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An Integrated Metro Operation Optimization to Minimize Energy Consumption

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Abstract

Energy efficient techniques are receiving increasing attention because of rising energy prices and environmental concerns. Railways, along with other transport modes, are facing increasing pressure to provide more intelligent and efficient power management strategies.

This paper presents an integrated optimization method for metro operation to minimize whole day substation energy consumption by calculating the most appropriate train trajectory (driving speed profile) and timetable configuration. A train trajectory optimization algorithm and timetable optimization algorithm are developed specifically for the study. The train operation performance is affected by a number of different systems that are closely interlinked. Therefore, an integrated optimization process is introduced to obtain the optimal results accurately and efficiently.

The results show that, by using the optimal train trajectory and timetable, the substation energy consumption and load can be significantly reduced, thereby improving the system performance and stability. This also has the effect of reducing substation investment costs for new metros.

Index Terms – Computer simulation; integrated optimization; railway operation; rail transportation.

1 Introduction

As urban populations have grown significantly over the past decade, metro systems have gained in popularity because of their convenience, efficiency and speed. In the meantime, metro operators are facing ever more pressure to save energy due to increasing environmental concerns. As two of the main foundations of metro operation, the train trajectory and timetable play a key role in metro energy consumption. An energy-efficient timetable is able

to minimize substation energy usage by taking full advantage of train regenerative braking energy. Appropriate train trajectories between stations can also provide a means of minimizing energy consumption during train operation. However, the timetable and train trajectory are not independent elements of metro operation and so should be considered jointly.

A number of researchers have studied various methods designed to improve railway operation performance. Chang introduced an appropriate coasting control method to optimize train movement using a genetic algorithm [1]. Bocharnikov presented a novel approach to calculate the best train coasting operation using a mixed searching method including a genetic algorithm in combination with fuzzy logic [2]. Lu developed a distance-step single train movement model, and implemented one exact algorithm (dynamic programming) and two exhaustive search methods (an ant colony optimization and a genetic algorithm) to optimize a single train trajectory. A comparison of the results has shown that the exact algorithm produces more accurate results but with a longer computation time than the exhaustive search methods [3]. In order to reduce the searching time, a number of researchers have developed mathematical models and computer programs to optimize the single train trajectory from a theoretical point of view [4-6]. The authors have previously presented a multiple train trajectory optimization paper to consider the balance between energy consumption and train delays [7]. However, only a small number of trains were included in the methodology. Therefore, the network is too small to be considered as a timetable. Methods have been proposed to obtain optimal synchronized timetables to minimize waiting times for passengers when transferring to other lines, or onto buses [8, 9]. Yang proposed a scheduling approach to optimize the metro timetable so that the regenerative braking energy from braking trains could be directly used by motoring trains within the same power network [10]. Bin presented an integrated method to optimize train headway by adjusting the train arrival time at platforms to improve train headway regulation [11]. The use of train regenerative braking is recognized as the main method to improve railway energy efficiency [12, 13]. In order to achieve a global optimality of driving strategy and optimal timetable, Shuai Su analyzed a hierarchy of energy-efficient train operation and proposed an integrated algorithm to generate a globally optimal operation schedule [14, 15]. Xiang and Hong developed a joint model to optimize timetable and train speed profile based on Genetic Algorithm. The results show at the maximum energy saving rate is around 25% [16, 17].

Most of the previous works have discussed train optimization for single-objective problems. In practice, train operation performance is affected by a number of different systems that are closely interlinked. For example, the inter-station journey time plays a key role in not only the

train trajectory optimization (energy-saving purpose), but also the timetable optimization (regenerative braking efficiency purpose). Therefore, the calculation of the inter-station journey time should be considered by both optimizations simultaneously. Furthermore, the timetable optimization should consider the performance of all the trains in the whole network in order to take the full advantage of the train regenerative braking. An integrated optimization method is therefore developed for this purpose.

In this paper, a vehicle movement modeling is first presented, followed by a description of the proposed integrated optimization method, which includes train trajectory optimization and timetable optimization. The aim of the method is to find the train movement mode sequence, inter-station journey times, and service intervals, which minimize the substation energy consumption for a whole day of metro operation.

2 Vehicle Movement Modeling

It is first necessary to consider the fundamental physics of train motion in order to develop the optimization algorithms. The methods used to solve the dynamic movement equations are based on the equations of motion of the railway vehicle subject to the constraints imposed on the vehicle by the route and driving style [18-20]. The general equation of vehicle motion, known as Lomonossoff's equation, can be written as Equation (1), which is based on Newton's second law of motion.

$$\begin{cases} M_{tr}\dfrac{d^2s}{dt^2} = F(v) - R(v) - F_{grad} \\ R(v) = a + b|v| + cv^2 \\ F_{grad} = M_{rs}gsin(\alpha) \\ M_{tr} = M_{rs}(1+\lambda_w) + M_p \end{cases} \tag{1}$$

where M_{tr} is the effective mass; M_{rs} is the rolling stock mass; M_p is the passenger mass; s is the train position; t is the time; v is the train speed; α is the gradient angle; λ_w is the rotary allowance; F is the traction force or braking force depending on the movement mode; F_{grad} is the force due to the gradient. R is the resistive force, the constants a, b, c being empirical and related to the track and aero-dynamic resistance known as the Davis equation [21].

In the vehicle movement model, time is the dependent variable. The state equation of the train motion can be presented as shown in Equation (2).

$$\begin{cases} \dot{s} = v \\ M_{tr}\dot{v} = u_f \cdot F_{tr}(v) - u_b \cdot F_{br}(v) - R(v) - F_{grad}(s) \end{cases} \tag{2}$$

where u_f and u_b are the control signals for forwards traction effort and backwards braking effort respectively; F_{tr} is the traction force; $F_{br}(v)$ is the braking effort at the current vehicle speed $v(t)$. The boundary condition, initial condition, final conditions are imposed as follows:

$$\begin{cases} v(0) = 0, s(0) = 0 \\ v(T) = 0, s(T) = S_t \end{cases} \tag{3}$$

where S_t is the train position at the terminal station.

Some other constraints are shown as follows:

$$\begin{cases} v \le v_{\lim}(s) \\ u_f \in [0,1] \\ u_b \in [0,1] \end{cases} \tag{4}$$

where $v_{lim}(s)$ is the train target speed or line speed limit (whichever is smaller) at the current position s.

Figure 1 and Table 1 show the four typical movement modes for train motion. In the motoring mode, traction power is used to achieve required acceleration rates to increase the train speed. In the cruising mode, the traction power is used to overcome the resistance and the effects of the gradient so that the train can keep at a constant speed. In the coasting mode, the traction power is shut down so the train speeds only affected by the resistance and the effects of the gradient. In the braking mode, the train applies the service brake or emergency brake to reduce the speed in order to stop at a station or a signal.

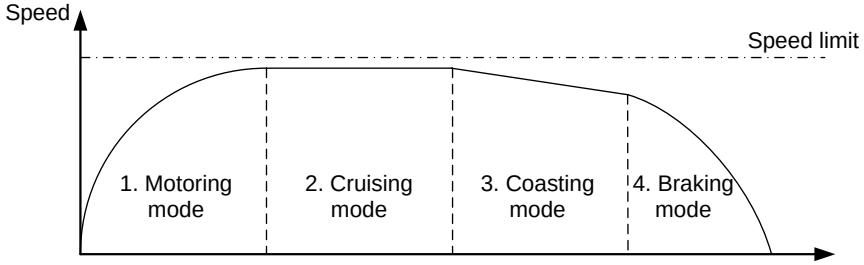

Figure 1. Four train movement modes.

In the coasting and braking modes, the forward tractive effort control signal equals zero and the tractive power is shut down. Therefore, there is no energy consumption in these modes. Furthermore, the backward braking effort control signal equals one in braking mode.

Table 1. Control signals in different movement modes.

Mode	u_f	u_b	Equations (5)
Motoring	1	0	$M_{tr}\dot{v} = F_{tr}(v) - R(v) - F_{grad}(s)$
Cruising	1	0	$M_{tr}\dot{v} = F_{tr}(v) - R(v) - F_{grad}(s) = 0$
Coasting	0	0	$M_{tr}\dot{v} = -R(v) - F_{grad}(s)$
Braking	0	1	$M_{tr}\dot{v} = -F_{br}[v(t)] - R(v) - F_{grad}(s)$

3 Operation Optimization

In the railway simulation, single train operation and multiple train operation are closely dependent parts, shown in Figure 2. Firstly, the inter-station journey times produced from the multiple train operation are an important constraint in the single train operation. Secondly, the train trajectory and the energy consumption calculated from the single train operation are the foundations of the multiple train operation. Therefore, the optimization in this study should integrate both single train trajectory optimization and timetable optimization, which are introduced in Sections 3.1 and 3.2 respectively.

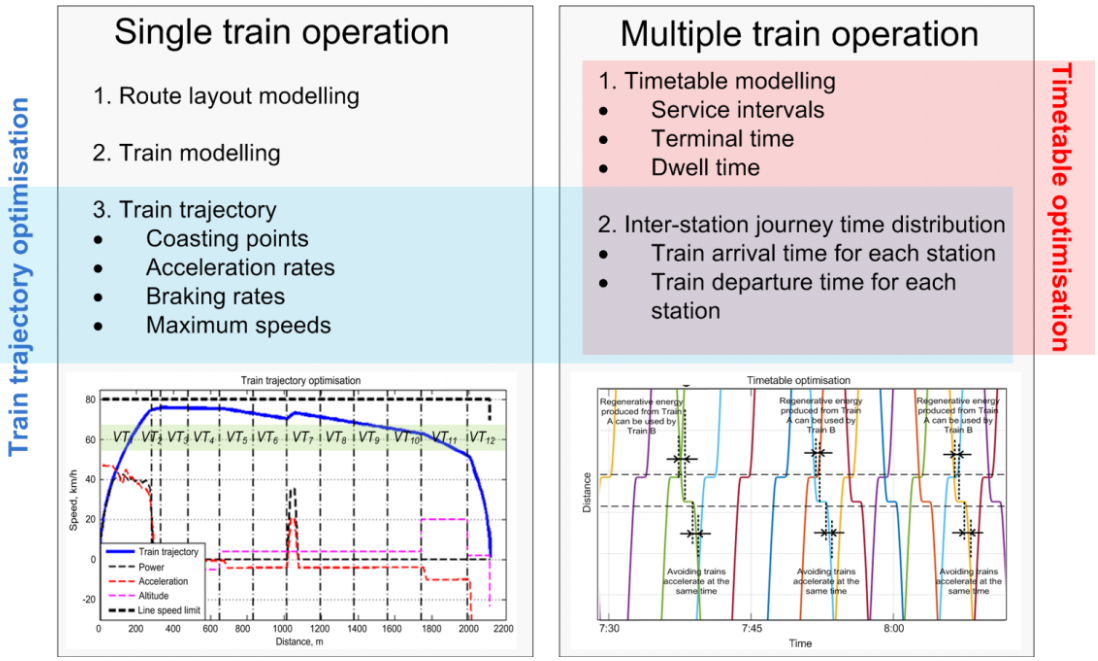

Figure 2. Train operation simulation and optimization.

3.1 Train Trajectory Optimization Algorithm

This section introduces a train trajectory optimization algorithm to minimize train traction energy consumption. In this study, the route is divided into a number of sections depending on the section length, gradient changes and line speed limit changes, as shown in Figure 3 (dot dash lines). The train trajectory and running performance can be controlled by using different movement modes in each section, as shown in Figure 3.

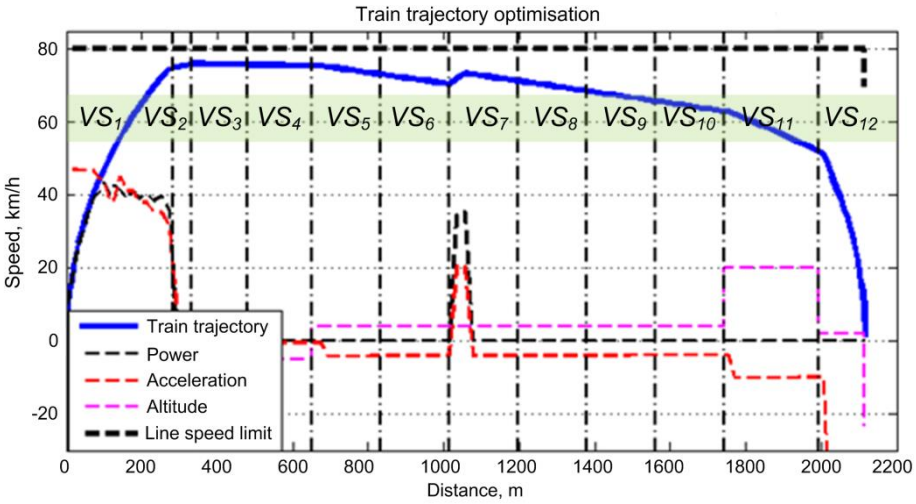

Figure 3. Train trajectory optimization.

The aim of the train trajectory optimization in this study is to calculate the most appropriate train movement mode sequence (VS=[VS_1, VS_2,…, VS_{si}]) to minimize train energy usage within a given scheduled single journey time (T_{sch}). The fitness function is shown as follows:

$$\begin{cases} \min \quad M_{opt} = E_{single} F_e, \ \ if\ T_{single} = T_{sch} \\ \qquad [IT, E_{it}] = g(VS) \end{cases} \tag{6}$$

where M_{opt} is the train traction energy composition that needs to be optimized for a single journey; F_e is the unit energy cost per kWh; IT is the inter-station journey time; E_{it} is the inter-station energy consumption; E_{single} and T_{single} are the train energy consumption and journey time of a single journey, which can be further calculated using Equation (7).

$$\begin{cases} T_{single} = \sum_{i=1}^{sn} (IT_i), \quad if\ |IT_i - IT_{si}| \in [0, IT_r] \\ E_{single} = \sum_{i=1}^{sn} \int_0^{IT_i} f[v(t)]v(t)dt \end{cases} \tag{7}$$

where $f[v(t)]$ is the maximum tractive effort at the current vehicle speed $v(t)$; si is the number of sections; sn is the number of stations; IT_s is the scheduled inter-station journey time; IT_r is the maximum variation between scheduled journey time and optimal journey time;

3.2 Timetable Optimization Algorithm

When air braking systems are used to slow down the vehicles by mechanical braking, the vehicle kinetic energy is converted to thermal energy and thus wasted. However, modern metro railways are usually equipped with regenerative braking systems, which work as an energy recovery mechanism. Such systems slow down the vehicles by converting kinetic energy to electrical energy, which can be used by other vehicles immediately via the power network, or stored if energy storage systems are provided. Regenerative braking systems improve the overall energy efficiency of metro networks and play a key role in timetable optimization.

Unfortunately, the application of energy storage systems is currently limited because of the high weight of batteries, short battery life, and insufficient overload capacity [22]. Therefore, most metro railways are not equipped with energy storage systems. If the regenerated electrical braking energy cannot be used immediately by other trains, the electricity will be converted into heat using resistances. Therefore, in this study, the aim of the algorithm is to create an optimal timetable to take full advantage of the regenerative braking energy. The timetable should meet the following requirements:

1. Braking synchronization: If a train is braking while another train in the same power supply network is motoring, the regenerative braking energy produced from the braking train can be used by the motoring train instantly, thereby reducing the overall energy consumption, as shown in Figure 4. The braking train and motoring train pair is defined as a braking synchronized group (*BSG*). The overlapping time of braking and motoring is defined as a braking synchronized time (*BST*). Due to the power supply network characteristic and transmission loss, the distance between the pair of trains should be as small as possible;
2. Motoring synchronization: If a train is motoring while another train in the same power supply network is also motoring at the same time, the substation load will become heavier due to the increase in power demand, as shown in Figure 4. The pair of motoring trains is defined as a motoring synchronized group (*MSG*). The overlapping time is defined as a motoring synchronized time (*MST*). However, due to

substation limitations and power flow stability concerns, trains should avoid motoring at the same time, especially trains that are very close to each other;

3. The proposed optimization aims to increase the braking synchronized groups and time to improve energy saving, and decrease the motoring synchronized groups and time to reduce the substation load.

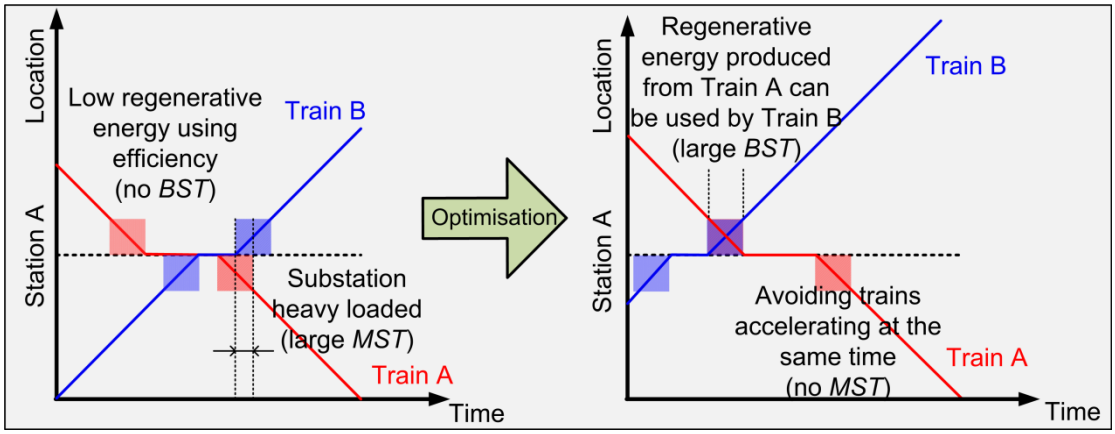

Figure 4. Regenerative braking energy usage comparison.

In this study, the optimal timetable aims to achieve a trade-off between the braking synchronization and the motoring synchronization by searching for the most appropriate inter-station journey time (*IT*) and service intervals (*ST*) with a fixed total journey time and dwell time. The fitness functions are shown as follows:

$$\begin{cases} \min\ SYT_{opt} = -\sum_{i=1}^{BSGn}(BST_i \times w_{bst}) + \sum_{j=1}^{MSGn}(MST_j \times w_{mst}) \\ [BSGn, BST_{set}, MSGn, MST_{set}] = f(IT_{set}, ST_{set}) \\ w_{bst} = \dfrac{sn}{|sn_A - sn_B|}, when\ Train\ A\ is\ braking\ and\ Train\ B\ is\ motoring \\ w_{mst} = \dfrac{sn}{|sn_A - sn_B|}, when\ both\ Train\ A\ \ and\ Train\ B\ are\ motoring \end{cases} \quad (8)$$

where SYT_{opt} is the total synchronized time that needs to be minimum; *BSGn* and *MSGn* are the number of braking synchronized groups and motoring synchronized groups respectively; sn_A and sn_B are the station ID which Train A and Train B are approaching to or leaving from respectively; the ω_{bst} and ω_{mst} are the weightings that are associated with the *BST* and *MST*. The weightings to each group may vary depending on the distance between the synchronized trains. For example, if the distance between the trains is large, the electricity transmission loss will become high, thus a small weighting will be calculated, as shown in Equation (8). The weighting becomes smallest if the two trains are stopping at two terminal stations at two ends

(very far). The number becomes greatest if the two trains are stopping at the same station (very closed).

The journey time and constraints are shown as follows:

$$\begin{cases} T_{single} = \sum_{i=1}^{sn} (IT_i), \quad if\ |IT_i - IT_{si}| \in [0, IT_r] \\ T_{day} = \sum_{i=1}^{tn} \left(T_{single} + ST_i\right), \quad if\ |ST_i - ST_{si}| \in [0, ST_r], where\ T_{day} = T_{sday}, ST \geq HD_{lmin} \end{cases} \tag{9}$$

where *tn* is the number of trains running in the network per day; T_{day} and T_{sday} are the simulated day journey time and scheduled day journey time for a whole day operation; ST_s is the scheduled service interval; ST_r is the maximum variation between scheduled service interval time and optimal service interval; HD_{lmin} is the minimum line headway time. The service interval should be larger than the minimum line headway time in order to avoid any train interactions.

As shown in Equations (9), variations (ST_r and IT_r) in the inter-station journey times and service intervals are applied in order to minimize the impact of the timetable rescheduling. Furthermore, it is important to note the dwell time will not be changed in the optimization in this study because the dwell time is specifically chosen to meet the passenger flow demand.

3.3 Optimization Integration

As shown in Equations (7) and (9), it is important to note that the inter-station journey time plays a key role in both the train trajectory optimization and the timetable optimization. Therefore, these two optimization objectives are expected to be processed at the same time, as an integrated optimization.

In this study, the integrated optimization aims to achieve minimum whole day substation energy consumption by searching for the most appropriate inter-station journey time (*IT*), service intervals (*ST*) and movement mode sequence (*VS*) with fixed journey times and dwell time. The fitness functions are shown as follows:

$$\begin{cases} \min\ E_{day} = (\sum_{i=1}^{tn} E_{singlei} - E_{reg} + E_{loss})F_e \\ T_{day} = \sum_{i=1}^{tn} \left(T_{single} + ST_i\right), if\ |ST_i - ST_{si}| \in [0, ST_r], where\ T_{day} = T_{sday}, ST \geq HD_{lmin} \\ [IT, E_{it}] = g(VS) \rightarrow OPTtrajectory \\ [BSGn, BST_{set}, MSGn, MST_{set}, E_{loss}] = f'\left(IT_{set}, ST_{set}, E_{single}, VS\right),\ IT \in OPTDATA_{trajectory} \end{cases} \quad (10)$$

where E_{reg} and E_{loss} are the effective regenerative braking energy and transmission loss of a single train respectively; function g and f' represent for train trajectory process and timetable process. Some constraints are shown follows:

$$\begin{cases} T_{single} = \sum_{i=1}^{sn} (IT_i),\ if\ |IT_i - IT_{si}|\ \in [0, IT_r],\ \ IT \in OPTDATA_{trajectory} \\ E_{single} = \sum_{i=1}^{sn} \int_0^{IT_i} f[v(t)]v(t)dt \\ E_{reg} = \sum_{i=1}^{BSG_n} \int_0^{BST_i} I_{re}(t)V_{re}(t)dt \\ E_{loss} = \sum_{i=1}^{tn} \int_0^{T_{single}} I_{tr}^2(t)R_{trans}dt \\ SYT = -\sum_{i=1}^{BSGn} (BST_i \times w_{bst}) + \sum_{j=1}^{MSGn} \left(MST_j \times w_{mst}\right) \end{cases} \quad (11)$$

where I_{re} and V_{re} are regenerative braking current and voltage respectively; I_{tr} is the traction current; R_{trans} is the transmission resistance; $OPTDATA_{trajectory}$ is a database created by the train trajectory process. The database includes optimal train trajectory results for every possible inter-station journey time (e.g. from 70 seconds to 85 seconds) for each inter-station running (e.g. a train running on a route with 9 stations will have 8 inter-station running).

Due to the complexity of such an integrated optimization problem, metaheuristic methods such as Genetic Algorithms (GA) are often applied to search for the optimum results. However, as shown in Figure 3, in this study, each inter-station journey is divided into a number of sections (e.g. 10 sections). A typical metro railway line usually contains 10 stations (9 inter-station journeys), therefore at least 90 variables need to be optimized in the single train trajectory optimization. Furthermore, as shown in Equation (10), gaining an optimal

timetable for such a typical metro line requires optimizing at least another 24 variables (20 different inter-station journey times for two directions and 4 different service intervals). Consequently, if implementing a genetic algorithm to solve such a complex integration optimization problem, the algorithm could easily obtain a local rather than global optimum.

Therefore, in this study, the train trajectory optimization and timetable optimization are processed separately. However, as the two optimizations are closely related to each other, it is necessary to ensure the train trajectory optimization produces all the potential trajectory results to be further used in the timetable optimization. Firstly, optimal train trajectory solution for every possible inter-station journey time for each inter-station running will be calculated and stored in a Database. Secondly, an optimal timetable will be produced considering the train synchronization performance and the results from the developed Database. Finally, the train whole day movement and the network energy consumption can be calculated using the corresponding optimal train trajectory and optimal timetable, as shown in Figure 5.

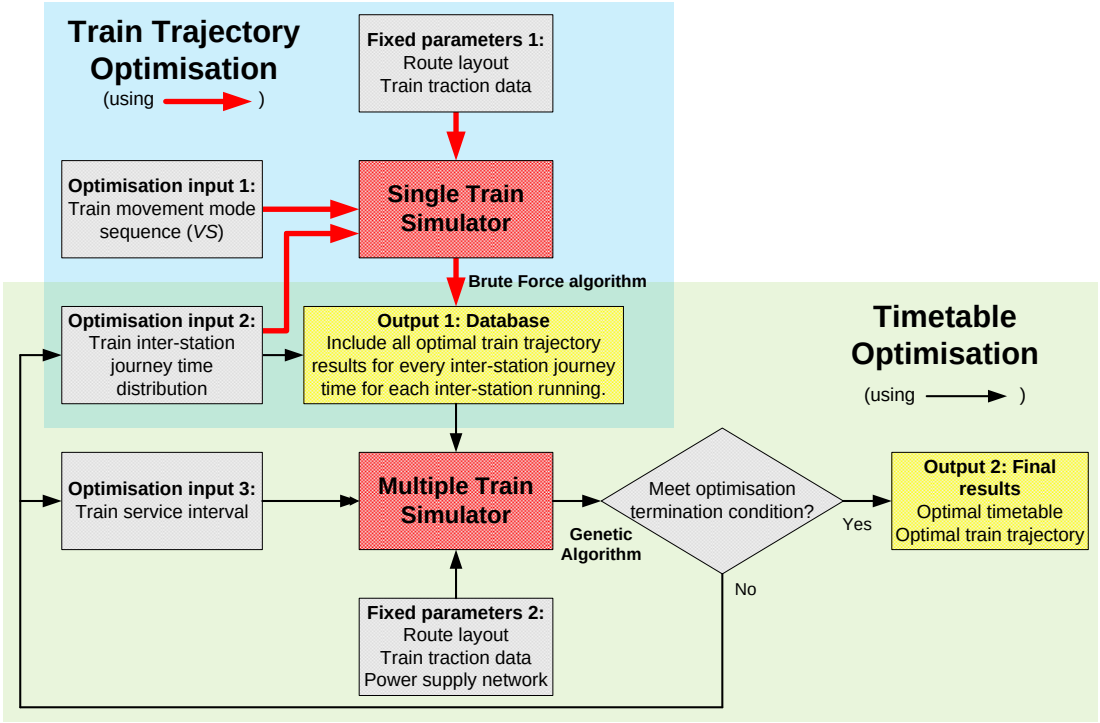

Figure 5. Flow chart of the integrated optimization.

In order to achieve this objective, in the single train trajectory optimization, a brute force algorithm is implemented. Such an algorithm enumerates all possibilities in the solution domain to find the optimum [7, 23]. The details are shown as follows:

Step 1: All possible solutions (movement mode sequences) will be implemented to calculate the train energy consumption and the journey time using Equation (10). The results along with the movement mode sequences will be stored in the database ($OPTDATA_{trajectory}$):

$$[IT, E_{it}] = \sum_{M_1=1}^{4} \sum_{M_2=1}^{4} \dots \sum_{M_{si}=1}^{4} g(VS) \tag{12}$$

$$OPTtrajectory \leftarrow [IT, E_{it}, VS] \tag{13}$$

where M is the movement mode code for each section; si is the number of sections.

Step 2: The database may include a number of solutions with the same inter-station journey time (IT) but different energy consumptions (E_{it}). This step aims to search the optimal movement mode sequence for every possible inter-station journey time for each inter-station running, as shown in Figure 6. The solutions in the database will be sorted by journey time and then energy consumption. Assume there are δ solutions in $OPTDATA_{trajectory}$, if:

$$E_{it\eta} \geq E_{it\eta-1} \; and \; IT_{\eta} = IT_{\eta-1}, \eta \in \delta \tag{14}$$

Then the solution η will be discarded as it is not the optimal solution for the inter-station journey time IT_{η}.

Step 3: After Step 2, only the optimal solution for every possible inter-station journey time for each inter-station running has been retained. The database will be implemented in the following timetable optimization.

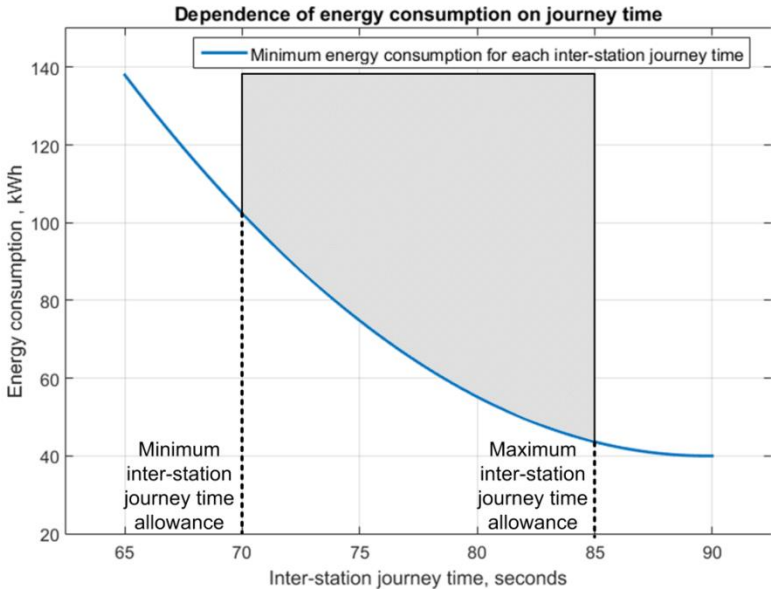

Figure 6. Dependence of energy consumption on journey time.

In the timetable optimization, in order to find the optimum results accurately and efficiently, a genetic algorithm is implemented to solve the problem. The algorithm is a search procedure which is based on the rules of natural selection and genetics. It presents a stochastic and iterative process which operates on a population of individuals. Each individual represents a potential solution (a set of *IT* and *ST* in this study) to a given problem. The algorithm runs as the following steps:

1. Initialization: A random initial population of the solutions is produced to form the first generation (*V*). The population includes a number of individuals (*pop_num*). The number of individuals at each population should be at least five times larger than the number of variables (e.g. 100 individuals in this study) [24]. Each individual represents a set of inter-station journey time (*IT*) and service interval (*ST*);

   Step 1: Set i=1;

   Step 2: if $i \leq pop_num$. Randomly generate a vector $Vi=(v_1, v_2,\ldots,v_{sn+stn})$ to represent for a solution, as shown in Figure 7. The solution should satisfy all constrains in Equation (10) and (11);

	v_1	v_2	...	v_{sn}	v_{sn+1}	v_{sn+2}	...	v_{sn+stn}
V_1	IT_1^1	IT_2^1	...	IT_{sn}^1	ST_1^1	ST_2^1	...	ST_{stn}^1
V_2	IT_1^2	IT_2^2	...	IT_{sn}^2	ST_1^2	ST_2^2	...	ST_{stn}^2
				.....				
V_{pop_num}	IT_1^{num}	IT_2^{num}	...	IT_{sn}^{num}	ST_1^{num}	ST_2^{num}	...	ST_{stn}^{num}

Figure 7. Initialization of the first population of the solutions.

Step 3: Set $i=i+1$. The algorithm returns back to Step 2 and produces another vector V_i until $i>pop_num$.

2. Evaluation: Each solution in the population needs to be evaluated to identify its performance.

   Step 1: The set of inter-station journey time (*IT*) and service interval (*ST*) will be used to form a complete whole day timetable for the multiple train simulation;

   Step 2: The inter-station journey time will simultaneously be used to search for the corresponding optimal train movement mode sequence (*VS*) and train energy consumption (E_{single}) from the Database ($OPTDATA_{trajectory}$). The searching results will be input to the multiple-train simulator, as shown in Figure 5. The simulators used in this study have been previously verified and tested in other studies [7, 25, 26];

   Step 3: For each solution, the multiple train simulator calculates the braking synchronized groups (*BSG*), braking synchronized time (*BST*), motoring synchronized group (*MSG*), motoring synchronized time (*MST*), regenerative braking energy and transmission loss. The whole day substation energy consumption (E_{day}) can then be calculated using Equation (10) as a fitness function. The solution with the lowest fitness value (E_{day}) represents for the best individual.

$$EVAL(V) = F\left(IT_{set}, ST_{set}, E_{single}, VS\right), \qquad IT \in OPTDATA_{trajectory} \quad (15)$$

   Step 4: The solutions in $EVAL(V)$ will be sorted by the fitness value.

3. Selection: A genetic operation will be implemented after the evaluation. The operation aims to choose appropriate individuals (parents) for breeding new individuals (offspring) in order to form a population for the next generation (V'). The first phase of the generic operation is the selection. In the selection operation, the first *top_num* top ranking individuals are retained for the next generation. The number is set as 10 in this study;

   Step 1: Set $j=1$;

   Step 2: if $j \leq top_num$, then $V'_j=EVAL(V_j)$;

   Step 3: Set $j=j+1$. The algorithm returns back to Step 2 and produces another V'_j until $j>top_num$.

4. Crossover and mutation: The second phase of the genetic operation is the crossover ans mutation. The next *cro_num* and *mut_num* ranking individuals will be selected for crossover and mutation. The crossover rate and mutation rate are 0.8 and 0.1 respectively (the numbers are 64 and 16 in this study), which have been tested and selected specifically for this study [27-29].

Step 1: Set k=1;

Step 2: if $k \leq cro_num/2$, then the crossover operation will randomly select one genetic from two chromosomes and exchange with each other. For example, assume two chromosomes $EVAL(V_m)=(v_1, v_2,\ldots, v_p, v_q,\ldots v_{sn+stn})$, $EVAL(V_n)= (v'_1, v'_2,\ldots, v'_p, v'_q,\ldots v'_{sn+stn})$, and genetic number p and q are selected. Then the new individuals are produced as $V'_k=(v_1, v_2,\ldots, v'_p, v'_q,\ldots v_{sn+stn})$, $V'_{k+1}=(v'_1, v'_2,\ldots, v_p, v_q,\ldots v'_{sn+stn})$;

Step 3: Set $k=k+2$. The algorithm returns back to Step 2 and produces another V'_k and V'_{k+1} until $k>cro_num/2$. This is because each crossover operation uses two individuals;

Step 4: Set r=1;

Step 5: if $r \leq mut_num$, then the mutation operation will randomly select one genetic from one chromosome and exchange the genetic with a random value, but the value should satisfy all constrains in Equation (10) and (11). For example, assume the chromosome $EVAL(V_o)= (v_1, v_2,\ldots, v_s, \ldots v_{sn+stn})$ and genetic s are selected. Then the new individual is produced as $V'_r=(v_1, v_2,\ldots, v^*_s,\ldots v_{sn+stn})$;

Step 6: Set $r=r+1$. The algorithm returns back to Step 5 and produces another V'_r until $r>mut_num$.

5. Replacement: The last phase of the genetic operation is the replacement. In this operation, the algorithm will produce *rep_num* new random individuals to take the place of the last *reo_num* ranking individuals in $EVAL(V)$;

   Step 1: Set t=1;

   Step 2: If $t \leq rep_num$, then $EVAL(V_t)$ will be discarded. The algorithm will randomly generate a new solution $V't=(v^*_1, v^*_2,\ldots, v^*_{sn+stn})$ to replace $EVAL(V_t)$, the solution should satisfy all constrains in Equation (10) and (11);

   Step 3: Set $t=t+1$. The algorithm returns back to Step 2 and produces another V'_t until $t>rep_num$.

6. New generation: Finally, after the genetic operation, a new generation (V') has been produced.

$$\begin{cases} V' = [V'_j, \ldots., V'_{top_num}, V'_k, \ldots, V'_{cro_num/2}, V'_r, \ldots, V'_{mut_num}, V'_t, \ldots, V'_{rep_num}] \\ pop_num = top_num + cro_{num}/2 + mut_num + rep_num \end{cases} \quad (16)$$

7. Termination: The algorithm returns back to the second step and is repeated until either of the following termination conditions are achieved: the cumulative change in the fitness function value is less than 1.0×10^{-4}, or the number of generations exceeds 100.

4 Case Study

The previous sections have described the development and implementation of an integrated metro operation optimization. In this paper, the case study is based on China's Guangzhou metro line 7, which is expected to start services in 2016. It is a suburb metro line connecting Guangzhou South Railway Station to the University City. The route is 17.5 km long and has 7 intermediate stations. The scheduled single journey time is 1372 s, including 1077 seconds running time and 295 seconds total dwell time, as shown in Table 2. The line speed limits and height profiles are shown in Figure 8.

Table 2. Scheduled timetable.

ID	Station name	Scheduled inter-station journey time	Platform center location, m	Dwell time, seconds
1	Guangzhou South Station		0	35
		00:01:20		
2	Shi Bi		1120	30
		00:02:01		
3	Xie Chun		3028	25
		00:02:13		
4	Zhong Chun		5200	25
		00:01:48		
5	Han Xi Chang Long		1266842	45
		00:02:12		
6	He Zhuang		8959	30
		00:02:22		
7	Guang Tang		11323	35
		00:02:23		
8	Nan Chun		13730	35
		00:03:38		
9	Da Xue Chen Nan		17508	35

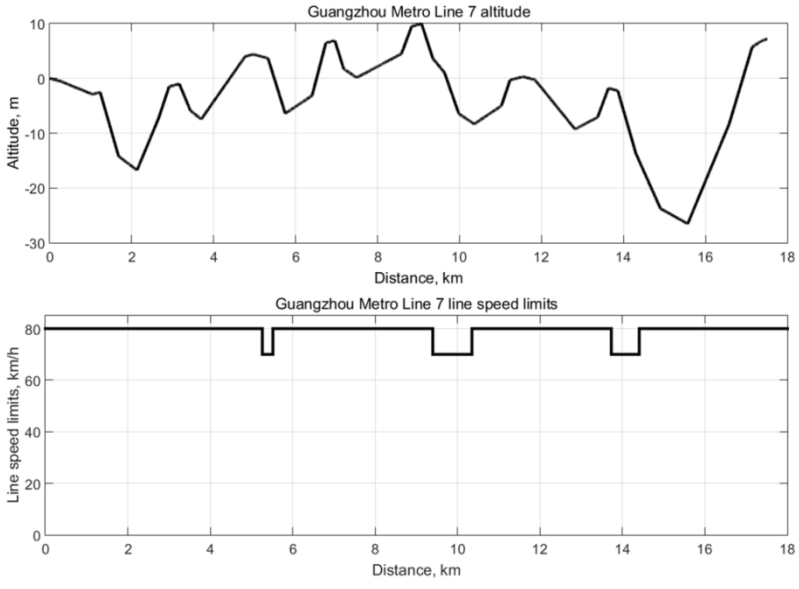

Figure 8. Guangzhou Metro Line gradient and speed limits.

In order to deal with variable passenger demands throughout a day, different train service intervals are used for the services on this metro line. For example, during peak time services (7 am to 9 am and 5 pm to 7 pm), the trains depart every 200 seconds. During off-peak time services, the service intervals are 300 seconds, 360 seconds or 600 seconds depending on demand. A 1500 V overhead line (OHL) power supply system supplies electrical energy to the trains. The traction characteristics of the train are shown in Table 3 and Figure 9. Each train set is composed of 6 cars, including 4 motor cars and 2 trailer cars. The maximum service speed and average operation speed are 80 km/h and 41 km/h respectively.

Table 3. Train traction characteristics.

Parameters	Value/Equation
Overall train mass, tonnes	279 (4M2T)
Train formation	4M2T
Train length, m	118
Rotary allowance	0.12
Resistance, N/tonne	$27+0.0042V^2$ (V: km/h)
OHL power	DC 1500V
Maximum power, kW	4227
Engine efficiency from electrical to mechanical power	87.88%
Max operation speed km/h	80
Max tractive effort kN	352 (AW2 4M2T)
Train control	Automatic Train Operation (ATO), human operation
Passenger number (AW2)	1860

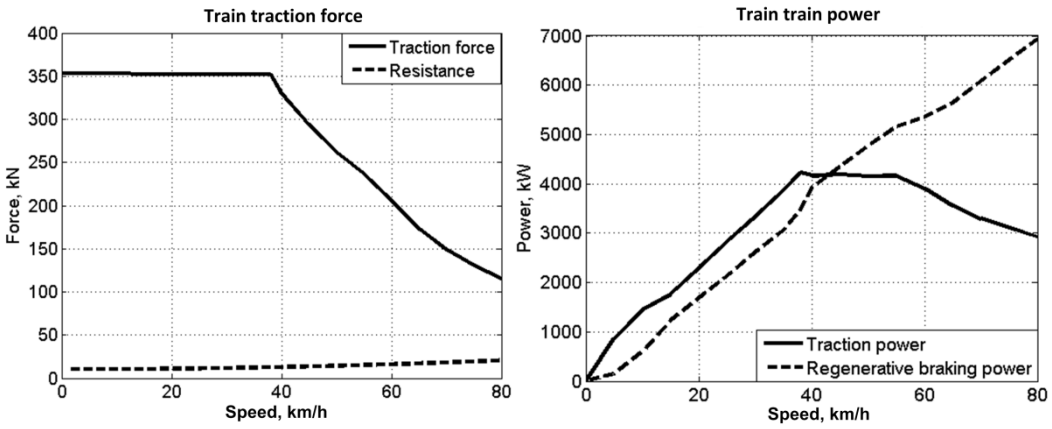

Figure 9. Train traction system characteristic.

Figure 10 shows the original train trajectory (without optimization) for the train running on this line proposed by the operator company. After the train reaches the target speeds, it is expected to keep at a constant speed (cruising mode), until it approaches a station stop. However, in reality it is difficult for ATO systems or train drivers to control the train at a constant speed due to the limitations of the train traction system. The train needs to switch between acceleration and braking frequently in order to track the given speed (trajectory). Therefore, such a simple driving strategy will cause more energy usage.

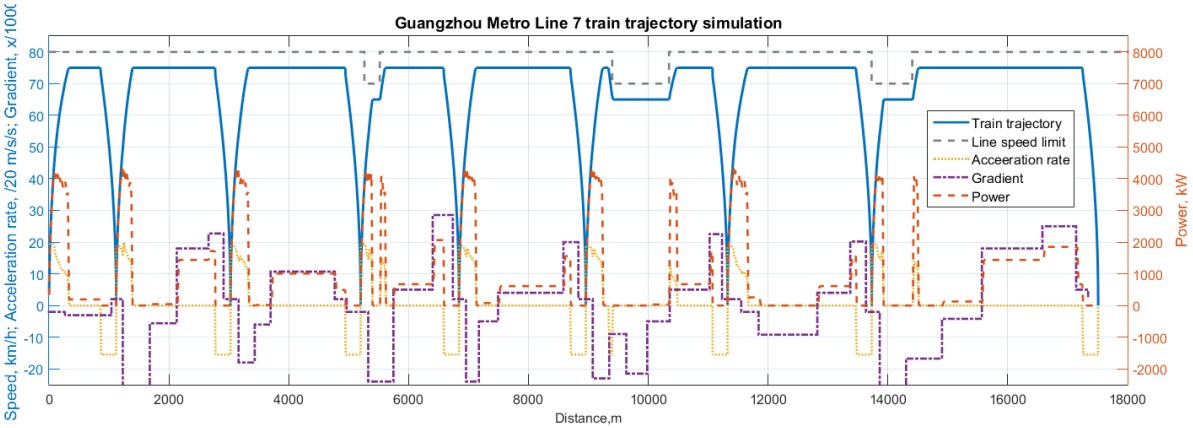

Figure 10. Original train trajectory (without optimization).

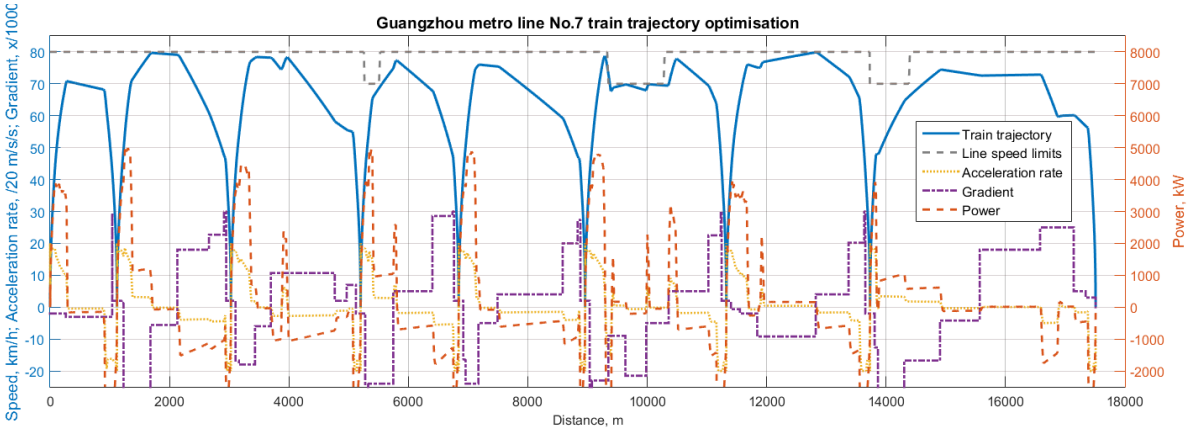

Figure 11. Optimal train trajectory.

By applying the proposed integration optimization, an optimized train trajectory and timetable have been obtained. Compared with the original train trajectory, the proposed optimal train trajectory does not consider the cruising mode, making the speed tracking much easier, as shown in Figure 11. Furthermore, the optimal train trajectory has applied the coasting mode and lengthened the coasting distance as long as possible in order to reduce the energy consumption. The maximum variation (IT_r) in the inter-station journey times is chosen at 5 seconds in the optimization because the train energy consumption increases rapidly when the reduction is over than 5 seconds, as shown in Figure 12.

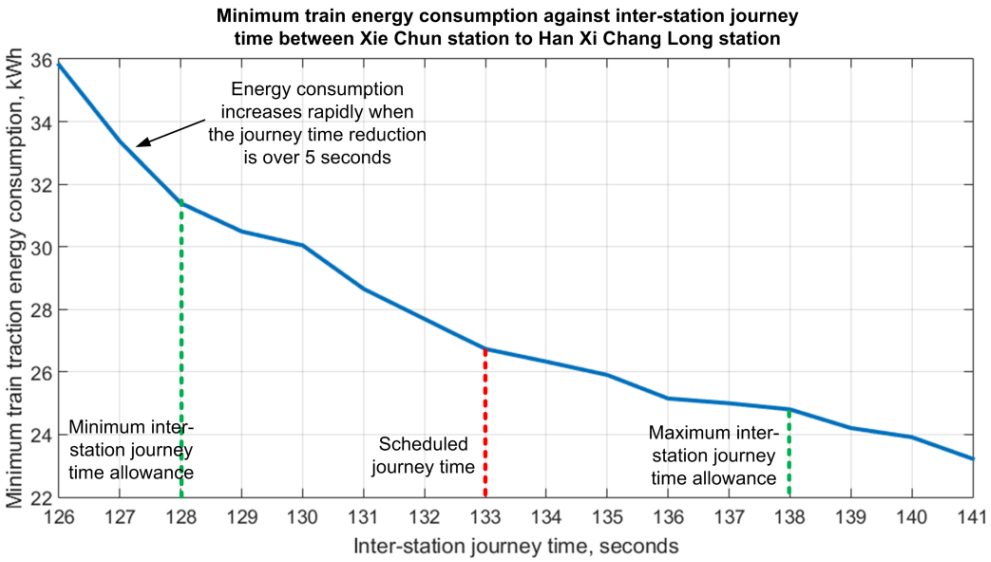

Figure 12. Dependence of energy consumption in Guangzhou Metro Line 7 case study.

The obtained optimized timetable is shown in Table 4. The largest change in the inter-station journey occurs between Guang Tang and Nan Chun stations. The maximum running speed in this section has increased from 75 km/h to 78 km/h, reducing the journey time by 5 seconds. The maximum variation (ST_r) in the service intervals is chosen at 30 seconds in this case study in order to minimize the impact of the timetable rescheduling and meet the requirement of the potential passenger flow. The single journey time, dwell time and the number of services remain the same, as shown in Figure 13.

Table 4. Schedule timetable and optimized timetable.

	Station	**Scheduled inter-station journey time**	**Optimal inter-station journey time**
Inter-station journey time	Guangzhou South Station	-	-
	Shi Bi	00:01:20	00:01:20
	Xie Chun	00:02:01	00:01:59
	Zhong Chun	00:02:13	00:02:16
	Han Xi Chang Long	00:01:48	00:01:48
	He Zhuang	00:02:12	00:02:15
	Guang Tang	00:02:22	00:02:22
	Nan Chun	00:02:23	00:02:18
	Da Xue Chen Nan	00:03:38	00:03:39
		-	-
	Single journey time, seconds	1077	1077
	Single dwell time, seconds	225	225

Peak time service interval, seconds	200	196
Off-peak time service interval 1, seconds	300	292
Off-peak time service interval 2, seconds	360	369
Off-peak time service interval 3, seconds	600	570
Number of services per day	404	404
Total daily operating time, seconds	67114	67114

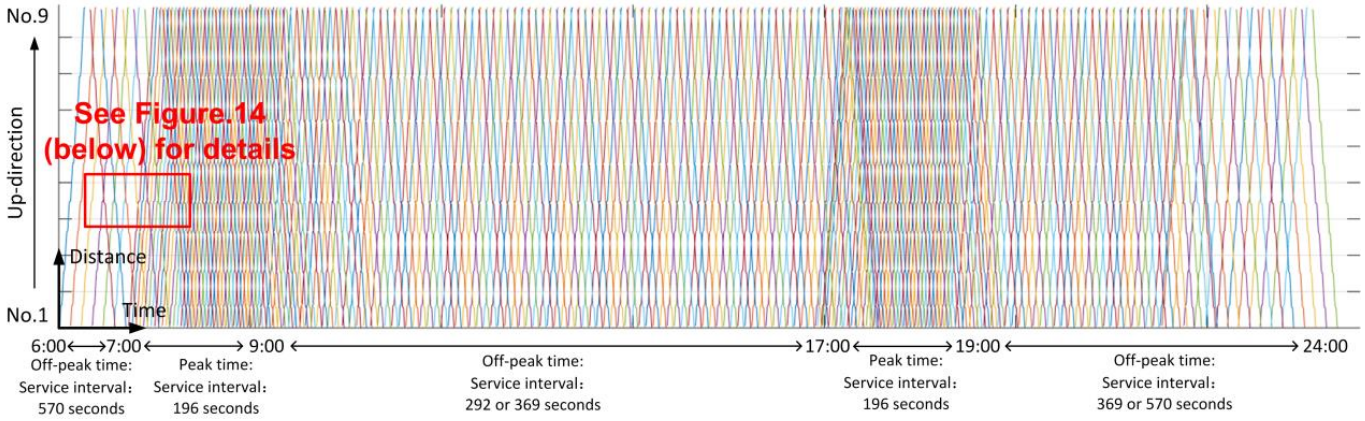

Figure 13. Train running diagram using optimal timetable.

Figure 14 is an enlarged view of a section from Figure 13 (shown in the red rectangle). It presents a comparison of the train movements near Han Xi Chang Long station from 7:00 am to 8:15 am with the scheduled timetable and the optimal timetable. When using the scheduled timetable, the up-direction train and the down-direction train usually arrive at the station at the same time. Therefore, the regenerative braking energy produced by both trains cannot be used by either train. Furthermore, both of the trains leave the platform at the same time, thereby increasing the substation load and resulting in a large motoring synchronized time (*MST*).

When using the optimal timetable, the up-direction train and the down-direction train arrive at different times. Therefore, the regenerative braking energy produced from the braking train can be used by the accelerating train within the braking synchronized time (*BST*). Moreover, both trains accelerate from the station at different times, thus the substation load could be significantly reduced.

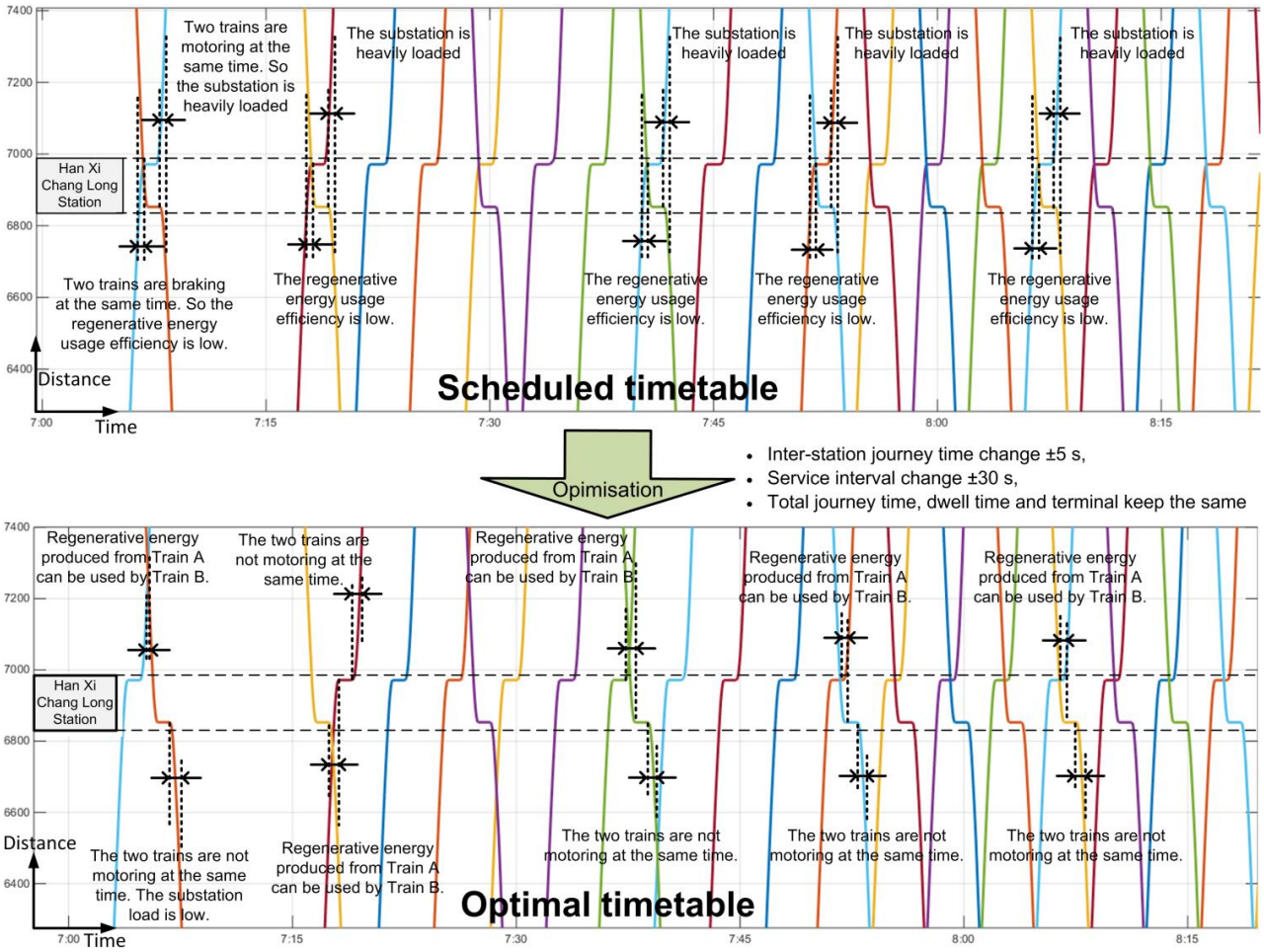

Figure 14. Comparison between original timetable (top) and optimal timetable (below).

Table 5 shows the improvement resulting from the optimal timetable. Compared with the scheduled timetable, using the optimal timetable increases the total braking synchronized time by 14.7%, resulting in a large energy saving. The maximum number of motoring trains at the same time has been reduced from 4 to 3, and the total motoring synchronized time is decreased by 68.5%, thereby reducing the substation load. Such an improvement could potentially reduce the maximum power demand on the substation, and lower the substation investment cost.

Table 5. Timetable optimization improvement.

	Schedule timetable	**Optimal timetable**
The number of braking synchronized groups (*BSG*) (more is better)	5561	5876 （+5.7%）
Total braking synchronized time (*BST*) (larger is better)	746974	857088 （+14.7%）
The number of motoring synchronized groups (*MSG*) (fewer is better)	6190	1474 （-74.2%）
Total motoring synchronized time (*MST*) (smaller is better)	742200	234114 （-68.5%）

The maximum number of motoring trains at the same time	3	4

Figure 15 demonstrates the procedure by which the objective function output evolves with the generation using the genetic algorithm. It can be observed that, the algorithm obtains better solutions in each new generation due to the effect of the heuristic guidance. The searching finally converges to the optimum in the 39th, whilst the best individuals achieve the fitness value -4774.

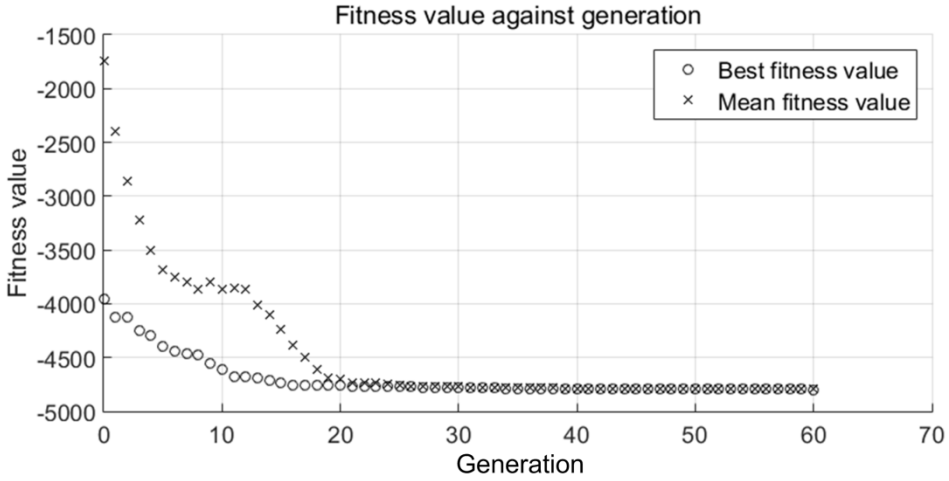

Figure 15. The mean and best fitness value at each generation.

Table 6 shows the optimization combinations for different operations. In reality, it is usually difficult to modify an existing timetable quickly because the timetable configuration is relevant to a large number of other systems. However, train trajectory (driving strategy) is relatively independent and more easily modified. Therefore, three operations are considered in this study, namely original operation (without any optimization), trajectory optimized operation (implement train trajectory optimization only) and timetable optimized operation (implement both train trajectory optimization and timetable optimization).

Table 6. Optimization combinations for different operations.

	Train trajectory optimization	**Timetable optimization**
Original operation	No	No
Trajectory optimized operation	Yes	No
Timetable optimized operation	Yes	Yes

As shown in Table 7, compared with the original operation, using the trajectory optimized operation could reduce the substation energy usage by 10,645 kWh (-23%) each day. Therefore, the potential annual substation energy saving could be up to £583k (£0.15 per

kWh). Note the train traction energy is provided by both substation and regenerative braking. In the trajectory optimized operation, the coasting mode has been applied. Therefore the train braking time, and thus the regenerative braking energy, has been reduced.

Compared with the trajectory optimized operation, in the timetable optimized operation, the regenerative braking energy is increased by 6%. Therefore, the substation energy saving can be further increased to 11,856 kWh per day (-25%), that is, £649k per year.

Table 7. Optimization and non-optimization operation results.

	Train running time, hours	**Substation energy usage, kWh**	**Train traction energy usage, kWh**	**Regenerative braking energy, kWh**
Original operation	18.6	46,588	93,447	50,813
Trajectory optimized operation	18.6	35,943 (-23%)	72,811 (-22%)	39,914 (-21%)
Timetable optimized operation	18.6	34,732 (-25%)	74,376 (-20%)	43,153 (-15%)

5 Conclusion

In this paper, an integrated optimization study has been presented that combines train trajectory optimization and timetable optimization. They are closely related to one another, and both of them play a key role in train operation. The integrated optimization aims to minimize substation energy consumption by calculating the most appropriate train movement modes, inter-station journey times and service intervals.

The proposed integrated method considers the train inter-station journey time in both train trajectory optimization and timetable optimization simultaneously. Such a process significantly increases the algorithm complexity but improve the overall metro network performance. Furthermore, when the algorithm is calculating the synchronized groups in the timetable optimization, it does not only consider the nearby two trains, but consider about all the trains running in the whole network throughout the whole day operation (e.g.: a 13.6 km long route with daily 200 trains and 18 hours operation). The huge amount of calculation increases the complexity, but improves the algorithm performance. Due to the complexity of such an integrated optimization problem, a brute force algorithm and a genetic algorithm are introduced, working together to obtain accurate results.

This study has identified that using the proposed integrated optimization could improve the train regenerative braking energy efficiency and significantly reduce the substation energy

consumption. Furthermore, the implemented optimal timetable is able to reduce the substation load, which improves the reliability of the railway power network and could potentially reduce the substation investment cost.

For a practical railway system, the real-time response requirements are usually very important because of safety concerns and system performance demands [30, 31]. Therefore, due to the significant computation time of the proposed integrated optimization method (approximately 10 minutes), it is not currently appropriate for real-time implementation. However, compared with mainline operation, metro operation is relatively simple. Therefore, the integrated algorithm is designed to produce optimal results offline, and then calculates less optimal results in real-time with reduced numbers of variables (e.g. headway only) if necessary. Furthermore, the computation time can be reduced by using high performance computing platforms.

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