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DOI:

[10.1093/mnras/stad464](https://doi.org/10.1093/mnras/stad464)

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Document Version

Publisher's PDF, also known as Version of record

Citation for published version (Harvard):

Gao, Y, Van roestel, J, Green, MJ, Fuller, J, Grishin, E & Toonen, S 2023, 'Observable tertiary tides in TIC242132789', *Monthly Notices of the Royal Astronomical Society*, vol. 521, no. 2, stad464, pp. 2114-2118. <https://doi.org/10.1093/mnras/stad464>

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Observable tertiary tides in TIC242132789

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Accepted 2023 February 3. in original form 2023 January 16

ABSTRACT

Many stars live in hierarchical triple systems, but the physics of such systems are still poorly understood. One understudied physical aspect of these systems is tertiary tides, wherein the tidal deformation of a tertiary in a hierarchical triple drains energy from the inner binary, causing the inner binary's orbital separation to decrease. This tidal process is difficult to observe directly, since such an observation requires a very compact hierarchical triple, the tertiary of which must be almost large enough to fill its Roche lobe at the epoch of observation. Concurrently, the recently discovered stellar system TIC242132789 is the fourth most compact observed hierarchical triple, and the most compact in which the tertiary is a giant. In this paper, we demonstrate that TIC242132789 provides a rare opportunity to place constraints on the model parameters for tertiary tides, and can even be a rare opportunity to directly observe tertiary-tides-induced orbital shrinkage of the inner binary. We calculate our expectations of how fast the inner orbit will shrink, and demonstrate that our estimates of this rate of shrinkage should be observable using current techniques. We conclude with a call for relevant observations of this system to commence.

Key words: binaries: close – binaries: eclipsing – stars: evolution.

1 INTRODUCTION

Stars reside in stellar systems, each consisting of one or more stars. Of these stellar systems, those which contain three stars are known as triples, and those triples which consist of two stars in a close Keplerian orbit, which is in turn in orbit with the third star around a much wider orbit, are known as hierarchical triples. In a hierarchical triple, the two stars in the close orbit are referred to as the inner binary, while the third star is called the tertiary.

Triples account for 13 per cent of all observed stellar systems (e.g. Tokovinin 2014), and this figure increases considerably for high-mass stars, of which a majority exist as hierarchical triples, and for which the average number of stars comprising their host systems is about three (Sana et al. 2012; Sana 2017).

Due to their numerical significance and potential for the third star to influence many astrophysical processes in its companions, much effort has been put forth to understand how hierarchical triples evolve (e.g. Toonen, Hamers & Portegies Zwart 2016; Toonen et al. 2020; Grishin & Perets 2022; Hamers, Glanz & Neunteufl 2022; Preece et al. 2022). However, due to their complexity, there are still many aspects of how these systems behave that are poorly understood.

One example of a poorly understood aspect of the behaviour of hierarchical triples is how the inner binary tidally interacts with the

tertiary. There is no question that, in certain cases, tertiary tidal effects can significantly alter the behaviour of the host hierarchical triple; such alteration was directly observed in the hierarchical triple HD 181068. HD 181068 (Derekas et al. 2011) is known to be a hierarchical triple system, the tertiary of which is a red giant, and was found to exhibit a series of luminosity oscillations, which could not be easily explained using conventional stellar evolution theory. The original discoverers of this luminosity variation attributed it to tidal perturbation by the inner binary, and it was indeed later found that such perturbation could explain the observed oscillations (Fuller et al. 2013). The authors who modelled the tidal perturbation effects also found that, according to their model, the tidal deformation of the tertiary, and its subsequent dissipation process is projected to shrink the orbital separation of the inner binary. This effect was later studied quantitatively and in detail by Gao et al. (2018, 2020), who subsequently named this tertiary tidal process, by which the inner binary orbits shrink due to tidal interactions with the tertiary, tertiary tides (TTs).

However, in all these studies of TTs, there exist free parameters in the models used to calculate the magnitude of their effects, which are not easy to ascertain using first principles. For example, the rate at which the inner binary orbital separation decreases as a function of time is dependent on the value of these free parameters. In order to constrain these parameters, it would be ideal to observe a hierarchical triple system in which TTs are acting so strongly that the inner binary orbital shrinkage can be measured via observations. However,

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Table 1. Observed and derived properties of TIC242132789, quoted from Rappaport et al. (2022), as well as the values used in our simulations.

Parameter	Value	Simulation
P_{in}	5.1287 ± 0.0013 d	–
P_{out}	$42.0317^{+0.0091}_{-0.0085}$ d	–
R_1	$1.207^{+0.029}_{-0.027} R_{\odot}$	0
R_2	$1.741^{+0.031}_{-0.030} R_{\odot}$	0
R_3	$12.22^{+0.12}_{-0.13} R_{\odot}$	$12.22 R_{\odot}$
m_1	$1.146^{+0.027}_{-0.034} M_{\odot}$	$1.15 M_{\odot}$
m_2	$1.346^{+0.031}_{-0.045} M_{\odot}$	$1.35 M_{\odot}$
m_3	$1.539^{+0.046}_{-0.060} M_{\odot}$	$1.54 M_{\odot}$
a_1	$16.98^{+0.13}_{-0.18} R_{\odot}$	$16.98 R_{\odot}$
a_2	$81.02^{+0.68}_{-0.91} R_{\odot}$	$81.02 R_{\odot}$
e_1	$0.01644^{+0.00041}_{-0.00042}$	0.0
e_2	$0.0055^{+0.0037}_{-0.0030}$	0.0

such an observation is yet to be achieved, as it would require a hierarchical triple with an extremely compact outer orbit for it not to be prohibitively time-consuming.

TIC242132789 is a recently discovered hierarchical triple. Like HD 181068, it has a red giant tertiary, which is being perturbed by a pair of less evolved stars in the inner binary. Observations made using *TESS* (Ricker et al. 2015) have managed to obtain detailed orbital parameters, as well as masses of all three objects, within TIC242132789 (Rappaport et al. 2022). To date, TIC242132789 is the fourth most compact hierarchical triple ever to be observed, and the most compact hierarchical triple in which the tertiary is a giant, which is not undergoing significant mass transfer with the inner binary, that has been discovered. Furthermore, its relatively wide inner orbit is also conducive to strong TTs. Should its inner binary orbit be measured to be rapidly shrinking, it would be an ultimate test of existing models of TTs, and help constrain several aspects of tidal theory in general besides.

In this paper, we introduce the characteristics of TIC242132789, and calculate the magnitude of the effects of TTs using the model of Gao et al. (2018, 2020) in Section 2. We then proceed to compare the results of these calculations with what is expected from the model previously used to investigate the tidal effects observed in HD 181068 (Fuller et al. 2013) in Section 3. Finally, we demonstrate that the orbital shrinkage in the binary might be directly observable under current conditions in Section 4. Finally, we discuss the implications of these results and call for observations of TIC242132789’s orbital shrinkage to commence in Section 5.

2 VISCOELASTIC MODEL OF TERTIARY TIDES IN TIC242132789

The physical parameters of TIC242132789, as found by Rappaport et al. (2022), are repeated in the central column of Table 1. Here, we have adopted the notation of Gao et al. (2018), citing the masses of the inner binary as m_1 and m_2 , the mass of the tertiary as m_3 , and the semimajor axes of the inner and outer orbits as a_1 and a_2 , respectively. The eccentricities of the inner and outer orbits are denoted e_1 and e_2 , respectively, but these are both close to zero for TIC242132789 (circular orbits).

The physical radii of the three stars m_1 , m_2 , and m_3 are R_1 , R_2 , and R_3 , respectively. The inner binary has an orbital period of P_{in} , whereas that of the outer orbit, between the tertiary and the inner

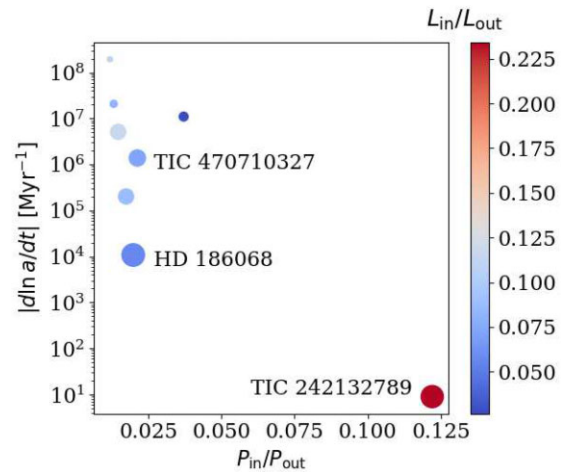


Figure 1. Period ratio ($P_{\text{in}}/P_{\text{out}}$) and the rate of decay due to TTs for known compact triples from equation (1) with a fiducial value of $\tau = 0.5$ yr. The colour bar indicates the angular momentum ratio, and the size of the marker is proportional to the radius of the tertiary. The circles are the six systems found in Rappaport et al. (2022), HD 186068 (Derekas et al. 2011; Fuller et al. 2013), and TIC470710327 (Eisner et al. 2022; Vigna-Gómez et al. 2022).

binary, is P_{out} . Under the parameters listed in Table 1, the Roche lobe radius of the tertiary is $27.37 R_{\odot}$, only slightly more than twice the physical radius of the tertiary itself.

To find the rate at which the inner binary orbit of this system shrinks as a result of TTs, we adopt the rapid calculation method developed in Gao et al. (2020), repeated below

$$-\frac{1}{a_1} \frac{da_1}{dt} = (2.22 \times 10^{-8} \text{ yr}^{-1}) \frac{4q}{(1+q)^2} \left(\frac{R_3}{100 R_{\odot}} \right)^{5.2} \times \left(\frac{a_1}{0.2 \text{ au}} \right)^{4.8} \left(\frac{a_2}{2 \text{ au}} \right)^{-10.2} \left(\frac{\tau}{0.534 \text{ yr}} \right)^{-1.0}, \quad (1)$$

where $q < 1$ is the inner binary mass ratio, and τ is the relaxation time as defined in Gao et al. (2018). Fig. 1 shows the expected decay rate of six recently discovered compact triple systems (Rappaport et al. 2022), including TIC242132789, together with the massive triple system TIC470710327 with a $6 + 6 M_{\odot}$ inner binary, and an $\sim 16 M_{\odot}$ main-sequence tertiary (Eisner et al. 2022), and the HD 1860168 system with an evolved tertiary giant (Derekas et al. 2011). The size of the markers is proportional to the radius of the tertiary. We see that due to the steep scaling with the tertiary radius R_3 in equation (1), only the systems with evolved tertiary radii (TIC242132789 and HD 186068) are able to decay due to TTs at a relatively rapid rate. The 5 d orbit of the inner binary of TIC242132789 is unique compared to all the other compact systems with an inner binary orbit of ~ 1 d, and allows a potential observation of the expected TTs decay rate of around $\sim 10^{-1} \text{ Myr}^{-1}$, and is the main object of study in this paper. Moreover, TIC242132789 has a non-negligible inner-to-outer angular momentum ratio, $L_{\text{in}}/L_{\text{out}} \gtrsim 0.2$, which will cause significant feedback on the tertiary’s orbit in addition to TTs. The inner orbit may also experience osculating oscillations in its orbital elements of order $P_{\text{in}}/P_{\text{out}}$, and accurate dynamical modelling will be required to disentangle the effect of TTs from osculating variations.

Fig. 1 demonstrates that TIC242132789 is a unique compact triple system, and we will presently demonstrate that it is currently the only one susceptible to direct observations of TT effects. Here, we explore its dynamical and tidal evolution in more detail. Since τ is difficult to calculate from first principles, we vary its value from 0.45

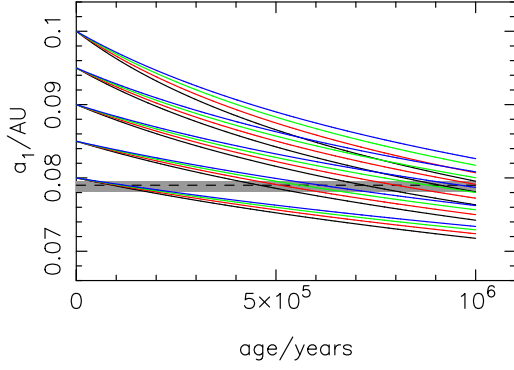


Figure 2. Evolution of the inner binary semimajor axis a_1 of TIC242132789, assuming different values of a_1 and τ . The different line colours denote different values of τ ; the black, red, green, and blue lines correspond to $\tau = 0.45, 0.50, 0.55,$ and 0.60 yr, respectively, calculated for different initial inner binary semimajor axes. The horizontal dashed line is the observed value of a_1 according to Rappaport et al. (2022), and the grey shaded area indicates the error bars of the corresponding observations.

to 0.60 yr (the range hitherto found in other stars in which TTs are most ineffective), and calculate the inner binary orbital shrinkage for a range of initial values for a_1 , ranging from 0.08 to 0.10 au. We find that, in all these cases, the inner binary semimajor axis is shortened drastically over a relatively short time-scale of 10^6 yr. This implies that the inner binary of TIC242132789 could potentially have been much less compact merely millions of years ago. We plot the results of these calculations in Fig. 2.

To check that these rapid calculations (equation 1) are valid for the orbital parameter range herein studied, we run a full viscoelastic model simulation (Gao et al. 2018) for the system at $\tau = 0.55$ yr, with an initial a_1 of 0.08 au, and compare the results with those obtained from the rapid calculation. The results differ by less than 5 per cent in terms of the amount of energy extracted from the inner binary orbit.

To obtain the current instantaneous rate of inner binary orbital shrinkage, we abstract the parameters given in the central column of Table 1 into those given in the rightmost column. From these parameters, it is then trivial to calculate the inner binary orbital shrinkage rate via equation (1), once a value of τ is assumed.

Since this shrinkage rate increases with decreasing τ , we assume a τ value of 0.60 yr, the longest of all the values in our range. We subsequently find the current inner binary orbital shrinkage rate of TIC242132789 to be

$$-\frac{1}{a_1} \frac{da_1}{dt} = 1.13 \times 10^{-7} \text{ yr}^{-1}. \quad (2)$$

3 DYNAMICAL TIDAL MODEL OF TERTIARY TIDES IN TIC242132789

As previously noted, Fuller et al. (2013) was a study of HD 181068 – a similar but less compact hierarchical triple – using a dynamical tidal model.

To obtain a rough estimate of how fast the inner binary shrinks using their model, we need an expression for the inner binary’s angular momentum L_{12} , as well as \dot{L}_{12} , which are

$$L_{12} = \mu_{12} a_1^2 \left(\frac{2\pi}{P_1} \right), \quad (3)$$

where $\mu_{12} = m_1 m_2 / (m_1 + m_2)$ is the reduced mass of the inner binary, and, noting equations 45 and 46 of Fuller et al. (2013),

$$\begin{aligned} \dot{L}_{12} &= \dot{L}_{12}(m=0) + \dot{L}_{12}(m=\pm 2) \\ &= 8 \frac{Gm_3^2}{R_3} \left(\frac{a_1}{a_2} \right)^2 \left(\frac{\mu_{12}}{m_3} \right)^2 \left(\frac{R_3}{a_2} \right)^6 \\ &\quad \times [(W_{2,0} Q_\alpha F_0)^2 \bar{\Gamma}_{\alpha,0} + (W_{2,2} Q_\alpha F_2)^2 \bar{\Gamma}_{\alpha,2} \\ &\quad + (W_{2,-2} Q_\alpha F_{-2})^2 \bar{\Gamma}_{\alpha,-2}], \end{aligned} \quad (4)$$

where $\dot{L}_{12}(m=x)$ denotes the value of \dot{L}_{12} under the $m=x$ oscillation modes.

Here, we have $F_0 = \frac{15}{8}$, $F_2 = \frac{35}{8}$, $F_{-2} = \frac{3}{8}$, $W_{2,0} = -\sqrt{\frac{\pi}{5}}$, $W_{2,\pm 2} = \sqrt{\frac{3\pi}{10}}$, and the evaluations of Q_α , $\bar{\Gamma}_{\alpha,0}$, $\bar{\Gamma}_{\alpha,2}$, and $\bar{\Gamma}_{\alpha,-2}$ are as follows.

To estimate Q_α , we use fig. 6 of Fuller et al. (2013). Admittedly, this figure was calculated for HD 181068, but it should be similar enough for TIC242132789, since both stars have similar radii, luminosities, and envelope structures. For an inner binary orbit of about 5 d, the tidal forcing frequency $\nu_m \approx 4\pi/P_{\text{in}}$, corresponding to $Q_\alpha^2 \sim 10^{-6}$.

The resonant detuning factor is $\bar{\Gamma}_{\alpha,m} = \Gamma_{\alpha,m} \omega_{\text{dyn}}^2$, where the dynamical frequency is $\omega_{\text{dyn}}^2 = \frac{Gm_3}{R_3^3}$, and the expression for $\Gamma_{\alpha,m}$ in equation 40 of Fuller et al. (2013) is

$$\Gamma_{\alpha,m} = \frac{\nu_m \gamma_\alpha}{(\omega_\alpha^2 - \nu_m^2)^2 + 4(\nu_m \gamma_\alpha)^2 + \gamma_\alpha^4}. \quad (5)$$

Here, γ_α and ω_α are the tidally forced mode’s damping rate and frequency, respectively.

The expected frequency detuning $\Delta\omega = \omega_\alpha - \nu_m$ of the most resonant modes (which dominate the tidal dissipation) are determined by the g-mode period spacing, which is measured to be about $\Delta P_g(\ell=1) \simeq 60$ s for $\ell=1$ modes in red giant stars similar to TIC242132789 (Vrard, Mosser & Samadi 2016). For tidally forced modes with $\ell=2$, the period spacing is smaller by a factor of $\sqrt{3}$. The typical frequency detuning is $\Delta\omega = \omega_\alpha^2 \Delta P_g / (2\pi)$, which we calculate to be $\Delta\omega \sim 4 \times 10^{-9} \text{ s}^{-1}$.

The mode damping rates are not easy to calculate from first principles, but can be extrapolated from mode lifetimes measured for red giant stars. From Vrard et al. (2018), mode damping rates of observed $\ell=0$ p modes for stars similar to TIC242132789 are $\gamma \sim 2 \times 10^{-7} \text{ s}^{-1}$. Those measurements are for modes with frequencies ~ 10 times higher than the resonant modes for TIC242132789, and thus, represent upper limits on the true values of γ_α . We use $\gamma_\alpha = 10^{-7} \text{ s}^{-1}$ in our estimates, extrapolating the measurements in fig. 5 of Vrard et al. (2018) to lower frequency. Although γ_α is quite uncertain, it is likely to be larger than the mode frequency spacing $\Delta\omega$, meaning that a reasonable estimate for the mode detuning parameter is $\Gamma_{\alpha,m} \sim 1/(4\nu_m \gamma_\alpha)$.

The result is that, for the parameters assumed above, the current rate at which the inner binary semimajor axis shrinks for TIC242132789 should presently be

$$-\frac{1}{a_1} \frac{da_1}{dt} = -2 \frac{\dot{L}_{12}}{L_{12}} = 8.70 \times 10^{-7} \text{ yr}^{-1}. \quad (6)$$

Should this orbital shrinkage rate hold, then the future evolution of the inner binary’s semimajor axis follows the thick solid black line in Fig. 3, where it can be seen that it is consistent with our previous results obtained using the viscoelastic model within an order of magnitude. However, it should be noted that we lack the means to reliably calculate the rate of decay for the inner binary orbit using this model under current conditions, and that this estimate is uncertain by more than an order of magnitude.

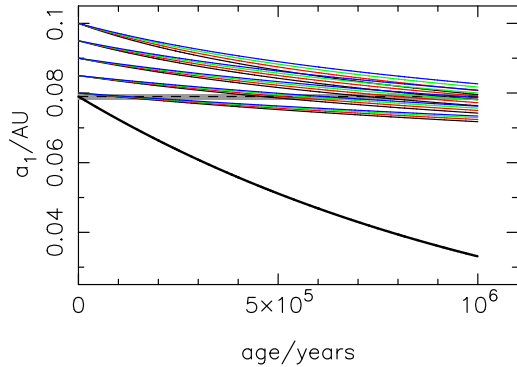


Figure 3. The results of Fig. 2 compared with the results obtained via the model used by Fuller et al. (2013, thick solid black line). This result is consistent with the models agreeing with each other to within an order of magnitude.

4 PROJECTED OBSERVATIONAL SIGNATURES

To observe the orbital shrinkage of a binary that is merging at a very slow pace, the method commonly known as eclipse timing measurements can be employed (e.g. Bours et al. 2016; Burdge et al. 2020). Below is a simplified cartoon account of this method intended to explain its basic principles.

First, the current orbital period P of the binary needs to be measured to a great degree of accuracy. Then, the amount of time, T , that it takes to complete N orbits under the assumption that P is constant is calculated as $T = NP$. Finally, the system needs to be subsequently observed for N orbits, and the amount of time required to complete these N orbits, which we will denote as T_{obs} , is recorded.

Assuming that the orbital period of the binary is shrinking at a rate of Δt per orbit, we have

$$\begin{aligned} T - T_{\text{obs}} &= \frac{N}{2} \left[\frac{1}{2} \Delta t + \left(N - \frac{1}{2} \right) \Delta t \right] \\ &= \frac{N^2}{2} \Delta t, \end{aligned} \quad (7)$$

from which it is trivial to calculate Δt , and hence the rate of orbital shrinkage

$$-\frac{1}{a_1} \frac{da_1}{dt} = \frac{2}{3} \frac{\Delta t}{P^2}. \quad (8)$$

Assuming that TIC242132789 undergoes a shrinkage rate equal to that in equation (2), $1.13 \times 10^{-7} \text{yr}^{-1}$, then it would take only 0.6 yr for $T - T_{\text{obs}}$ to reach 1 s, which is the threshold at which such an approach is considered feasible for the purposes of achieving a detection of the orbital shrinkage. By comparison, HD 181068 has an orbital shrinkage rate roughly 10^{-3} of this value, and would require roughly 20 yr of continued observations to achieve the same results.

5 DISCUSSION

Of all the sources of uncertainty that plague our models, the values of τ in the viscoelastic model and γ_α in the dynamical model are by far the most influential. Should τ be much greater than the range we have tested, it is possible that the TTs in even TIC242132789 will be too weak to be observable. However, we point out that, of all the systems previously studied, published τ values have never exceeded the range tried in this paper for TTs (Gao et al. 2018); therefore, it is possible that this trend will not be broken by TIC242132789. Yet since this uncertainty cannot be fully accounted for, and there is

still the possibility that TIC242132789 may end up with a τ or γ_α value that renders TTs undetectable, it behooves us to consider the merits of our proposed observations in light of the possibility of a non-detection.

Assuming that our models and calculations are correct, and that we can indeed observe the inner binary orbital period decrease of TIC242132789, such an observation would help confirm the validity of current pioneering models of TTs. If it fails to find such a decrease, it would help to constrain the parameters (such as τ) of our current models. In either case, the results of the observation shall help constrain not only aspects of tides in hierarchical triples, but also the more general tidal theories from which the models for TTs were derived, which still suffer from great amounts of uncertainty (e.g. Ogilvie 2014).

Merits of such an observation aside, it should be noted that the account of eclipse timing measurements in Section 4 does not tell the entire story: many sources of error need to be accounted for during these measurements, most notably irregularities introduced by magnetic fields (also known as Applegate’s mechanism), exoplanets, and precession. However, these issues are beyond the scope of this paper, and the methods and techniques relating to them will be covered in a follow-up paper, which will provide details on the observational side of this project. With that being said, assuming that these obstacles can be overcome, we propose that such an observation run on TIC242132789 should go ahead.

ACKNOWLEDGEMENTS

YG is a Royal Society K. C. Wong International Fellow, and as such acknowledges funding from the Royal Society and the K. C. Wong Education Foundation.

ST acknowledges support from the Netherlands Research Council NWO (VENI 639.041.645 and VIDI 203.061 grants).

MJG acknowledges funding by the European Research Council (ERC) under the European Union’s FP7 Programme, grant no. 833031 (PI: Dan Maoz).

DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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