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Innovative Applications of O.R.

Elementary modelling and behavioural analysis for emergency evacuations using social media

John Fry[a,*], Jane M. Binner[b]

[a] Sheffield University Management School, Conduit Road, Sheffield S10 1FL, UK
[b] Birmingham Business School, University House, Edgbaston Park Road, Birmingham B15 2TY, UK

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ABSTRACT

Social media usage in evacuations and emergency management represents a rapidly expanding field of study. Our paper thus provides quantitative insight into a serious practical problem. Within this context a behavioural approach is key. We discuss when facilitators should consider model-based interventions amid further implications for disaster communication and emergency management. We model the behaviour of individual people by deriving optimal contrarian strategies. We formulate a Bayesian algorithm which enables the optimal evacuation to be conducted sequentially under worsening conditions.

1. Introduction

There has been increasing attention paid to evacuation problems in recent years (see e.g. Bish and Sherali, 2013; Bretschneider & Kimms, 2012; Lim, Zangeneh, Baharnemati, & Assavapokee, 2012). In addition to several high profile events (Schadschneider et al., 2008) this has been accompanied by increases in the frequency and severity of natural disasters coupled with population growth in various high-risk areas of the world (Bozorgi-Amri, Jabalameli, Mirzapour, & e Hashem, 2013). One of the key themes that has emerged in recent years is that emergency evacuations may be greatly aided by social media.

As evidenced by recent testimony to the UK Parliament (Preston, Branicki, & Binner, 2014a) social media usage in emergency evacuations represents an important practical problem to which our paper adds quantitative insight. The use of social media to co-ordinate emergency evacuations is already in its operational infancy (Chen & Xiao, 2008; Nakajima, Yamane, Hattori, & Ishida, 2008; Preston, Binner, Branicki, Ferrario, & Kolokitha, 2011; Preston, Binner, Branicki, Galla, & Jones, 2014b). Further real world examples include survey evidence of social media usage to gain information about emergencies (American Red Cross, 2010) and the Personal Localised Alerting Network (PLAN) in New York that has emerged out of a collaboration between the federal communications commission, the federal management agency and industry.

Within emergency evacuations a behavioural approach to OR is key (Hämäläinen, Luoma, & Saarinen, 2013; Tamura, 2005). Human aggression and terrorism (Annlin Jin & Paramasivan, 2012; Oh, Agrawal, & Rao, 2011), panic in response to perceived dangers ("flight panics") and greedy behaviour ("acquisitive panics" or "crazes") may all have important implications for real evacuations (Schadschneider et al., 2008). In our context behavioural and social processes are extremely important (Drake, Gerde, & Wasieleski, 2009). Social media has been variously used to document the London 7/7 bombings and the Fukushima radiological disaster amid several more localised tragedies (Preston et al., 2014b). Further, anecdotal evidence from practitioners at the UK cabinet office suggests that individuals continue to use social media, and may do so excessively, even in the throes of an emergency evacuation.

Both behavioural OR (Hämäläinen et al., 2013) and evacuations (Zheng, Zhong, & Liu, 2009) present interesting multidisciplinary challenges for which multiple methodologies are needed. On a similar theme our paper presents a network flow model in Sections 2–3, a game theoretic model in Section 4 and computational modelling in Sections 5–6. More intricate modelling of specialised evacuations problems is possible (Abdelghany, Abdelghany, Mahmassani, & Alhalabi, 2014) but our model is specifically based on discussions we had with practitioners about how real crowds were managed at major events in London. Our model thus seems to apply very generally. For additional discussion and model justification see Sections 2–3.

Whilst a large body of work discusses mathematical modelling of evacuations (Burstedde, Klauck, Schadschneider, & Zittarz, 2001; Ferscha & Zia, 2010; Helbing, Farkas, & Vicsek, 2000) comparatively very little work has so far been published on the role of social media

* Corresponding author. Tel.: +44 1142223384.
E-mail addresses: John.Fry@sheffield.ac.uk (J. Fry), J.M.Binner@bham.ac.uk (J.M. Binner).

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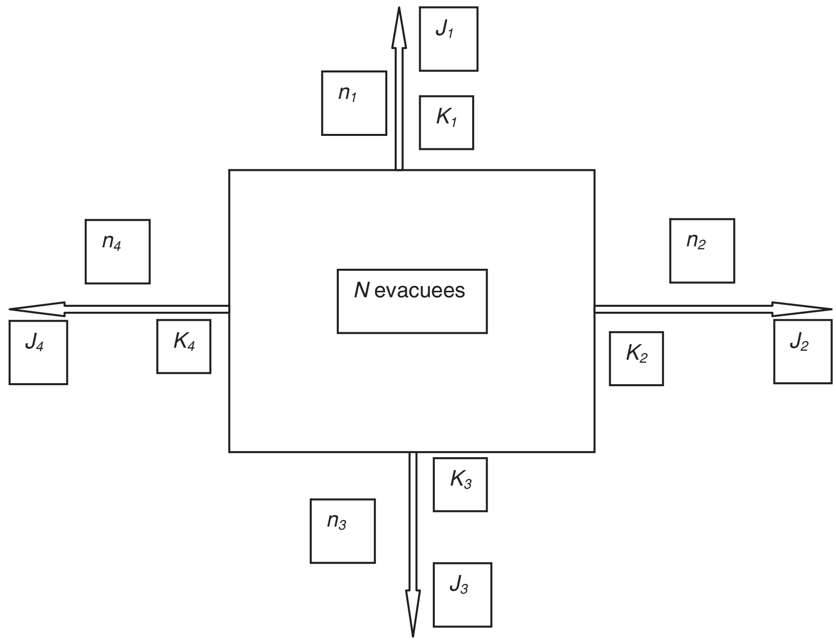

Fig. 1. Schematic of the model described in the main text. The illustration is for the special case of $B = 4$ exit routes, each characterised by a processing capacity, K_i, and an offset waiting time, J_i.

in such evacuations. The main contributions of this paper are thus as follows. First, we provide several ways to measure the usefulness of information from social media in the context of emergency evacuations - to date an under-explored issue. Second, our hand-calculation model provides a blueprint for those, such as civil engineers, social planners, etc, tasked with dealing with the mechanics of real-life evacuations. Third, we incorporate behavioural aspects in our discussion. We discuss optimal model-based interventions. We address the issue of over-crowding. We describe optimal contrarian strategies in which an individual may be best advised to eschew busy popular routes in favour of less popular but theoretically slower routes. This basic optimality result may also have further implications for psychological aspects of evacuations (Muir, Bottomley, & Marrison, 1996). We provide a Bayesian algorithm allowing a facilitator to continuously update the optimal evacuation strategy as new information arrives.

The layout of this paper is as follows. In Section 2 we give a mathematical description of the evacuation problem. In Section 3 we discuss model-based interventions. Section 4 models individual behaviour. Section 5 derives an optimal online allocation algorithm under worsening conditions. Section 6 discusses an empirical application pertaining to the potential emergency evacuation of the City of Sheffield in the United Kingdom. Section 7 concludes. A mathematical appendix can be found at the end of the paper.

2. A mathematical description of the evacuation problem

In this section we consider a macroscopic version of the evacuation problem. A related statistical mechanics model is considered in Galla (2011). However, rather than explicitly modelling a flow process we consider a model with a group of N agents who arrive simultaneously in a central location and are awaiting evacuation through one of several exit routes. Fig. 1 shows a model with B = 4 such routes or branches, but any number is possible. Fig. 2 indicates how this model may be tailored to a more realistic practical setting.

In the sequel we label branches by $i = 1, \ldots, B$. A branch, i, is defined by two parameters: a capacity K_i and a 'baseline' journey time to a place of safety, J_i. The parameter J_i subsumes information pertaining to distance/average speed - both of which should have a clear physical interpretation in the context of a practical problem. The quantity J_i is an offset, and reflects the time needed to traverse the exit route. K_i is the number of evacuees which can be 'processed' by exit route i per unit time, reflecting for example the width of the corresponding exit or other operational constraints. This simple formulation is intended to capture the observation that whilst bottlenecks (represented by decreasing K_i) and delays (represented by increasing J_i) are extremely important in real evacuations often the exact geometry of the bottleneck is of only minor importance (Schadschneider et al., 2008). If n_i people are allocated to Route i then the evacuation time of the last individual exiting through route i becomes

$$\frac{n_i}{K_i} + J_i, \tag{1}$$

i.e. the cohort of evacuees waits a total of n_i/K_i units of time before all are processed. Safety is then reached J_i periods of time later. Evacuees are processed sequentially, i.e. the first agent evacuating through exit i reaches safety at time $1/K_i + J_i$, the second agent at time $2/K_i + J_i$ and so on. An allocation of evacuees to exit routes is a tuple $(n_1, \ldots, n_B)$ satisfying $\sum_{i=1}^{B} n_i = N$, $n_i \geq 0$. The n_i denote the number of agents evacuating through exit i. Here, we make a continuum approximation under the assumption that the total number of evacuees is large, $N \gg 1$.

In addition to minimising the final evacuation time there is also interest in two related solution concepts (Fry, Galla, & Binner, 2014; Jarvis & Ratliff, 1982):

(i) The number of people evacuated via Route i by time t is given by

$$f_i(t) = \begin{cases} 0 & t < J_i, \\ K_i(t - J_i) & t \in \left[J_i, \frac{n_i}{K_i} + J_i\right], \\ n_i & t \geq \left[J_i, \frac{n_i}{K_i} + J_i\right]. \end{cases} \tag{2}$$

(ii) The total exposure prior to evacuation is given by

$$S = \sum_{i=1}^{B} \int_0^{n_i} \left(J_i + \frac{x_i}{K_i}\right) dx_i = \sum_{i=1}^{B} \left(n_i J_i + \frac{n_i^2}{2K_i}\right). \tag{3}$$

Eqs. (1)–(3) thus present a linear model which might be deemed physically unrealistic in relation to nonlinearity brought about by

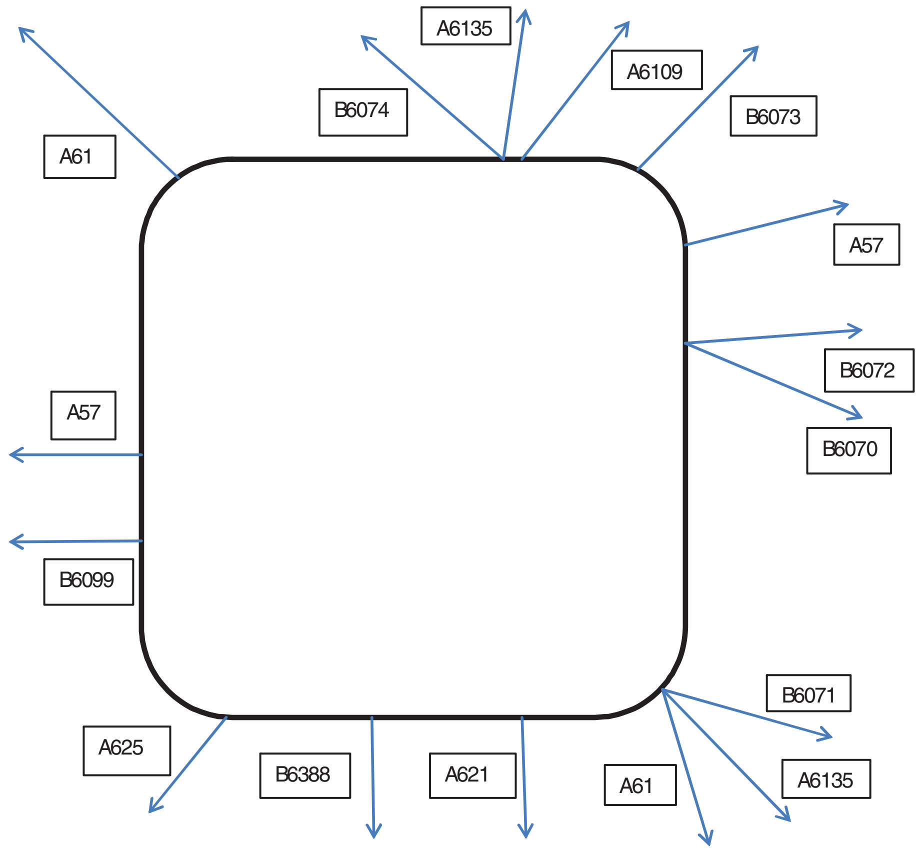

Fig. 2. Heuristic model of the Sheffield Inner Ring Road that encompasses Sheffield City Centre.

congestion effects etc. However, presenting the model in this way tallies with discussions we had with practitioners about how crowds were managed at major events in London. Further, we envisage that using information from social media parameter values may be updated sequentially thus capturing any nonlinear effects (Zobel, 2014). Moreover, even if our model appears unreallistic research in humanitarian logistics suggests that despite their inherent unpredictability preliminary preparations for such events may still be crucial in securing a faster response (Balcik & Beamon, 2008; Campbell & Jones, 2011; Yi & Kumar, 2007).

Proposition 1 shows how the optimal evacuation process may be achieved. Further, this solution appears to hold a degree of robustness (Fry et al., 2014).

Proposition 1 (Fry et al., 2014)**.** *The allocation $n_1, n_2, \ldots, n_B$ defined by*

$$\frac{n_1}{K_1} + J_1 = \frac{n_2}{K_2} + J_2 = \cdots = \frac{n_B}{K_B} + J_B = \lambda \tag{4}$$

(i) *Minimises the time of the last evacuee,*
(ii) *Maximises the number of people evacuated by time t,*
(iii) *Minimises the total exposure prior to evacuation,*

where λ denotes a Lagrange multiplier defined by

$$\lambda = \frac{\sum_{i=1}^{B} K_i J_i + N}{\sum_{i=1}^{B} K_i}. \tag{5}$$

Suppose instead that the J_i and K_i are assumed to be unknown parameters. In this case an alternative solution can be found by appealing to Eq. (3) and minimising the expected total exposure

$$\textit{Expected Total Exposure} := E\left(\sum_{i=1}^{B} n_i J_i + \frac{n_i^2}{2K_i}\right). \tag{6}$$

This leads to the following solution.

Proposition 2 (Fry et al., 2014)**.** *Suppose that J_i and K_i are randomly distributed. The expected total exposure prior to evacuation is minimised by*

$$n_i = \frac{1}{E[1/K_i]}\left(\frac{N + \sum_{j=1}^{B}\frac{E[J_j]}{E[1/K_j]} - E[J_i]\sum_{j=1}^{B}\frac{1}{E[1/K_j]}}{\sum_{j=1}^{B}\frac{1}{E[1/K_j]}}\right). \tag{7}$$

A related result is available that serves as a basic proof of concept and illustrates how social media may be used to improve emergency evacuations (see Proposition 3). Suppose that as in (Chen & Xiao, 2008; Nakajima et al., 2008; Preston et al., 2011) the J_i and K_i have to be estimated by a facilitator using information from social media. In the sequel we label these estimates $\hat{J}_i$ and $\hat{K}_i$. Throughout this process we assume only that evacuees have access to social media via a suitable handheld device e.g. mobile phone/tablet and make no assumptions regarding the platform (e.g. Facebook, Twitter, news websites etc.) that individuals may use. This fits well with the rise of so-called citizen journalism (Allan & Thorsen, 2009) practised by non-professional journalists via social media platforms. We envisage that facilitators may be able to use social media to gain information regarding delays and congestion via evacuees (in similar spirit to citizen journalism) or via strategically placed agents. This may be made possible in applications using sensory or cellular phone data (Chiu, Zheng, Villalobos, & Gautam, 2007), gate counts (ODA, 2011) or via automated probability elicitation techniques (see e.g. Gosling, Oakley, & O'Hagan, 2007). In addition to a plethora of recent applications, the dynamic and imperfect nature of real evacuations (Schadschneider et al., 2008) reinforces the potential significance of social media in these situations. The following proposition shows that, in principal, if social media can be used to accurately track system parameters then the final evacuation time can be reduced

Proposition 3. *Better information reduces the final evacuation time in the following sense. Using information from social media the difference between the actual and optimal final evacuation time satisfies the inequality*

$$\max_i \left\{ \left| \frac{\hat{K}_i\hat{\lambda} - \hat{J}_i}{K_i} + J_i - \lambda \right| \right\} \leq \max_i \left\{ \left| \frac{\hat{K}_i\hat{\lambda}}{K_i} - \lambda \right| \right\} + \max_i \left\{ \left| J_i - \frac{\hat{K}_i\hat{J}_i}{K_i} \right| \right\}, \tag{8}$$

where λ *and* $\hat{\lambda}$ *denote Lagrange multipliers defined by Eqs. (5) and (9) respectively*

$$\hat{\lambda} = \frac{\sum_{i=1}^{B} \hat{K}_i\hat{J}_i + N}{\sum_{i=1}^{B} \hat{K}_i}. \tag{9}$$

Proof. The allocation made using information from social media satisfies $\hat{n}_i = \hat{K}_i(\hat{\lambda} - \hat{J}_i)$ leading to the final evacuation time

$$\frac{\hat{n}_i}{K_i} + J_i = \frac{\hat{K}_i}{K_i}(\hat{\lambda} - \hat{J}_i) + J_i.$$

The optimal final evacuation time is given by λ in Eq. (5). As a consequence of the triangle inequality the difference between the two allocations satisfies

$$\begin{aligned} \max_i \left\{ \left| \frac{\hat{K}_i\hat{\lambda} - \hat{J}_i}{K_i} + J_i - \lambda \right| \right\} &= \max_i \left\{ \left| \left(\frac{\hat{K}_i\hat{\lambda}}{K_i} - \lambda \right) + \left(J_i - \frac{\hat{J}_i\hat{K}_i}{K_i} \right) \right| \right\} \\ &\leq \max_i \left\{ \left| \frac{\hat{K}_i\hat{\lambda}}{K_i} - \lambda \right| + \left| J_i - \frac{\hat{J}_i\hat{K}_i}{K_i} \right| \right\} \\ &\leq \max_i \left\{ \left| \frac{\hat{K}_i\hat{\lambda}}{K_i} - \lambda \right| \right\} + \max_i \left\{ \left| J_i - \frac{\hat{J}_i\hat{K}_i}{K_i} \right| \right\} \end{aligned}$$

□

The first term on the RHS of Eq. (8) shows that as knowledge of the optimal system processing capacity increases the actual final evacuation time becomes closer to the optimum value. The second term on the RHS of Eq. (8) shows that as knowledge of the true journey times and route capacities improves the actual final evacuation time also gets closer to the optimum value.

In view of the above, some further discussion is needed regarding the ease with which data can be gathered from social media. Inter alia location data from mobile phones allows for a comprehensive study of mobility patterns (Bekhor, Cohen, & Solomon, 2013). In road networks smart devices can be used to obtain high-quality flow and travel-time data. Roggen, Wirz, Troster, and Helbing (2011) use mobile-phone location data to model the behaviour of pedestrian crowds. Sakaki, Matsuo, Yanagihara, Chandrasiri, and Nawa (2012) and Kosala and Adi (2012) discuss the harvesting of real-time traffic information from Twitter to inform users about traffic congestion. Thompson, White, Dougherty, Albright, and Schmidt (2010) consider the use of smartphones to detect car accidents. Mohan, Padmanabhan, and Ramjee (2008) detail the use of mobile smartphones to monitor more general road conditions. Recent work in Chen, Fan, and Sun (2015) and Ni, He, and Gao (2014) extracts statistically significant signals from big social media datasets. Indeed, research presented to the US Congress suggests that it is the volume, rather than the paucity, of information on social media systems that presents the real challenge (Lindsay, 2011). Finally, empirical evidence of social media usage to track road closures in real-life emergency evacuations is documented in Bird, Ling, and Haynes (2012) and Palen (2008) together with other uses (Oxendine & Waters, 2014).

3. Model-based interventions

Model-based interventions in complex systems are very challenging (Brocklesby, 2009; Luoma, Hämäläinen, & Saarinen, 2010). Within our specific setting there are also further complications as real-time analysis of social media is difficult to manage and subject to abuse (Preston et al., 2014a). One possibility is that networks may be maliciously seeded with poor quality information (Annlin Jin & Paramasivan, 2012). A further complication is that terrorists may themselves have access to social media (Oh et al., 2011). Amidst such uncertainty how might facilitators proceed and decide if information from social media is sufficient to indicate that a change in evacuation strategy is required (Chen & Xiao, 2008; Nakajima et al., 2008). From a network perspective (Vojnović, Gupta, Karagiannis, & Gkantsidis, 2008) a natural benchmark is an oblivious strategy such as a uniform evacuation strategy. If using information from social media outperforms such a strategy then the information it provides clearly has a degree of value. Proposition 4 thus provides a simple benchmark to help decide when information from social media suggests that a change in strategy is merited.

Proposition 4 (Comparison with uniform allocation strategies). *Suppose that* $E[\hat{J}_i] = J_i$, $E[\hat{K}_i] = K_i$ *and* $E[\hat{K}_i\hat{\lambda}] = K_i\lambda$. *Let* T_{unif} *denote the total exposure corresponding to a uniform evacuation strategy with* $n_i = \frac{N}{B}$, *and* T_{opt} *denote the unknown optimal strategy obtained using* $n_i = K_i(\lambda - J_i)$. *We have the following:*

There exists a threshold such that if

(i) *High-quality information paradigm. If*

$$\sum_i \left(\frac{\mathrm{var}(\hat{n}_i)}{2K_i} \right) < T_{unif} - T_{opt} \tag{10}$$

then evacuating using information from social media performs better on average than a uniform allocation strategy.

(ii) *Low-quality information paradigm.*

$$\sum_i \left(\frac{\mathrm{var}(\hat{n}_i)}{2K_i} \right) > T_{unif} - T_{opt} \tag{11}$$

then evacuating using information from social media performs worse on average than a uniform allocation strategy where

$$T_{unif} = \frac{N}{B}\sum_i J_i + \frac{N^2}{2B^2}\sum_i \left(\frac{1}{K_i} \right),$$

and T_{opt} *denotes the optimal minimum exposure*

$$T_{opt} = \lambda^2 \left(\frac{\sum_i K_i}{2} \right) - \sum_i \left(\frac{J_i^2 K_i}{2} \right). \tag{12}$$

Proof. The condition $E[\hat{K}_i\hat{\lambda}] = K_i\lambda$ means that $E[\hat{n}_i] = n_i$ and $E[\hat{n}_i^2] = n_i^2 + \mathrm{var}(\hat{n}_i)$, where n_i is the optimal allocation satisfying Eq. (12). Using social media the expected total exposure is

$$\begin{aligned} \sum_i \left(J_i E[\hat{n}_i] + \frac{E[\hat{n}_i^2)]}{2K_i} \right) &= \sum_i \left(J_i n_i + \frac{n_i^2}{2K_i} + \frac{\mathrm{var}(\hat{n}_i)}{2K_i} \right) \\ &= T_{opt} + \sum_i \frac{\mathrm{var}(\hat{n}_i)}{2K_i} \end{aligned}$$

Formulas (10)–(11) follow by comparison noting that under a uniform evacuation strategy with $n_i = N/B$ the total exposure is given by

$$T_{unif} = \frac{N}{B}\sum_i J_i + \frac{N^2}{2B^2}\sum_i \frac{1}{K_i}.$$

□

Proposition 4 contributes, at least in part, to wider debates on resilience and information quality (Kolfal, Patterson, & Yeo, 2013; Zobel & Khansa, 2012). If Eq. (10) applies then evacuating using information from social media performs better on average than a uniform allocation strategy and provides tangible benefits in terms of added control of the evacuation process. If Eq. (11) applies then a uniform allocation strategy is better on average and using information from social media may be detrimental in extreme cases. If $\mathrm{var}(\hat{n}_i) = \sigma^2$ then the threshold in Eq. (11) simplifies to

$$\sigma^2 \geq 2\left(\sum_i \left\{\frac{1}{K_i}\right\}\right)(T_{unif} - T_{opt}). \tag{13}$$

Eqs. (10)–(13) remain a little complicated to apply directly but do at least suggest that they may be applied in practice by using automated probability elicitation techniques (see e.g. Gosling et al., 2007).

The condition $E[\hat{K}_i\hat{\lambda}] = K_i\lambda$ relates to the ability to accurately assess knowledge of the true optimal route allocation and true route capacities since $\hat{K}_i\hat{\lambda} = \hat{n}_i + \hat{J}_i\hat{K}_i$. It is easy to show that this condition is satisfied if $\hat{K}_i = K_i$ a.s. and only the $\hat{J}_i$ have to be estimated. However, the condition $E[\hat{K}_i\hat{\lambda}] = K_i\lambda$ is not in general satisfied if $\hat{J}_i = J_i$ a.s. and only the $\hat{K}_i$ have to be estimated. This suggest that Eqs. (10)–(11) are less likely to apply if evacuations are subject to considerable uncertainties regarding congestion. Over-confidence in assessing congestion may thus be a serious problem in real-world evacuations and there may be interesting parallels here with over-confidence in financial markets during bubbles (Fry, 2012). A more general version of Eq. (11) assuming only $E[\hat{J}_i] = J_i$ and $E[\hat{K}_i] = K_i$ is

$$\sum_i \left(J_i E[\hat{n}_i] + \frac{E[\hat{n}_i^2]}{2K_i}\right) \geq T_{unif} - T_{rand}. \tag{14}$$

Finally, Eqs. (10)–(14) also offer a slight refinement upon recently published results in Fry et al. (2014).

When is it worthwhile to use incoming information from social media to update previous strategies? If Proposition 4 answers this question relative to a naive uniform evacuation strategy Proposition 5 answers this question relative to existing pre-planned strategies that do not make use of the most recent information available. This is thus intended to be closer in spirit to real-world applications where some emergency pre-planning has already been undertaken (Balcik & Beamon, 2008; Campbell & Jones, 2011; Yi & Kumar, 2007).

Proposition 5 (When to update evacuation strategies)**.** *Define $n_i = \lambda(K_i - J_i)$ and suppose that the true underlying parameter values J_i and K_i have now changed to $J_{i,new}$ and $K_{i,new}$. Suppose further that $E[\hat{J}_i] = J_{i,new}$, $E[\hat{K}_i = K_{i,new}]$ and $E[\hat{K}_i\hat{\lambda}_i] = K_{i,new}\lambda_{new}$.*

(i) *If*

$$\sum_i \left(n_i J_{i,new} + \frac{n_i^2}{2K_{i,new}}\right) \geq T_{opt,new} + \sum \frac{\mathrm{var}(\hat{n}_{i,new})}{2K_{i,new}}, \tag{15}$$

then using updated information is optimal.

(ii) *If*

$$\sum_i \left(n_i J_{i,new} + \frac{n_i^2}{2K_{i,new}}\right) \leq T_{opt,new} + \sum \frac{\mathrm{var}(\hat{n}_{i,new})}{2K_{i,new}}, \tag{16}$$

then it is optimal to retain use of the original pre-planned strategy.

Proof. Using the original optimal strategy without the benefit of incoming information from social media gives $n_i = \lambda(K_i - J_i)$. The total exposure is given by $\sum_i \left(J_{i,new} n_i + \frac{n_i^2}{2K_{i,new}}\right)$. Using an updated strategy the expected total exposure becomes

$$\begin{aligned}\sum_i \left(J_{i,new}E[\hat{n}_i] + \frac{E[\hat{n}_i^2)]}{2K_{i,new}}\right) &= \sum_i \left(J_{i,new}n_{i,new} + \frac{n_{i,new}^2}{2K_{i,new}} + \frac{\mathrm{var}(\hat{n}_{i,new})}{2K_{i,new}}\right)\\ &= T_{opt} + \sum_i \frac{\mathrm{var}(\hat{n}_{i,new})}{2K_{i,new}}\end{aligned}$$

and Eqs. (15)–(16) follow by comparison. □

4. Modelling individual behaviour

It is clear that during evacuations it is not just the authorities who are directing the evacuation who can access information from social media. Individual evacuees can also access this information freely and make their own decisions accordingly. This may be particularly important in relation to over-crowding and jamming which appear to be key features of both real evacuations (Schadschneider et al., 2008) and computer simulation results (Smyrnakis & Galla, 2012). Emergency evacuations take place in socio-technical systems (Bonen, 1979; Cliff & Northrop, 2012; Hollnagel, Pariès, Woods, & Wreathall, 2011) whereby interactions between human agents and new (social media) technologies may have wide-ranging consequences beyond the technical specifications of infrastructure. For instance, suppose during an emergency evacuees observe information from social media and all rush to the same exit. How might individual evacuees and emergency planners best proceed?

Within the context of this study we analyse individual behaviour as follows. We consider optimal contrarian strategies by which an individual may be able to reduce their expected evacuation time based on how the rest of the crowd will act. Such contrarian strategies are well-documented in finance and investing (Chan, 1988) but may also lead to optimal evacuation strategies here.[1]

We view the evacuation problem from the perspective of an individual known as A who has to compete with a crowd of size m. This results in a one-shot two-player game in which we can analyse A's best strategy response. Suppose the crowd is of size m. Of this crowd m_i elect to evacuate through Route i. If A chooses to evacuate through Route i the $m_i + 1$ evacuees are stacked and are randomly ordered. A's waiting time prior to being processed and traversing the route is distributed according to $U(0, m_i + 1)$ and the expected waiting time is $(m_i + 1)/2$. Thus the expected final evacuation time via route i is

$$\frac{m_i + 1}{2K_i} + J_i. \tag{17}$$

A seeks to evacuate as quickly as possible and so must choose

$$\min_i E\left\{\frac{m_i + 1}{2K_i} + J_i\right\}. \tag{18}$$

Rational expectations (Muth, 1961), Eqs.(17)–(18) and physical considerations mean that we may assume that m_i and $1/K_i$ are negatively correlated since if $1/K_i$ decreases then Route i becomes wider and a more attractive choice of exit. Similarly, if $1/K_i$ increases then Route i becomes narrower and a more difficult route from which to escape. We have the following Proposition which illustrates the tradeoff between choosing more populous faster routes and slower less populous routes faced by individual agents.

Proposition 6. *Optimal contrarian strategies*

(i) *A's optimal route of exit satisfies*

$$\min_i \left\{\frac{1}{2}\mathrm{Cov}\left(m_i, \frac{1}{K_i}\right) + \frac{1}{2}E(m_i)E\left(\frac{1}{K_i}\right) + \frac{1}{2}E\left(\frac{1}{K_i}\right) + E(J_i)\right\}. \tag{19}$$

(ii) *If $\mathrm{Cor}(m_i, 1/K_i) = -1$ A's optimal route of exit satisfies*

$$\min_i \left\{-\frac{1}{2}s.d(m_i)s.d.\left(\frac{1}{K_i}\right) + \frac{1}{2}E(m_i)E\left(\frac{1}{K_i}\right) + \frac{1}{2}E\left(\frac{1}{K_i}\right) + E(J_i)\right\}, \tag{20}$$

where s.d denotes standard deviation.

[1] As a simple illustration the authors are regular train commuters and we often find ourselves adopting ad hoc contrarian strategies to avoid busy carriages and stairwells etc. Doubtless the reader can think of similar times when they adopt such contrarian strategies in their own daily lives.

(iii) *If* $\text{Cor}(m_i, 1/K_i) = 0$ *A's optimal route of exit satisfies*

$$\min_i \left\{ \frac{1}{2}E(m_i)E\left(\frac{1}{K_i}\right) + \frac{1}{2}E\left(\frac{1}{K_i}\right) + E(J_i) \right\}. \tag{21}$$

Proof. From Eq. (17) it follows that

$$\begin{aligned}\text{Optimal Exit Route} &= \min_i \left\{ \frac{1}{2}E\left[\frac{m_i}{K_i}\right] + \frac{1}{2}E\left[\frac{1}{K_i}\right] + E[J_i] \right\}, \\ &= \min_i \left\{ \frac{1}{2}\text{Cov}\left(m_i, \frac{1}{K_i}\right) + \frac{1}{2}E[m_i]E\left[\frac{1}{K_i}\right] \right. \\ &\quad \left. + \frac{1}{2}E\left[\frac{1}{K_i}\right] + E[J_i] \right\}.\end{aligned}$$

Eqs. (20)–(21) follow by using $\text{Cov}(m_i, \frac{1}{K_i}) = -s.d.(m_i)s.d.(\frac{1}{K_i})$ and $\text{Cov}(m_i, \frac{1}{K_i}) = 0$ respectively. □

Proposition 6 shows that A's optimal choice of exit should address the physical parameters of the problem – the $E(J_i)$ and $E(1/K_i)$ terms – but should also include a penalty for popular choices of route – the $1/2E(m_i)E(1/K_i)$ term. The $\text{Cov}(m_i, 1/K_i)$ term is a measure of how crowds respond to congestion. Eq. (19) gives the most general result. Eq. (20) describes the idealised scenario whereby crowds rationally incorporate information about congestion into their collective actions. Eq. (21) describes a zero-intelligence or stampede scenario when the behaviour of crowds is not moderated by such congestion and may be closer to the dynamics of non-adaptive behaviour in real-life evacuations (Schadschneider et al., 2008).

Beyond narrow mathematical definitions of rationality Propostion 6 also highlights several further potential issues. A combination of heuristics and other psychological factors (Muir et al., 1996) may mean that the behaviour of evacuees in real evacuations systematically departs from Eq. (19). Inter alia there may be links to insight problems which, though analytically tractable, may still mislead large numbers of people (Mayer & Davidson, 1995). If real evacuees do tend to behave according to Eq. (19) there may still be important implications for the communication of risk during evacuations so that bottlenecks may be avoided (Smyrnakis & Galla, 2012). Individual outcomes and the system-level behaviour as a whole ultimately depend on the actions of everybody else. If agents located close to each other on the network form similar opinions and replicate each other's actions this may lead to bottlenecks and congestion which is something that emergency planners should pay close attention to.

5. On-line allocation algorithm

In this section we construct an algorithm to discuss the on-line monitoring of an evacuation problem – so that evacuees may be better directed using updated information on capacities (over-crowding) and delayed journey times. Thus, this represents on alternative treatment of a mass-evacuation problem considered in Chiu et al. (2007) pand complements existing studies on the simulation and analysis of evacuations (Schreckenberg & Sharma, 2002), pedestrian motion (Schadschneider, Pöschel, Kühne, Schreckenberg, & Wold, 2006) and road traffic (Kerner, 2004). Further, if social media usage leads to congestion and jamming then the algorithm presented here shows how evacuees may be better directed. This is significant as real-time analysis of social media is extremely challenging and subject to abuse (Preston et al., 2014a).

The parameters K_i and J_i are assumed to be random variables as under the Bayesian paradigm (Bernardo & Smith, 2000). Uncertainty about J_i and K_i is expressed by the posterior distributions which can be updated sequentially given information on the number of evacuees n_{t_j} at time t_j that have evacuated along route *i*.

It is assumed that the most likely values for J_i and K_i are $\hat{J}_i$ and $\hat{K}_i$ respectively. In the context of a real evacuation it is assumed that the $\hat{J}_i$ and $\hat{K}_i$ represent optimal values so that $J_i \geq \hat{J}_i$ and $K_i \leq \hat{K}_i$. We choose the prior distributions

$$\pi(K_i) = \frac{2K_i}{\hat{K}_i^2}\ (0 \leq K_i \leq \hat{K}_i),$$
$$\pi(J_i) = 2\hat{J}_i^2 J_i^{-3}\ (J_i \geq \hat{J}_i). \tag{22}$$

Under the Specification (22) we have that $E[J_i] = 2\hat{J}_i$ and $E[1/K_i] = 2/\hat{K}_i$ and minimising the expected total evacuation time yields the same optimum strategy as assuming that $J_i = \hat{J}_i$ and $K_i = \hat{K}_i$ a.s. If we choose n_i to minimise the expected total exposure the functional to be minimised becomes

$$E\left(\sum_{i=1}^{B} n_i J_i + \frac{n_i^2}{2K_i}\right) = 2\left(\sum_{i=1}^{B} n_i \hat{J}_i + \frac{n_i^2}{2\hat{K}_i}\right),$$

i.e. the original functional up to a constant of proportionality.

Unforeseen delays are liable to be an intrinsic feature of real evacuations (Schadschneider et al., 2008) and it is unlikely that evacuees can move faster during evacuations compared to conventional performance levels (Pauls, 1995). Hence, we assume the conditional density (likelihood):

$$f(n_{t_j}|n_{t_{j-1}}) = \frac{2(n_{t_j} - n_{t_{j-1}})}{(K_i(t_j - t_{j-1})^2} = \frac{2\Delta n_{t_j}}{K_i^2(\Delta t_j)^2}. \tag{23}$$

This means that the number of evacuees n_{t_j} observed at time t_j cannot exceed $K_i(t_j - J)$ and that no more than $K(t_{j+1} - t_j)$ individuals can evacuate via Route *i* during the interval (t_j, t_{j+1}). Our basic online algorithm is as follows:

1. Observe n_{t_j} the number of evacuees through Route *i* by time t_j
2. Update the distributions of J_i and K_i given the data n_{t_j}
3. Calculate the expectations $E[J_i|n_{t_j}]$ and $E[1/K_i|n_{t_j}]$ and allocate evacuees according to Eq. (7) – the optimal evacuation strategy under parametric uncertainty

In the sequel we suppress the *i* subscript and replace J_i, K_i by J, K. Eq. (24) shows that the parameters for each route can be updated in parallel – the calculations for each route are essentially independent. This feature of our algorithm is important and may lead to significant time savings in applications. From the likelihood function in Eq. (23) we can use Bayes' Theorem (Bernardo & Smith, 2000) to form a posterior probability distribution for J and K given the observed evacuees n_{t_j} at time j:

$$\pi(J, K|n_{t_j}) \propto \frac{2^{T+2}\hat{J}^2 n_{t_1}}{\hat{K}^2 J^3 K^{2T-1}(t_1 - J)^2} \cdot \frac{\prod_{j>1} \Delta n_{t_j}}{\prod_{j>1} (\Delta t_j)^2}, \tag{24}$$

where $\Delta n_{t_j} = n_{t_j} - n_{t_{j-1}}$. Further, we have that $n_1/(t_1 - J) \leq K \leq \hat{K}$, $\hat{J} \leq J \leq t_1 - n_1/\hat{K}$. The posterior expectations $E[J|n_{t_j}]$ and $E[1/K|n_{t_j}]$, required as part of the optimal allocation according to Proposition 2, may be calculated as follows:

Proposition 7 (Posterior expectations)**.** *Let* $t_j = t_1, \ldots, t_T$. *We have that*

$$E\left[\frac{1}{K}|n_{t_j}\right] = \frac{2T-2}{\xi^{2-2T} - \hat{K}^{2-2T}} \frac{\xi^{1-2T} - \hat{K}^{1-2T}}{2T-1} \quad (T > 1), \tag{25}$$

$$E\left[\frac{1}{K}\right] = \frac{1}{\ln(\hat{K}/\xi)} \int_{\xi}^{\hat{K}} K^{-2} dK = \frac{1}{\ln(\hat{K}/\xi)} \left(\frac{1}{\xi} - \frac{1}{\hat{K}}\right) \quad (T = 1), \tag{26}$$

$$E[J|n_{t_j}] = \frac{2t_1}{3} + \frac{1}{3\text{Const.}}\left[\frac{1}{A^2(t_1 - A)} - \frac{1}{B^2(t_1 - B)}\right], \tag{27}$$

where

$$\xi = \min\left\{\frac{n_{t_1}}{t_1 - \hat{J}}, \min_{j>1}\left\{\frac{\Delta n_{t_j}}{\Delta t_j}\right\}\right\}$$

$$A = t_1 - \frac{n_{t_1}}{\xi}$$
$$B = t_1 - \frac{n_{t_1}}{\hat{K}}$$
$$\text{Const}^{-1} = \left[\frac{1}{2t_1}\left[\frac{1}{B^2(t_1-B)} - \frac{1}{A^2(t_1-A)}\right] + \frac{3}{t_1^4}\ln\left(\frac{A(t_1-B)}{B(t_1-A)}\right) + \frac{3(A-B)}{2t_1^3}\left[\frac{1}{BA} + \frac{1}{(t_1-A)(t_1-B)}\right]\right]. \quad (28)$$

Proof. See the Appendix. □

6. Empirical application

Sheffield is England's third largest metropolitan authority and fourth largest city with an estimated population of 555,500. The geography of the city renders it vulnerable to flooding and the city has experienced very bad flooding in recent years, especially in 2007 (Sheffield City Council, 2013).

A heuristic map of the Sheffield Inner Ring Road that encompasses Sheffield city centre is shown in Fig. 2. For the purposes of this study we assume that A-roads are dual carriage ways with an average speed of 64 kilometers per hour (40 miles per hour) and B-roads are single carriage way with an average speed of 48 kilometers per hour (30 miles per hour). In line with some of the themes expressed in Sheffield City Council (2013) we assume that the purpose is to evacuate the city centre to a distance of 5 kilometers. This gives the parameters $J_A = (60\times 5)/64 = 4.6875$ minutes and $J_B = (60\times 5)/48 = 6.25$ minutes. Similarly, we can use data on stopping distances published in Highway Code (2014) to estimate the K_i. For A-routes one car requires 40 meters of road (36 meters stopping distance at 40 miles per hour plus 4 meters for the average car). The distance travelled in 1 minute is $64,000/60 = 1066.667$ meters giving $K_A = \lfloor 1066.667/40\rfloor \times 2$ Carriage Ways $= 52$. For B-routes one car requires 27 meters of road (23 meters stopping distance at 30 miles per hour plus 4 meters for the length of the average car). The distance travelled in 1 minute is $48,000/60 = 800$ meters giving $K_B = \lfloor 800/27\rfloor = 29$.

Suppose that we have to evacuate

1. 24,000 vehicles,
2. 15,000 vehicles.

Given Sheffield's size and the combined capacity of the city's two football stadia these figures appear reasonable. We have that $\sum K_i = 9(52) + 7(29) = 671$ and $\sum K_i J_i = 9(52\times 4.6875) + 7(29\times 6.25) = 3462.5$. Under Scenario 1. we have from Eq. (5) that the final evacuation time λ is given by

$$\lambda = \frac{3462.5 + 24000}{671} = 40.92771982$$

This gives

$$\frac{n_A}{52} + 4.6875 = 40.92771982;$$
$$n_A = 52(40.92771982 - 4.6875) = 1884.491$$
$$\frac{n_B}{29} + 6.25 = 40.92771982;$$
$$n_B = 29(40.92771982 - 6.25) = 1005.654$$

This means that in order to obtain the optimal solution 1884 cars should be allocated to each A road and 1006 cars should be allocated to each B road. Under Scenario 2. similar reasoning gives

$$\lambda = \frac{3462.5 + 15000}{671} = 27.51490313,$$
$$\frac{n_A}{52} + 4.6875 = 27.51490313;$$
$$n_A = 52(27.51490313 - 4.6875) = 1187.025,$$
$$\frac{n_B}{29} + 6.25 = 27.51490313;$$
$$n_B = 29(27.51490313 - -6.25) = 616.682.$$

In this case 1187 cars should be allocated to each A road and 617 to each B road. In the sequel we will refer to both solutions as the "Original Strategy".

In the sequel we introduce random error into the system. Suppose $K_{i,new} = K_i$, so that the conditions satisfying Propositions 4–5 are satisfied, and suppose further that $J_{i,new} = J_i v_i$ where $v_i \sim U(1, T_{end})$ and the estimator $\hat{J}_i = J_{i,new}e^{Z_i}$ where $Z_i \sim N(-\sigma^2/2, \sigma^2)$. Set up in this way the error terms satisfy $E(e^{Z_i}) = 1$. In computer simulations we compare the effect of using information from social media against the original strategy under Scenarios 1–2. above and a naive uniform evacuation strategy.

Simulation results are shown in Table 1. First, for both perfect information ($\sigma^2 = 0$) and low noise ($\sigma^2 = 0.03$) updating strategies using information from social media perform best and may lead to significant time savings. For noisier observations ($\sigma^2 = 0.06, \sigma^2 = 0.15$) updating strategies perform worse for less severe disruptions (T_{end} low) but may still be very worthwhile when disruptions are more severe (T_{end} high). For higher values of σ^2 using information from social media is not as effective as the original strategy but still out performs the naive uniform strategy. Throughout the simulations the original strategy out-performs the uniform strategy. This simple result nonetheless reaffirms the importance of emergency planning – even in the presence of manifold uncertainty (Balcik & Beamon, 2008; Yi & Kumar, 2007).

7. Conclusions, policy implications and discussion

This paper explores evacuation problems in a way that to date has been fundamentally under-explored – namely modelling the effects of social media. As evidenced by recent testimonies to the UK Parliament this represents an important practical problem to which we lend quantitative insight. Our models identify several key features of real evacuations, may purposefully inform policy debates and may ultimately guide the construction of more elaborate and applicable models. However, the contributions of this paper extend beyond the purely operational and we provide an algorithm through which the optimal network allocation may be updated online. We will consider the real-world application of our models in later work. In future work it might also be interesting to compare results with a different form of information spreading e.g. in the form of evacuation assistants (Schadschneider, Eilhardt, Nowak, Wagoum, & Seyfried, 2011).

There is an important behavioural dimension to evacuation problems (Hämäläinen et al., 2013). Issues highlighted by our study include over-confidence, psychological factors, heuristic decision making and insight problems. Information quality is an important issue associated with real world networks and evacuations (Preston et al., 2014b; Preston et al., 2014a). We can show that as information quality increases the potential benefits brought by social media increase. However, when information quality from social media is less good its use comes with risks attached and other strategies may prove more effective. This leads to a complex model-based intervention problem (Luoma et al., 2010). This is particularly pertinent as both simulation results (Smyrnakis & Galla, 2012) and empirical evidence (Schadschneider et al., 2008) emphasise the role of over-crowding and jamming in evacuations. We offer two different solutions to this problem. From an individual perspective we derive an optimal contrarian strategy which enables an individual to offset technically faster routes against their propensity for over-use by other agents in the system. This approach may also hold some wider significance for a behavioural treatment of evacuation problems. From a facilitator's perspective we derive an online allocation algorithm for updating the optimal evacuation strategy. Results from a computer simulation

Table 1
Simulation results: mean final evacuation time (standard deviation) based on 1,000,000 simulations.

T_{end} ($N = 24000$)	Social media $\sigma^2 = 0$	Social media $\sigma^2 = 0.03$	Social media $\sigma^2 = 0.06$	Social media $\sigma^2 = 0.15$	Social media $\sigma^2 = 0.27$	Social media $\sigma^2 = 0.7$	Uniform strategy	Original strategy
2	43.507 (0.376)	45.847 (0.824)	46.674 (1.011)	48.094 (1.327)	49.188 (1.587)	51.129 (2.304)	63.443 (0.689)	46.448 (0.591)
3	46.089 (0.752)	49.346 (1.375)	50.508 (1.646)	52.521 (2.100)	54.095 (2.459)	56.946 (3.390)	68.914 (1.376)	51.961 (1.178)
4	48.666 (1.127)	52.867 (1.939)	54.370 (2.296)	56.996 (2.892)	59.049 (3.357)	62.806 (4.499)	74.383 (2.067)	57.471 (1.767)
5	51.246 (1.505)	56.393 (2.509)	58.242 (2.951)	61.467 (3.678)	64.010 (4.254)	68.671 (5.617)	79.844 (2.757)	62.973 (2.359)
T_{end} ($N = 15000$)	Social media $\sigma^2 = 0$	Social media $\sigma^2 = 0.03$	Social media $\sigma^2 = 0.06$	Social media $\sigma^2 = 0.15$	Social media $\sigma^2 = 0.27$	Social media $\sigma^2 = 0.7$	Uniform strategy	Original strategy
2	30.095 (0.376)	32.434 (0.823)	33.259 (1.012)	34.681 (1.327)	35.778 (1.587)	37.721 (2.307)	44.047 (0.688)	33.036 (0.589)
3	32.676 (0.753)	35.933 (1.375)	37.095 (1.648)	39.105 (2.099)	40.684 (2.461)	43.532 (3.392)	49.515 (1.378)	38.545 (1.178)
4	35.254 (1.127)	39.450 (1.938)	40.958 (2.297)	43.581 (2.891)	45.631 (3.355)	49.385 (4.507)	54.984 (2.063)	44.054 (1.765)
5	37.836 (1.504)	42.979 (2.509)	44.830 (2.944)	48.065 (3.682)	50.602 (4.254)	55.262 (5.625)	60.449 (2.756)	49.563 (2.354)

suggest that when delays are sufficiently large even noisy information from social media may still be very valuable. Finally, irrespective of the potential impact of social media, results from the computer simulations reaffirm the importance of emergency planning (Balcik & Beamon, 2008; Campbell & Jones, 2011; Yi & Kumar, 2007).

In order to maximise the benefits of social media various capacity issues need to be addressed (Preston et al., 2014a). This requires people in the right places having the appropriate mix of technical and soft skills. However, in seeking to build new capacities it is important to recognise the conflicting aims facing businesses in terms of profit maximisation and resilience planning (Hamel & Välikangas, 2003). It is important to understand how people behave in emergency situations (CSaP, 2014; Schadschneider et al., 2008) and in large crowds (ODA, 2011). For instance the issue of poor quality signage appears to be commonly over-looked (ODA, 2011). Further, communication with a diverse public about emergency situations remains a challenging issue (CSaP, 2014). From a behavioural perspective it is interesting to note that social media can also be used to support purely emotional crises – see e.g. Simm, Ferrario, Gradinar, and Whittle (2014) who design a digital anxiety device to support autistic adults.

Empirical evidence of real-world evacuation problems leads to several real-world policy implications (CSaP, 2014; Schadschneider et al., 2008). Congestion and blockages tend to occur at "hotspots" in the network (ODA, 2011). This may have important implications for both passenger flow control (discussed here) and the structural design of networks and buildings. There may also be important administrative challenges that need to be overcome in terms of co-ordinating action between different agencies and between different branches of the transport network (ODA, 2011). The amount of data involved in evacuation problems can be very large (ODA, 2011). Thus, future behavioural analyses of emergency evacuations using social media will require tools and techniques from Big Data analytics.

Our paper offers an intriguing blue print which we hope will allow others to better conduct real-world emergency evacuation planning once the relevant physical and geographical parameters are included. This potential utility underpins the value of our contribution. Evacuation problems are well-studied although many outstanding problems remain (Schadschneider et al., 2008). Here, our ultimate objective remains to create a reliable tool that allows emergency planners to leverage social media to protect the public at large – enabling smarter evacuations. The current authors are already actively engaged with policymakers, scientists and businesses to investigate the calibration of our model and its real world implementation (Binner et al., 2013; Busayawan, Whittle, Binner, & Lawlor-Wright, 2015; Preston et al., 2014b; Schmidt and Binner, 2014). Ultimately, the value of work such as ours occurs when these models prove directly responsible for lives being saved.

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Appendix

Proof of Proposition 7. We have that $K \geq \frac{\Delta n_{t_j}}{\Delta t_j}$ and hence that $K \geq \min_j \left\{ \frac{\Delta n_{t_j}}{\Delta t_j} \right\}$. Further, since

$$K > \frac{n_1}{t_1 - J}, \tag{29}$$

and the RHS of (29) is an increasing function of J we must also have

$$K > \frac{n_{t_1}}{t_1 - \hat{J}}.$$

Hence, it follows that

$$\pi(K|n_{t_j}) \propto K^{1-2T} \text{ over the range } \left(\min \left\{ \frac{n_{t_1}}{t_1 - \hat{J}}, \min_{j>1} \left\{ \frac{\Delta n_{t_j}}{\Delta t_j} \right\} \right\}, \hat{K} \right). \tag{30}$$

We have that

$$E\left[\frac{1}{K}\right] = \frac{2T-2}{\xi^{2-2T} - \hat{K}^{2-2T}} \int_{\xi}^{\hat{K}} K^{-2T} dK = \frac{2T-2}{\xi^{2-2T} - \hat{K}^{2-2T}} \frac{\xi^{1-2T} - \hat{K}^{1-2T}}{2T-1}.$$

For $T = 1$ we have that

$$E\left[\frac{1}{K}\right] = \frac{1}{\ln(\hat{K}/\xi)} \int_{\xi}^{\hat{K}} K^{-2} dK = \frac{1}{\ln(\hat{K}/\xi)} \left(\frac{1}{\xi} - \frac{1}{\hat{K}} \right).$$

Similarly, it follows that the posterior density for J can be written as

$$\pi(J|n_{t_j}) \propto J^{-3}(t_1 - J)^{-2} \quad \text{on} \quad \left(t_1 - \frac{n_{t_1}}{\xi}, t_1 - \frac{n_{t_1}}{\hat{K}}\right). \tag{31}$$

Define

$$I_{m,n} = \int \frac{dx}{x^m(bx+c)^n}. \tag{32}$$

We have that (32) satisfies the recursion (Riley, Hobson, & Bence, 2010b)

$$-c(m-1)I_{m,n} = \frac{1}{x^{m-1}(bx+c)^{n-1}} + b(m+n-2)I_{m-1,n}. \tag{33}$$

Since $\pi(J|n_{t_j})$ is a normalized probability density we need to find the normalisation constant in order to calculate $E[J|n_{t_j}]$. It follows from (31)–(33) with $m = 3$, $n = 2$, $b = -1$ and $c = t_1$ that

$$-2t_1 I_{m,n} = \left[\frac{1}{J^2(t_1 - J)}\right]_B^A - 3I_{m-1,n}. \tag{34}$$

Next divide (34) by $-2t_1$Const. where Const. denotes the normalisation constant satisfying

$$\frac{[I_{m,n}]_B^A}{\text{Const.}} = 1.$$

Thus, we obtain

$$1 = -\frac{1}{2t_1\text{Const.}}\left[\frac{1}{A^2(t_1 - A)} - \frac{1}{B^2(t_1 - B)}\right] + \frac{3}{2t_1}E[J];$$

$$E[J] = \frac{2t_1}{3} + \frac{1}{3\text{Const.}}\left[\frac{1}{A^2(t_1 - A)} - \frac{1}{B^2(t_1 - B)}\right].$$

Next, we need to reapply the Recursion (33) in order to find the normalisation constant. From (33) we have that

$$I_{3,2} = -\frac{1}{2t_1}\left[\frac{1}{J^2(t_1 - J)}\right]_B^A + \frac{3}{2t_1}I_{2,2}, \tag{35}$$

and $I_{3,2}^{-1}$ is the appropriate normalisation constant. Using partial fractions

$$I_{2,2} = \int_B^A \frac{dJ}{J^2(t_1 - J)^2} = \frac{2}{t_1^3}\ln\left(\frac{A(t_1 - B)}{B(t_1 - A)}\right) + \frac{A - B}{t_1^2}\left[\frac{1}{BA} + \frac{1}{(t_1 - A)(t_1 - B)}\right], \tag{36}$$

Combining Eqs. (35)–(36) Eq. (28) follows. □

References

PLAN:Personal Localized Alerting Network [http://transition.fcc.gov/pshs/services/plan.html].

Abdelghany, A., Abdelghany, K., Mahmassani, H., & Alhalabi, W. (2014). Modelling framework for optimal evacuation of large-scale crowded pedestrian facilities. *European Journal of Operational Research, 237*, 1105–1118.

Allan, S., & Thorsen, E. (Eds.). (2009). *Citizen journalism: global perspectives*. New York: Peter Lang.

American Red Cross (2010). American Red Cross Survey: social media in disasters and emergencies.

Annlin Jin, S. V., & Paramasivan, B. (2012). False data injection attack and its countermeasures in wireless sensor networks. *European Journal of Scientific Research, 82*, 248–257.

Balcik, B., & Beamon, B. M. (2008). Facility location in humanitarian relief. *International Journal of Logistics Research and Applications, 11*, 101–121.

Bekhor, S., Cohen, Y., & Solomon, C. (2013). Evaluating long distance travel patterns in israel by tracking cellular phone positions. *Journal of Advanced Transportation, 47*, 435–446.

Bernardo, J. M., & Smith, A. F. M. (2000). *Bayesian Theory*. Chichester: Wiley.

Binner, J. M., Nasu, B., Simm, W., Ferrario, M. A., Whittle, J. Ariyatum, B. (2013). Evaluating public confidence in policing services; a new decision support tool. Preprint.

Bird, D., Ling, M., & Haynes, K. (2012). Flooding facebook the use of social media during the queensland and victorian floods. *Australian Journal of Emergency Management, 27*, 27–33.

Bish, D. R., & Sherali, H. D. (2013). Aggregate-level demand management in evacuation planning. *European Journal of Operational Research, 224*, 79–92.

Bonen, Z. (1979). Evolutionary behaviour of complex socio-technical systems. *Working Paper*. Alfred P. Sloan School of Management, MIT.

Bozorgi-Amri, A., Jabalameli, M. S., Mirzapour, S. M. J., & e Hashem, A. (2013). A multi-objective robust stochastic programming model for disaster relief logistics under uncertainty. *OR Spectrum, 35*, 905–933.

Bretschneider, S., & Kimms, A. (2012). Pattern-based evacuation planning for urban areas. *European Journal of Operational Research, 216*, 57–69.

Brocklesby, J. (2009). Ethics beyond the model: how social dynamics can interfere with ethical practice in operational research/management science. *Omega, 37*, 1073–1082.

Burstedde, C., Klauck, K., Schadschneider, A., & Zittarz, J. (2001). Simulation of pedestrian dynamics using a two-dimensional cellular automaton. *Physica A, 295*, 507–525.

Busayawan, A., Whittle, J. N., Binner, J. M., & Lawlor-Wright, T. (2015). Better service design for greater civic engagement. *The Design Journal*.(forthcoming)

Campbell, A. M., & Jones, P. C. (2011). Prepositioning supplies in preparation for disasters. *European Journal of Operational Research, 209*, 156–165.

Chan, K. C. (1988). On the contrarian strategy. *The Journal of Business, 61*, 147–163.

Chen, Y., & Xiao, D. (2008). Real-time traffic management under emergency evacuation based on dynamic traffic assignment. In *Proceedings of the ieee international conference on automation and logistics (ical) 2008* (pp. 1376–1380).

Chen, Z.-Y., Fan, Z.-P., & Sun, M. (2015). Behavior-aware user response modelling in social media: Learning from diverse heterogeneous data. *European Journal of Operational Research, 241*, 422–434.

Chiu, Y.-C., Zheng, H., Villalobos, J., & Gautam, B. (2007). Modelling no-notice mass evacuation using a dynamic traffic flow optimization model. *IEE Transactions, 39*, 83–94.

Cliff, D., & Northrop, L. (2012). The global financial markets: an ultra-large-scale systems perspective. In R. Calinescu, & D. Garlan (Eds.), *Monterey workshop 2012, lncs 7539* (pp. 29–70).

CSaP (2014). [http://www.csap.cam.ac.uk/media/uploads/files/1/policychallenges-booklet-web-final.pdf]. Policy challenges briefing: emergency behaviours, Centre for Science and Policy, University of Cambrdige

Drake, M. J., Gerde, V. W., & Wasieleski, D. M. (2009). Socially responsible modelling: a stakeholder approach to the implementation of ethical modeling in operations research. *OR Spectrum, 33*, 1–26.

Ferscha, A., & Zia, K. (2010). Crowd evacuation based on vibro-tactile guidance. *Pervasive Computing, 9*, 33–42.

Fry, J. (2012). Exogenous and endogenous shocks as phase transitions in complex financial systems. *European Physical Journal B, 85*, 405.

Fry, J., Galla, T., & Binner, J. M. (2014). Quantitative decision making rules for the next generation of smarter evacuations. In J. Preston, J. M. Binner, L. Branicki, T. Galla, N. Jones, J. King, M. Kolokitha, & M. Smyrnakis (Eds.), *City Evacuations: an interdisciplinary approach* (pp. 63–88). Springer.

Galla, T. (2011). Optimizing evacuation flow in a two-channel exclusion process. *Journal of Statistical Mechanics: Theory and Experiment, 2011*, P09004.

Gosling, J. P., Oakley, J. E., & O'Hagan, A. (2007). Non-parametric elicitation for heavy-tailed prior distributions. *Bayesian Analysis, 2*, 693–718.

Hämäläinen, R., Luoma, J., & Saarinen, E. (2013). On the importance of behavioural operational research: the case of understanding and communcating about dynamic systems. *European Journal of Operational Research, 228*, 623–634.

Hamel, G., & Välikangas, L. (2003). The quest for resilience. *Harvard Business Review, 81*, 52–63.

Helbing, D., Farkas, I., & Vicsek, T. (2000). Simulating dynamical features of escape panic. *Nature, 407*, 487–490.

Highway Code (2014). The United Kingdom Highway Code. https://www.gov.uk/highway-code.

Hollnagel, E., Pariès, J., Woods, D. D., & Wreathall, J. (Eds.). (2011). *Resilience engineering in practice: a guidebook*. Farnham, Surrey: Ashgate.

Jarvis, J. J., & Ratliff, H. D. (1982). Some equivalent objectives for dynamic network flow problems. *Management Science, 28*, 106–109.

Kerner, B. (2004). *The physics of traffic*. Springer.

Kolfal, B., Patterson, R. A., & Yeo, L. M. (2013). Market impact on it security spending. *Decision Sciences, 44*, 517–556.

Kosala, R., & Adi, E. (2012). Harvesting real time traffic information from twitter. *Procedia Engineering, 50*, 1–11.

Lim, G. J., Zangeneh, S., Baharnemati, M. R., & Assavapokee, T. (2012). A capacitated network flow optimization approach for short notice evacuation planning. *European Journal of Operational Research, 223*, 234–245.

Lindsay, B. R. (2011). Social media and disasters: current uses, future options and policy considerations. Congressional research service report for Congress R41987.

Luoma, J., Hämäläinen, R., & Saarinen, E. (2010). Acting with systems intelligence: integrating complex responsive processes with the systems perspective. *Journal of the Operational Research Society, 62*, 3–11.

Mayer, R. E., & Davidson, J. E. (1995). The search for insight: Grappling with Gestalt psychology's unanswered questions. In J. E. Davidson (Ed.), *The nature of insight* (pp. 3–32). Cambridge, Massachussets: MIT Press.

Mohan, P., Padmanabhan, V. N., & Ramjee, R. (2008). Nericell: Rich monitoring of road and traffic conditions using mobile smartphones. In *Proceedings of the 6th acm conference on embedded network sensor systems* (pp. 323–336).

Muir, H. C., Bottomley, D. M., & Marrison, C. (1996). Effects of motivation and cabin configuration on emergency aircraft evacuation behaviour and rates of egress. *International Journal of Aviation Psychology, 6*, 57–77.

Muth, J. A. (1961). Rational expectations and the theory of price movements. *Econometrica, 29*, 315–335.

Nakajima, Y., Yamane, S., Hattori, H., & Ishida, T. (2008). Evacuation guide system based on massively multi-agent systems. In *Proceedings of the 7th international joint conference on autonomous agents and multi-agent systems, 2008* (pp. 1653–1654).

Ni, M., He, Q. Gao, J. (2014). Using Social Media to predict traffic flow under special event conditions. Preprint.

ODA (2011). Westfield Opening Observation Exercise. In *Unpublished report*. London: Olympic Delivery Authority.

Oh, O., Agrawal, M., & Rao, H. R. (2011). Information control and terrorism: tracking the mumbai terrorist attack through twitter. *Information Systems Frontiers, 13*, 33–43.

Oxendine, C. E., & Waters, N. (2014). No-notice urban evacuations: using crowdsourced mobile data to minimize risk. *Geography Compass, 8*, 49–62.

Palen, L. (2008). Online social media in crisis events. *Educase Quarterly, 31*, 76–78.

Pauls, J. L. (1995). Movement of people. In P. DiNenno, J. Beyler, C. L., R. L. P. Custer, W. D. Walron, J. M. Watts Jr., D. Drysdale, & J. R. Hall Jr. (Eds.), *SFPE handbook of fire protection engineering* (2nd). Quincy Massachussets: National Fire Protection Association.

Preston, J., Binner, J. M., Branicki, L., Ferrario, M., & Kolokitha, M. (2011). Multiple attacks on transport infrastructure: an inter-disciplinary exploration of the impact of social networking technologies upon real time information sharing and recovery. *Journal of Homeland Security*.

Preston, J., Binner, J. M., Branicki, L., Galla, T., & Jones, N. (Eds.). (2014b). *City evacuations: an interdisciplinary approach*. Springer.

Preston, J., Branicki, L., & Binner, J. M. (2014a). *Real time social media analysis in emergency management: written evidence* p. SMD0005. Published by the House of Commons Science and Technology Select Committee, May 2014.

Riley, K. F., Hobson, M. P., & Bence, S. J. (2010b). *Mathematical methods for physics and engineering.*. Cambridge: Cambridge University Press.

Roggen, D., Wirz, M., Troster, G., & Helbing, D. (2011). Recognition of crowd behavior from mobile sensors with pattern analysis and graph clustering methods. *Networks and Heterogeneous Media, 6*, 521–544.

Sakaki, T., Matsuo, Y., Yanagihara, T., Chandrasiri, N. P., & Nawa, K. (2012). Real-time event extraction for driving information from social sensors. In *Proceedings of the 2012 ieee international conference on cyber technology in automation, control and intelligent systems* (pp. 221–226).

Schadschneider, A., Eilhardt, C., Nowak, S., Wagoum, A. U. K., & Seyfried, A. (2011). HERMES: An evacuation assistant for large sports arenas based on microscopic simulations of pedestrian dynamics. In V. V. Kozlov, A. P. Buslaev, A. S. Bugaev, M. V. Yashina, A. Schadschneider, & M. Schreckenburg (Eds.), *Traffic and Granular Flow '11* (pp. 287–298). Springer.

Schadschneider, A., Klingsch, W., Klüpfel, H., Kretz, T., Rogsch, C., & Seyfried, A. (2008). Evacuation dynamics; emprical results, modelling and applications. In B. Meyers (Ed.), *Encyclopaedia of complexity and system science*. Berlin: Springer.

Schadschneider, A., Pöschel, T., Kühne, R., Schreckenberg, M., & Wold, D. E. (Eds.). (2006). *Traffic and granular flow*. Berlin: Springer.

Schmidt, V. A., & Binner, J. M. (2014). A semi-automated display for geotagged text. In J. Preston, J. M. Binner, L. Branicki, T. Galla, & N. Jones (Eds.), *City evacuations: an interdisciplinary approach* (pp. 107–116). Springer.

Schreckenberg, M., & Sharma, S. D. (Eds.). (2002). *Pedestrian and evacuation dynamics*. Berlin Hiedelberg: Springer.

Sheffield City Council (2013). Strategic Environmental Assessment: Sheffield local flood risk management strategy, Version: 01. [https://www.sheffield.gov.uk/environment/flooding/strategy.html].

Simm, W., Ferrario, M. A., Gradinar, A., & Whittle, J. (2014). Prototyping 'clasp'. implications for designing digital technology for and with adults with autism. *DIS 14 June 21–25 Vancouver, BC, Canada*, 345–354.

Smyrnakis, M., & Galla, T. (2012). Effects of communication and utility-based decision making in a simple model of evacuation. *European Physical Journal B, 85*, 378.

Tamura, H. (2005). Behavioral models for complex decisions. *European Journal of Operational Research, 166*, 655–665.

Thompson, C., White, J., Dougherty, B., Albright, A., & Schmidt, D. C. (2010). Using smartphones to detect car accidents and provide situational awareness to emergency responders. In Y. T. MingluLi, & J. X. C. Giannelli (Eds.), *Mobile Wireless Middleware, Operating Systems, and Applications* (pp. 29–42). Berlin Heidelberg: Springer.

Vojnović, M., Gupta, V., Karagiannis, T., & Gkantsidis, C. (2008). Sampling strategies for epidemic-style information dissemination. In *Proceedings of the Ieee infocom 2008*.

Yi, W., & Kumar, A. (2007). Ant colony optimization for disaster relief operations. *Transportation Research Part E, 43*, 660–672.

Zheng, X., Zhong, T., & Liu, M. (2009). Modelling crowd evacuation of a building based on seven methodological approaches. *Building and Environment, 44*, 437–445.

Zobel, C. W. (2014). Quantitatively representing nonlinear disaster recovery. *Decision Sciences, 45*, 1053–1082.

Zobel, C. W., & Khansa, L. (2012). Quantifying cyberinfrastructure resilience against multi-event attacks. *Decision Sciences, 43*, 687–710.

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