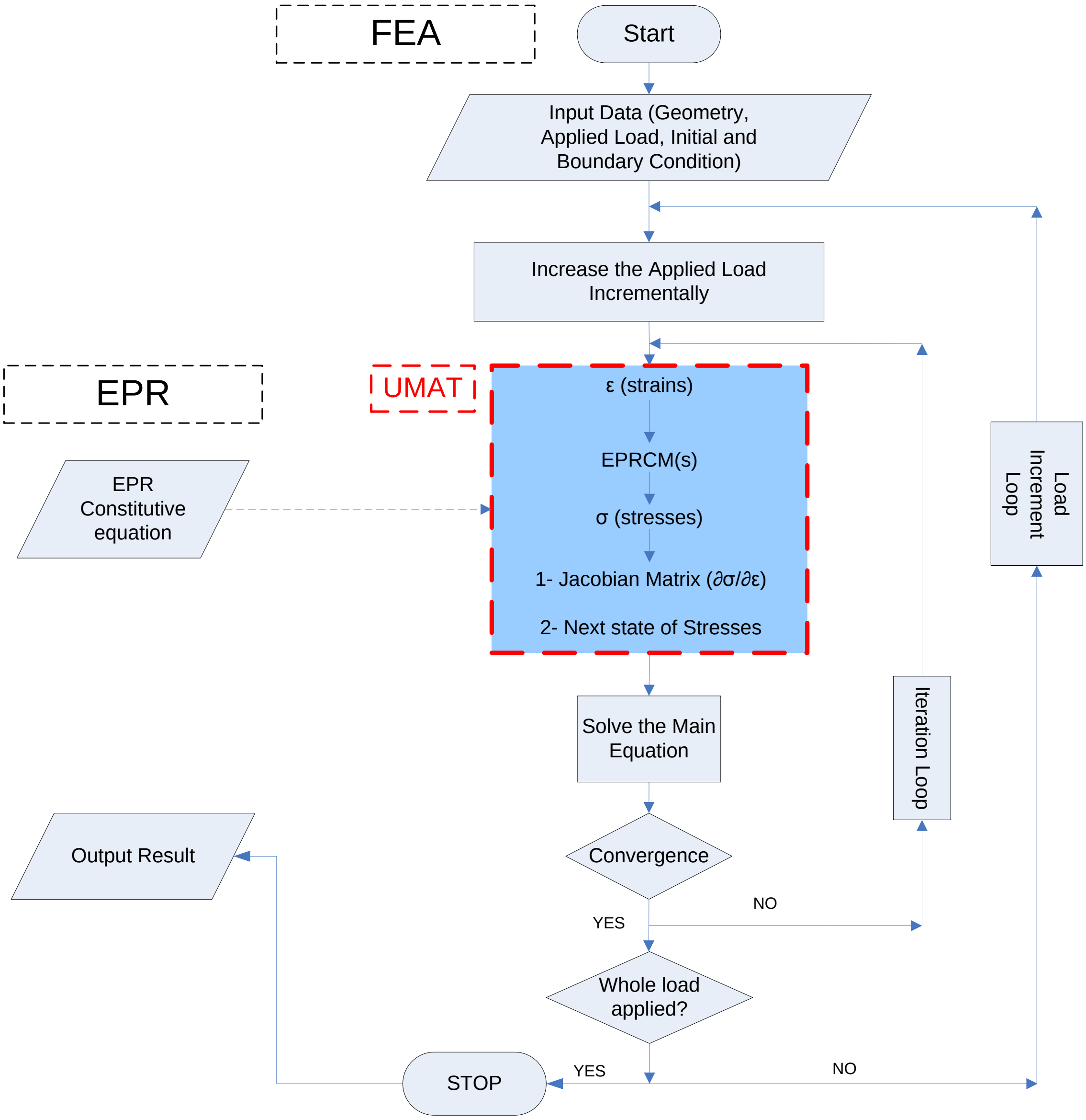


Figure 1: The incorporation of EPRCM in ABAQUS finite element software

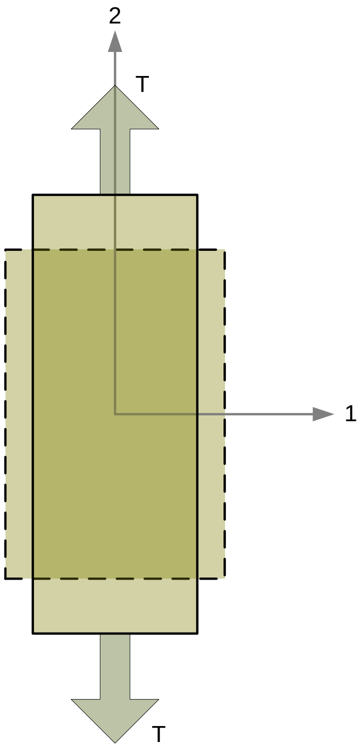


Figure 2: A plane stress sample of a material under tension along axis 2

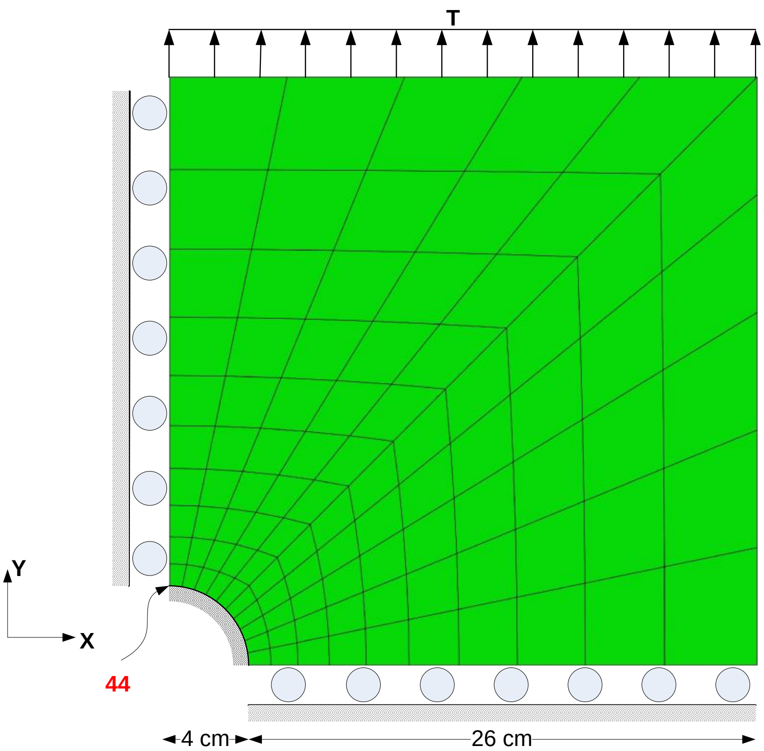


Figure 3: A plate with circular hole at its centre under tension loading along Y direction

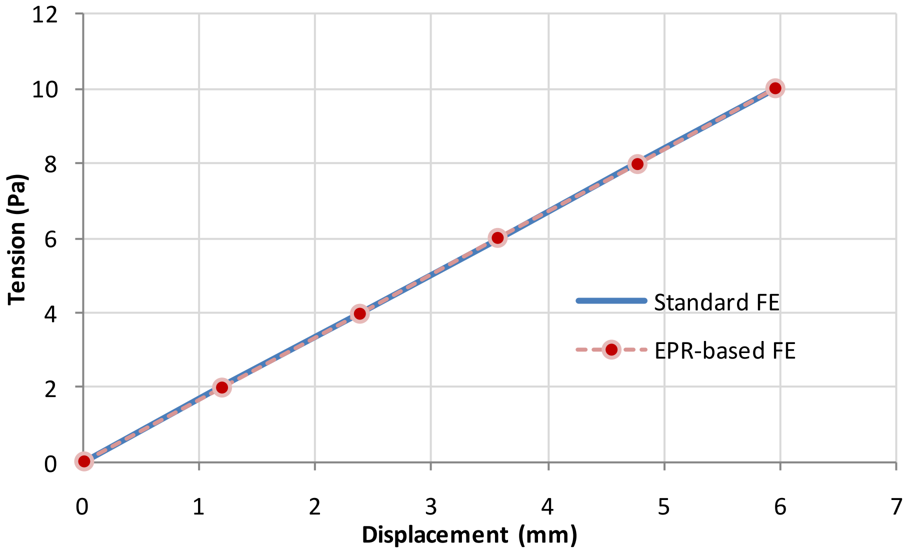


Figure 4: Tension-displacement curve of node 44 for Standard FE and EPR-based FE

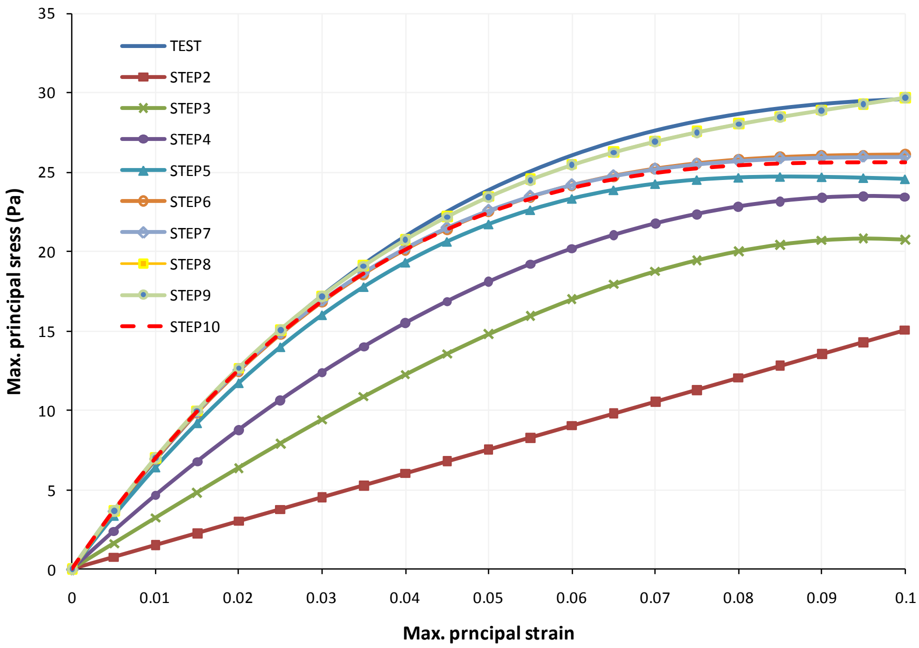


Figure 5: Stress-strain curve, obtained from different evolutionary steps

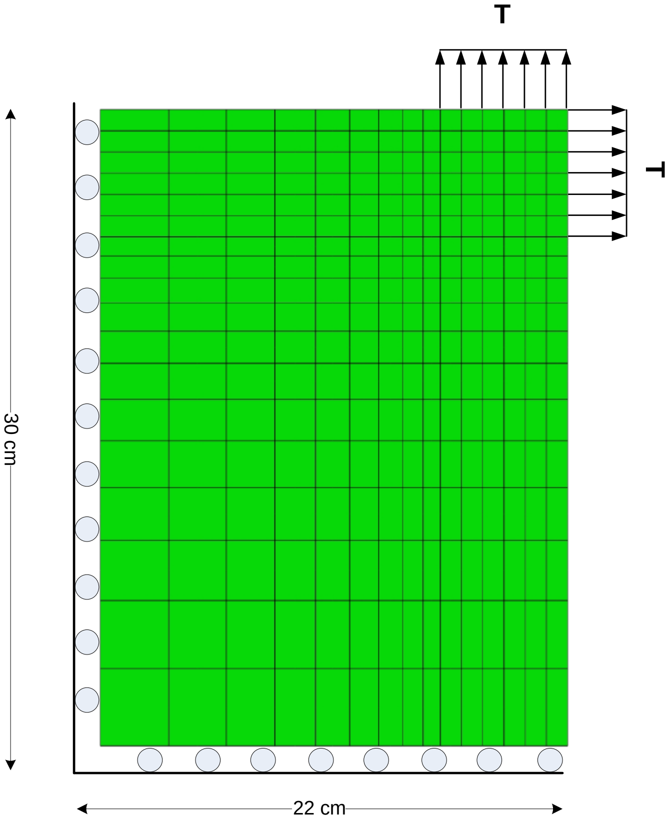
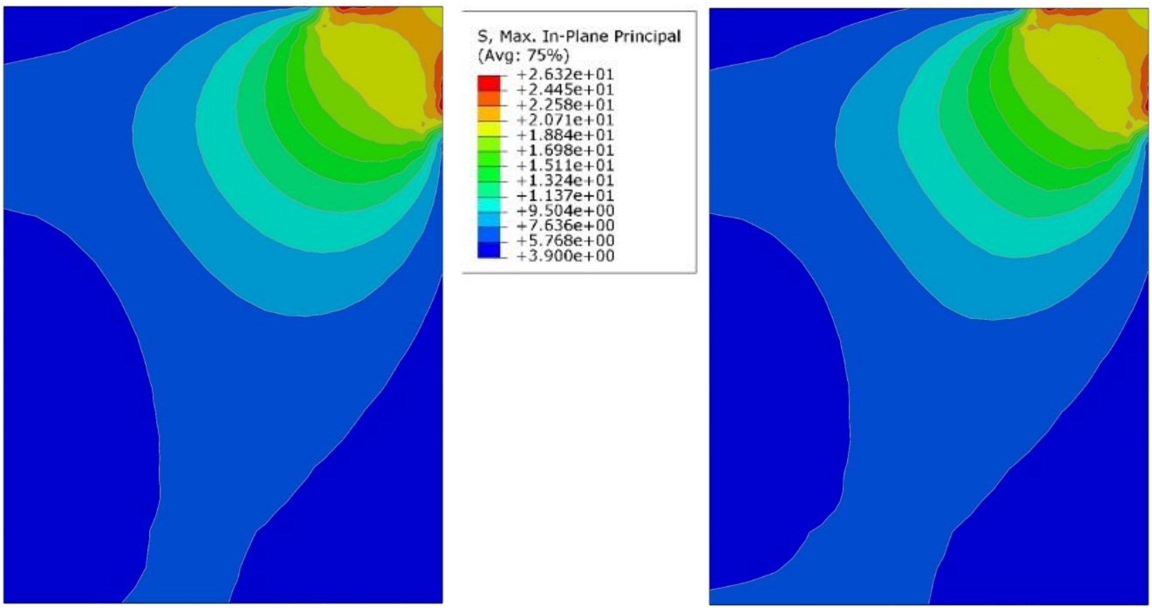
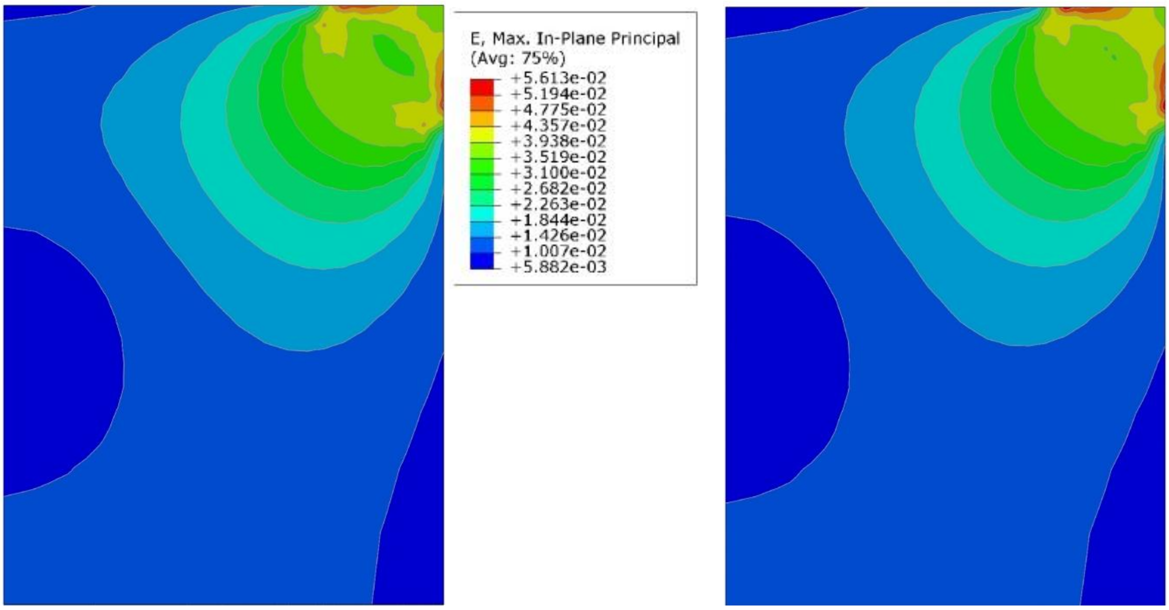


Figure 6: The FE mesh and geometry of plane stress panel under biaxial tension loading



(a) (b)

Figure 7: Contours of max principal stress (a) standard FE (b) EPR-FEM



(a) (b)

Figure 8: Contours of max principal strains (a) standard FE (b) EPR-FEM

Table 1: Summary of results obtained for EPR based models for material with linear behaviour

Equation	Model No.	CoD for training (%)	CoD for validation (%)
(8)	Model 1	0.07	5.3
(9)		93.87	86.96
(10)		100	100
(11)	Model 2	0.07	6.74
(12)		93.90	77.97
(13)		100	100
(14)	Model 3	0.07	-
(15)		100	100

Table 2: CoD values of training and validation data set for all equations developed for all three models

Eq. No	CoD (%)					
	Model 1		Model 2		Model 3	
	Training (%)	Testing (%)	Training (%)	Testing (%)	Training (%)	Testing (%)
2-1	0.07	5.97	0.07	7.37	0.07	-
2-2	74.96	72.77	74.96	72.34	91.76	0.97
2-3	94.14	93.15	94.12	94.43	99.05	81.99
2-4	97.56	97.16	97.54	97.56	99.76	96.72
2-5	99.25	99.44	99.25	99.54	100.00	99.96
2-6	99.96	99.97	99.96	99.98	99.99	99.98
2-7	99.99	99.99	99.99	100.00	100.00	100.00
2-8	99.99	100.00	99.99	100.00	100.00	100.00
2-9	100.00	100.00	100.00	100.00	100.00	100.00
2-10	100.00	100.00	100.00	100.00	100.00	100.00