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Effects of Product Substitutability and Power Relationships on Performance in Triadic Supply Chains

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Abstract**—**With the development of the retail industry and the increasing complexity of the power relationship between common retailers and manufacturers, research on product substitutability has become increasingly critical for operational management and decision-making regarding substitutable products. We investigated the effects of product substitutability on retail prices, profits, and social welfare for a triadic supply chain comprised of a retailer and two competing manufacturers. We then extended our analysis to the Nash bargaining game to evaluate the impact of product substitutability and bargaining power on equilibrium in a multiunit bilateral negotiation. The findings revealed that product homogeneity can harm the profits of manufacturers and the overall supply chain. In contrast, product substitutability's impact on the profits of retailers depends on the inter-firm power relationship. Moreover, the retailer's profit was found to consistently increase with respect to substitutability in a manufacturer Stackelberg model, but not so in the vertical Nash and retailer Stackelberg models. We also explored the effect of power structure on supply chain performance. Our results provide valuable insights that can help manufacturers and retailers decide on pricing, sourcing, and brand positioning to improve economic and social performance, as well as assist the government in deciding whether product differentiation should be encouraged.

Keywords: supply chain management; product substitutability; power structure; Nash bargaining game

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1. Introduction

Common retailers have made great progress in improving product diversity to cover various market segments. These firms typically offer multiple substitutable products in most product categories. For instance, department stores resell both Levi's and Gap clothing; Suning and GOME sell multiple home appliance brands, such as Gree and Haier; and Huawei and Xiaomi dominate smartphone and wearables sales on Tmall (Beier and Ivan, 2020). These retailing practices extend beyond supermarkets and department stores to encompass a diverse array of marketplaces, including specialty stores. (e.g., those focused on sales of smartphones, sporting merchandise, or consumer electronics). Due to the substitutable nature of the products offered by retailers, competition is mainly between in-store brand manufacturers rather than across stores. Understanding the effects of product substitutability can aid in the coordination of product pricing and improve the financial performance of retailers and manufacturers (Besanko et al., 2005).

The distinct power positions that manufacturers and retailers hold in their respective supply chains also tend to influence their operational and strategic decisions in dealing with their supply chain partners. For example, Samsung—one of the largest global consumer electronics and semiconductor manufacturers—often acts as the supply chain leader due to its greater level of market power compared to retailers in electronics supply chains (Luo et al., 2018). Dominant retailers, such as Carrefour and Walmart, often hold leadership roles in supply chains when working with most of their upstream suppliers (Ertek and Griffin, 2002; Hu, 2019). However, this may not be the case when working with manufacturing giants, such as Unilever and Nestle, as they hold similar market power, thus leading to a more balanced supply chain power relationship where members compete in a vertical Nash game (Bian et al., 2017).

Among the few prior studies to have examined the impact of product substitutability on supply chain decision-making and performance, Hsieh and Wu (2009) discovered that product substitutability and uncertainty had varied impacts on supply chain performance. Zheng et al. (2020) examined the impact of product substitutability on the profits of manufacturers and retailers, with a focus on the supply network size. Other researchers have found it appropriate to conduct individual negotiations between upstream and downstream firms for input prices in "tight" oligopolies (Horn and Wolinsky, 1988; Iozzi and Valletti, 2014; Shang and Cai, 2021).

The findings of prior research have revealed that an increase in product substitutability negatively affects manufacturer' profits while benefiting those of retailers (Hsieh and Wu, 2009; Zheng et al., 2020). In contrast, some common retailers (e.g., JD.com) present specific distinctions among similar products on their display pages, even offering functionalities for side-by-side comparisons of various parameters and features of substitutable products to highlight their differences. In sum, in the context of an exponential surge in retail products and the increasing complexity of power relationships between common retailers and manufacturers, investigating the interactive effect of inter-firm power relationships and product substitutability holds paramount significance for supply chain management. Therefore, this study was conducted to address the following questions:

(1) How does product substitutability affect supply chain firms' operational decisions, profitability, and social welfare under different power structures?

(2) How does the power structure impact pricing policies and economic and social performance in a triadic supply chain setting?

(3) How does product substitutability influence the decisions and performance of firms with varying levels of bargaining power in a triadic supply chain?

To solve these questions, we studied the impact of product substitutability on a triadic supply chain, comprised of two manufacturers and one common retailer, under three supply chain power structures: manufacturer Stackelberg (MS), vertical Nash (VN), and retailer Stackelberg (RS). We explored the impact of each power structure on pricing decisions, firms' profits, and social welfare.

In addition, manufacturers and retailers often have varying levels of bargaining power in business practice, which may not be captured by the above three power structures. For example, retailers often negotiate with manufacturers (e.g., Mattel and Apple) during the holiday season after advertising prices and collecting consumer bookings to meet demand (Guo and Iyer, 2013). At the same time, multiunit bilateral bargaining can reflect a scenario in which each competing firm independently negotiates with a common retailer, considering a disagreement point in case negotiations break down. This scenario requires that information exchanges do not occur between the two negotiation processes, which can happen if the corresponding party to one negotiation infers the wholesale price of the other negotiation while making decisions (Hsu et al., 2017). Furthermore, decision-making can be influenced by bargaining power using the Nash bargaining game. Therefore, we added a multiunit bilateral negotiation under Nash bargaining to examine these effects. We further analyzed the effectiveness of triadic supply chains, referring to the demand structure generated in previous economic studies (Lus and Murie, 2009; Shubik and Levitan, 2013).

This research offers the following significant contributions. Our research complements the existing supply chain management literature on product substitutability (Choi, 1991; Hsieh and Wu, 2009; Zheng et al., 2020) by employing appropriate demand functions that characterize the substitute product. Moreover, this study systematically examines the effects of both product substitutability and inter-firm power relationships on supply chain decision-making and operational performance.

In examining the effects of product substitutability, Zheng et al. (2020) focused on the supply network size. We extend their work and validate some of their findings on comprehensive supply chain power relationships. Additionally, we generalize the power structure model to incorporate a parameterization of power into the wholesale contract negotiation through a bargaining game.

The non-appropriation of the overall surplus of consumers leads to low output, while insufficient product substitutability results in overproduction, thus highlighting product substitution's impact on social welfare (Bester and Petrakis, 1993). Cellini et al. (2004) modeled a free-entry equilibrium in a differentiated oligopoly and found that quantity competition can lead to greater social welfare as product substitutability decreases. Moreover, Chen and Nie (2014) revealed that total product quantity and social welfare increase with a growth in market power, yet decrease with a rise in substitutability. Understanding how product substitutability affects social welfare is beneficial to policymakers in formulating and implementing appropriate policies (Scrimitore, 2011; Wang et al., 2019). Thus, we evaluate the impact of product substitutability and power relationships on social welfare to provide comprehensive theorical and practical insights under an increasingly complex power relationship.

Our modeling results provide valuable managerial insights. For instance, increased product substitutability reduces demand coverage due to product homogeneity, but increases demand because of intensified competition and the consequent retail price reduction. Our results differ from those of previous research showing that an increase in the degree of product substitutability leads to a rise in retailer profits (Choi, 1991; Hsieh and Wu, 2009). We found that the impact of product substitutability on the profits of common retailers is contingent upon the power relationship with manufacturers and the magnitude of product substitutability. Specifically, a retailer's economic performance decreases in relation to substitutability when the manufacturers' bargaining power and product substitutability are small, but increases when these are high.

Previous studies have revealed that an increase in the extent of product substitutability harms social welfare (Scrimitore, 2011; Chen and Nie, 2014; Wang et al., 2019). In contrast, our study enhances the understanding of product substitutability's impact on social welfare by incorporating its interactive effect and the inter-firm power relationship. We found that the relationship between product substitutability and social welfare is moderated by the power structure. Furthermore, we observed that higher product substitutability may result in a reduction of overall social welfare under a balanced power structure (i.e., VN). However, in cases where either the manufacturers or retailers are Stackelberg led, product substitutability's effect on social welfare is more intricate. In such scenarios, an increase in product substitutability may result in either an increase or decrease in social welfare. Derived from our comprehensive analysis, these new insights can support supply chain firms in decisions about pricing, sourcing, and brand positioning to help optimize their strategies, and enhance their overall performance and social welfare under varying market conditions and product substitutability levels.

The remainder of this study is organized as follows. A review of the relevant research is presented in Section 2. The MS, VN, and RS models are presented in Section 3. Section 4 offers an analysis of how product substitutability influences optimal operational decisions, firms' economic performance, and social welfare under the three considered power structure models. Section 5 discusses the impacts of various power structures on the triadic supply chain, and Section 6 describes how we extended the analysis to the Nash bargaining case and examined the impact of substitutability on the equilibrium results in multiunit bilateral bargaining. Finally, Section 7 summarizes the main research results, discusses managerial implications, and recommends further research.

2. Literature Review

Understanding how product substitutability affects supply chain performance is critical for many businesses. This literature review focuses on three research streams: (1) product substitutability's impact on supply chain management, (2) the effect of the power relationship on supply chain management, and (3) the influence of bargaining theory on the supply chain. If increasing the price of one product results in heightened sales for another, then those products are considered substitutes (Deaton et al., 1980; Russell and Petersen, 2000). In addition, product substitutability in the common retailer supply chain can be referred to as the degree of similarity of the products from different brands (Slade, 1995; Sudhir, 2001).

Practically speaking, not only the manufacturers' competition intensity but also the demand expansion effect can be reflected in product substitutability (Lus and Muriel, 2009; Zheng et al., 2020), which significantly affects the supply chain's performance. This has attracted considerable attention in the operational management literature. For instance, McGuire and Staelin (1983) explored the influence of product substitutability on the equilibrium distribution framework, wherein manufacturers of each distribution channel sell their commodities through a specialized or integrated seller. Choi (1991) was among the first to research the influence of substitutability on a common retailer supply chain. However, the research yielded counterintuitive findings that both prices and profits increase with product substitutability, and emphasized the importance of an appropriate demand function in modeling a triadic supply chain (Choi, 1991).

It is worth noting that Choi's (1991) demand function construed the cross-price sensitivity parameter as indicative of product fungibility, which violated some widely-accepted facts for differentiated products. In particular, product substitutability also reflects the demand effect in addition to competition intensity (Lus and Muriel, 2009). Zhao et al. (2012) employed game theory to explore the pricing of substitutable merchandise, treating customer requirements and the manufacturing cost of each product as fuzzy variables.

Xia et al. (2013) showed that product substitutability between enterprises affects a manufacturer's channel strategy and that businesses demonstrated a greater motivation to add indirect channels when product substitutability is high. In addition, Zhao et al. (2014) developed centralized and decentralized pricing models to study the effect of the manufacturers' competitive strategies and supply chain power structures on pricing decisions for substitutable products in common retailer supply chains. Yang et al. (2015) examined how asymmetric substitutability affects the equilibrium channel structures of competing manufacturers, finding that manufacturers tend to sell directly when there is sufficient asymmetry in substitutability.

Xiao et al. (2017) examined a supply chain composed of two manufacturers providing substitutable products through two retailers, observing that whether supply chain members benefit from a two-part tariff contract depends on substitutability and demand uncertainty. Meanwhile, Wang et al. (2019) explored endogenous product substitutability and characterized its impacts. The authors revealed that individual companies, and society in general, can benefit from reducing product substitutability through investment. Moreover, they suggested that policies should aim to promote such investment among firms in the market.

Taleizadeh et al. (2019) examined how to optimize order quantities and prices to maximize total profit for a portfolio of complementary and substitutable merchandise. Additionally, Zheng et al. (2020) showed that, under the MS power structure, the impact of substitutability on firms' profits can depend on the degree of the supply coverage. Moreover, increasing substitutability exerts competition and demand effects on firms' equilibrium profits (Zheng et al., 2020). Hotkar and Gilbert (2021) considered a setup in which nonexclusive resellers purchased alternative products from two upstream suppliers, one of which introduced a direct channel. They concluded that, when the substitutability of merchandise is sufficiently high, the reseller and the supply chain cannot benefit from such a direct channel. Moreover, their findings show that the substitutability of merchandise considerably impacts channel policies, pricing, and firms' profits (Hotkar and Gilbert, 2021). Xia et al. (2023) investigated how product substitutability and demand uncertainty affect whether two rival manufacturers ought to invest in environmental quality improvements for a common supplier. However, among the above-mentioned studies, few have combined various power structures and the impact of bargaining power on firms' operational and strategic decisions.

Previous research has thoroughly studied the influences of power structures on numerous areas of supply chains, and shown that they influence decision-making and the performance of supply chain members (Shi et al., 2013; Wang et al., 2017; Li et al., 2018; Chen et al., 2017). Wu et al. (2012) used six game models with various vertical and horizontal power relationships to analyze pricing decisions in a competitive context in a supply chain involving two retailers and a supplier. Analyzing a supply chain facing disruption risks, Li et al. (2016) examined how power structures influence the supplier's endogenous reliability enhancement and the pricing strategies in equilibrium for the firms. They revealed that the supply chain achieved a higher level of reliability under the supplier–leader game, but not necessarily higher overall payoffs.

In an examination of the impact of power structures on pricing strategies and benefits for supply chain members, Luo et al. (2017) investigated a supply chain where two manufacturers offered differentiated brands to retailers. The authors found that announcing pricing decisions led to lower profits, which could explain why competitors are often reluctant to disclose prices, bid rates, and contract decisions. Luo et al. (2018) also investigated a two-stage supply chain composed of a retailer and a supplier producing two generations of alternative products, finding that power structures have a significant influence on pricing policies and economic performance.

Li et al. (2020) explored the influence of power structure on production and pricing strategies in a system of subcontracted assembly characterized by decentralization, comprising two suppliers and one manufacturer. The authors discovered that the assembly system benefits greatly under the KAS power structure (wherein the two suppliers make decisions simultaneously). Meanwhile, Hu et al. (2021) investigated the channel structure in the context of online and offline retailers competing in the e-commerce industry. The authors researched whether online retailers should include express delivery services, finding that vertical integration improves the online channel's profitability only in the online retailer Stackelberg game.

Liu et al. (2021) focused on decision analysis in sustainable supply chains with varying subsidies under numerous power structures. Their results suggest that the vertical Nash model yields the highest overall supply chain profit. Zhang et al. (2022) explored four situations regarding green advertising investment under various power structures, finding that bearing the costs of such advertising is not necessarily unfavorable for the investing firm. The follower in the Stackelberg game theoretical model benefits from investing in green advertising when there is only one firm making the investment (Zhang et al., 2022).

Some studies, such as those of Cachon and Harker (2002) and Lovejoy (2010), found that the Stackelberg game—where the supplier or manufacturer takes all of the trade surpluses—

does not reflect reality. In fact, both companies have some bargaining power in supply chain contract negotiations. Since it was first introduced by Nash (1950), bargaining theory has been widely adopted in many aspects of supply chain operation management. For example, considering the asymmetric Nash bargaining of wholesale prices, Baron et al. (2016) examined the impact of bargaining in a single industry chain. The results led to several empirically-supported predictions, including the finding that, in competitive industries, chains function in a manner similar to that under MS contracts.

Chen et al. (2019) compared the equilibriums of the competition model and the two patterns of cooperation in a supply chain involving two suppliers who manufacture substitutive products using the Nash bargaining game. They discovered that the optimal strategy for cooperation depends on the extent of substitutability, the power relationship between enterprises in cooperative contract bargaining, and productivity differences (Chen et al., 2019). Finally, Chen et al. (2022) evaluated how low-carbon technology transfer influenced the performance of two competing firms using Nash bargaining. The authors reported that contractual choices for the licensing of low-carbon technologies depend on the trade-off between the benefits derived from technology licensing and the drawbacks stemming from competition, which are influenced by such factors as inter-firm power relations.

Whereas the above-described studies used one-on-one Nash bargaining to examine supply chain management, several studies have considered multiunit bilateral bargaining. For example, Feng and Lu (2012) adopted a multiunit bilateral bargaining game in one-to-two channels to study the sourcing decisions of competing manufacturers that outsourced to a common supplier. They found that a decline in a manufacturer's bargaining power may increase profits under outsourcing (Feng and Lu, 2012). Meanwhile, Iozzi and Valletti (2014) used Nash bargaining between one upstream supplier and several competing retailers to set the input price to study the countervailing buyer power. Shang and Cai (2021) used game theory and multiunit bilateral bargaining to compare price matching with simultaneous negotiation through the co-seller's dual-buyer differentiation Bertrand competition model. Unlike Shang and Cai (2021), and the research mentioned above, our study focuses on how the effect of product substitutability is impacted by power structures and bargaining power in a multiunit bilateral bargaining game.

In sum, the literature review highlights the significance of understanding the impact of product substitutability, power structures, and Nash bargaining theory on supply chain decisions and performance. Previous research has extensively investigated how product substitutability influences pricing decisions, equilibrium channel structures, and overall supply chain performance. These studies have typically been limited to analyzing specific market structures or evaluating the impact of power structures on supply chains' decisions and economic performance. However, there is a lack of research examining the interplay between product substitutability and the inter-firm power relationship on supply chain decision-making and performance. Moreover, the applications of the Nash bargaining theory have provided insights into cooperative contracts, technology transfer, and buyer-seller negotiations in various supply chain contexts. However, once more, the joint effect of product substitutability on supply chain management seems absent from the literature.

Table 1 provides a comprehensive overview of the literature closely related to our research. In contrast to the counterintuitive findings of previous studies (Choi, 1991) arguing that firms' profits decrease as products became more differentiated, we employed an appropriate parameterization to characterize the effect of product substitutability in the demand function. Building on the results that retailers can benefit from more substitutability (Zheng et al., 2020), we focused on various supply chain power structures and bargaining negotiations instead of the size of the upstream supply network in order to extend the investigation of the effects of product substitutability on supply chain decision-making and performance. While previous studies (Hsieh and Wu, 2009; Chen et al., 2019) on product substitutability using power structure models have foregrounded firms' economic performance, we also sought to evaluate the effects of product substitutability and supply chain power relationships on social welfare, which can benefit policymaking.

TABLE 1. Summary of the related literature

Article	**Common retailer supply chain**	**Demand function**	**Different power structure**	**Effect of product substitutability**	**Nash bargaining game**
Choi (1991)	Yes	Direct	Yes	Numerical analysis	No
Hsieh and Wu (2009)	Yes	Direct	No	Numerical analysis	No
Lus and Muriel (2009)	No	Inverse	No	Numerical analysis	No

Zhao et al. (2012)	Yes	Direct	No	No	No
Xia et al. (2013)	No	Inverse	No	No	No
Zhao et al. (2014)	Yes	Direct	Yes	Numerical analysis	No
Luo et al. (2017)	Yes	Direct	Yes	No	No
Fang et al. (2018)	Yes	Direct	Yes	No	No
Wang et al. (2019)	No	Inverse	No	Analytical analysis	No
Chen et al. (2019)	No	Inverse	No	Analytical analysis	Yes
Zheng et al. (2020)	Yes	Inverse	No	Analytical analysis	No
Hsiao et al. (2022)	No	Inverse	No	Numerical analysis	No
This research	Yes	Inverse	Yes	Analytical analysis	Yes

3. Models and Equilibrium Analysis

3.1. Supply Chain Model

We construct a stylized supply chain model including two manufacturers (manufacturer i and j, $j = 3 - i$) and one common retailer. The manufacturers were symmetrical, with the same unit manufacturing cost c, and they sold substitutable products through a common retailer denoted by r at wholesale price w_i $(i = 1,2)$. The retailer sold the alternative products to customers at price p_i $(i = 1,2)$. Table 2 lists the variables and parameters.

TABLE 2. Notation

p_i	Unit retail price for product i, $i = 1,2$
q_i	Order quantity for product i, $i = 1,2$
w_i	Unit retail wholesale price for product i, $i = 1,2$
c	Unit manufacturing cost
a	The maximum retail price for the retailer
γ	Degree of substitutability between products, $0 \leq \gamma \leq 1$
θ	Each manufacturer's bargaining power, $0 \leq \theta \leq 1$
π_i	Profit for manufacturer i, $i = 1,2$
π_r	Profit for the retailer
π	Profit for the supply chain, $\pi = \pi_r + \sum_{i=1}^{2} \pi_i$
sw	Social welfare

Following the literature (Lus and Muriel, 2009; Chen et al., 2019; Zheng et al., 2020), we adopted inverse demand functions, derived from the optimization problem of a representative consumer characterized by a quadratic and strictly concave utility function, as follows: $U(q_i, q_{3-i}) = a\sum_{i=1}^{2} q_i - \frac{1}{2}(\sum_{i=1}^{2} q_i{}^2 + \sum_{i=1}^{2} \gamma q_i q_{3-i})$, $i = 1, 2$. From the maximization of $U(q_i, q_{3-i}) - \sum_{i=1}^{2} p_i q_i$, we then obtained the inverse demand function: $p_i = a - q_i - \gamma q_{3-i}$, $i = 1, 2$. Linear inverse demand functions of this type are extensively used in marketing, economics, and operation management (e.g., Fang et al., 2018; Zheng et al., 2020) to model competition among alternative products due to their ability to capture the practical price–demand relationship and simplify the analytical process.

It should be noted that the retail price of a manufacturer's merchandise decreases with the quantity produced by itself and its competitors. In our study, a represents the maximum retail price for the retailer, and γ $(0 \le \gamma \le 1)$ is a parameter describing the degree of substitution between the products provided by manufacturers i and j ($i = 1, 2$ and $j = 3 - i$) (Wang et al., 2013; Qing et al., 2017). When $\gamma = 0$, the products provided by the manufacturers are independent and irreplaceable. At the other extreme, when $\gamma = 1$, the products are perfectly substitutable. When $0 < \gamma < 1$, the products are imperfectly substitutable, indicating that the manufacturers have produced somewhat differentiated products. The γ parameter can also reflect the degree of market competition, with a high value indicating a high level of market competition.

Considering the wholesale price contract, we provide the profits functions as follows:

Manufacturer i's profit $\pi_i(w_i)$ is:

$$\pi_i(w_i) = (w_i - c)q_i, \ i = 1,2. \tag{1}$$

Retailer *r*'s profit $\pi_r(q_1, q_2)$ is:

$$\pi_r(q_1, q_2) = (p_1 - w_1)q_1 + (p_2 - w_2)q_2 \tag{2}$$

In what follows, we present three various supply chain power relationships and the corresponding decision orders.

3.2. Manufacturer Stackelberg Model (MS)

In the MS model, the manufacturers are the Stackelberg leader, and the retailer is the follower. The decision-making order is as follows. First, each manufacturer reveals its wholesale price to the retailer. Next, the retailer decides the ordering quantities based on the manufacturers' wholesale prices. Eventually, depending on consumer demand, the retailer and two manufacturers receive their profits, as shown below:

$$\max_{w_i} \pi_i^{MS}(w_i) \rightarrow \max_{q_1,q_2} \pi_r^{MS}(q_1, q_2)$$

3.3. Vertical Nash Model (VN)

In the VN model, the manufacturers and the retailer make decisions simultaneously, with the following decision sequence. Each manufacturer determines its wholesale price to maximize its own profit in anticipation of the retailer's order quantity, and the retailer determines its ordering quantities to maximize its profit in anticipation of the manufacturers' wholesale prices. Eventually, depending on consumer demand, the retailer and the two manufacturers receive their profits, as shown below:

$$\begin{cases} \max_{w_i} \pi_i^{VN}(w_i) \\ \max_{q_1,q_2} \pi_r^{VN}(q_1, q_2) \end{cases}$$

3.4. Retailer Stackelberg Model (RS)

In the RS model, the retailer is the Stackelberg leader, and the manufacturers are the followers in the decision-making order. First, the retailer notifies the manufacturers of the quantity desired, anticipating each manufacturer's wholesale pricing. Each manufacturer then sets the wholesale price in order to maximize its profits. Eventually, depending on consumer demand, the retailer and the two manufacturers receive their profits, as shown below:

$$\max_{q_1,q_2} \pi_r^{RS}(q_1, q_2) \rightarrow \max_{w_i} \pi_i^{RS}(w_i)$$

We assumed that the manufacturers and the retailer pursue their own self-interests, and thus set their wholesale price and ordering quantity decisions to maximize their profits. Table 2 presents the optimal wholesale prices for the two manufacturers and the optimal ordering quantities for the retailer. The derivation process of these optimal solutions is provided in the Appendix.

TABLE 3. Optimal solutions for the three supply chain models

Models	MS	VN	RS
w_i	$\frac{a+c-a\gamma}{2-\gamma}$	$\frac{a-a\gamma+2c}{3-\gamma}$	$\frac{c(3-\gamma)+a(1-\gamma)}{2(2-\gamma)}$
q_i	$\frac{a-c}{4+2\gamma-2\gamma^2}$	$\frac{a-c}{(3-\gamma)(1+\gamma)}$	$\frac{a-c}{4+2\gamma-2\gamma^2}$
p_i	$\frac{3a+c-2a\gamma}{4-2\gamma}$	$\frac{c+a(2-\gamma)}{3-\gamma}$	$\frac{3a+c-2a\gamma}{4-2\gamma}$
π_i	$\frac{(a-c)^2(1-\gamma)}{2(2-\gamma)^2(1+\gamma)}$	$\frac{(a-c)^2(1-\gamma)}{(3-\gamma)^2(1+\gamma)}$	$\frac{(a-c)^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)}$
π_r	$\frac{(a-c)^2}{2(2-\gamma)^2(1+\gamma)}$	$\frac{2(a-c)^2}{(3-\gamma)^2(1+\gamma)}$	$\frac{(a-c)^2}{2(2+\gamma-\gamma^2)}$
π	$\frac{(a-c)^2(3-2\gamma)}{2(2-\gamma)^2(1+\gamma)}$	$\frac{2(a-c)^2(2-\gamma)}{(3-\gamma)^2(1+\gamma)}$	$\frac{(a-c)^2(3-2\gamma)}{2(2-\gamma)^2(1+\gamma)}$

4. Effects of Product Substitutability on the Supply Chain

This section examines how product substitutability affects the optimal retail prices and maximum profits for the two manufacturers and the common retailer. It also considers social welfare under the three supply chain models.

4.1. The Effect of Substitutability on Decision-making

First, we examined the impact of substitutability on products' retail prices, wholesale prices, and order quantities, from which we derived the following lemma.

Lemma 1: ***(a).*** p_i^{MS}, p_i^{VN}, ***and*** p_i^{RS} ***decrease in*** γ $(i=1,2)$;

(b). w_i^{MS}, w_i^{VN}, ***and*** w_i^{RS} ***decrease in*** γ $(i=1,2)$;

(c). If $0\le\gamma<\gamma^b$, ***then*** q_i^{MS}, q_i^{RS} ***decrease in*** γ; ***if*** $\gamma^b<\gamma\le 1$, ***then*** q_i^{MS}, q_i^{RS} ***increase in*** γ; ***if*** $0\le\gamma\le 1$, q_i^{VN} ***decreases in*** γ ***, where*** $\gamma^b=\frac{1}{2}$.

Lemma 1 indicates that the products' retail and wholesale prices decrease apropos of the product substitutability in the MS, VN, and RS models. This suggests that the retailer will set lower retail prices and drive down wholesale prices for both products amidst high substitutability.

Intuitively, high product substitutability will intensify the manufacturers' competition, which serves to drive down wholesale prices and leads to decreased product retail prices under all three supply chain models. This lemma demonstrates that product homogeneity can stimulate demand from price-sensitive consumers, as noted in previous studies (Talluri and Van Ryzin, 2005). However, the impact of substitutability on the optimal order quantities is complex. In the VN model, quantity is consistently a decreasing function of product substitutability (γ), whereas in the MS and RS models, the quantity decreases (increases) with product substitutability (γ) when γ is small (large).

In the VN model, the retailer can only speculate on the manufacturer's wholesale price decision when determining quantity, meaning that quantity is directly affected by substitutability. However, there are both direct and indirect effects of substitutability on product quantity in the MS and RS models. Heterogeneity in consumer preferences (Bayus and Putsis, 1999) causes product proliferation (Dhingra and Morrow, 2019), but retail and wholesale prices indirectly lead to higher quantities by decreasing product substitutability. We found that the direct (indirect) influence is dominant when substitutability is small (large).

4.2. The Effect of Substitutability on Profits

Next, we examined the effect of substitutability on the maximum profits of the manufacturers and the retailer, and derived the following proposition:

Proposition 1:

(1) $\pi_i^{MS}(w_i^{MS})$, $\pi_i^{VN}(w_i^{VN})$ ***and*** $\pi_i^{RS}(w_i^{RS})$ ***decrease with*** γ ($i = 1, 2$)***;*** π^{MS}, π^{VN}, ***and*** π^{RS} ***decrease with*** γ***.***

(2) a. $\pi_r^{MS}(q_1^{MS}, q_2^{MS})$ ***increases with*** γ***.***

b. If $0 \leq \gamma < \gamma^a$***, then*** $\pi_r^{VN}(q_1^{VN}, q_2^{VN})$ ***decreases with*** γ***; if*** $\gamma^a < \gamma \leq 1$***, then*** $\pi_r^{VN}(q_1^{VN}, q_2^{VN})$ ***increases with*** γ***, where*** $\gamma^a = \frac{1}{3}$***.***

c. If $0 \leq \gamma < \gamma^b$***, then*** $\pi_r^{RS}(q_1^{RS}, q_2^{RS})$ ***decreases with*** γ***; if*** $\gamma^b < \gamma \leq 1$***, then*** $\pi_r^{RS}(q_1^{RS}, q_2^{RS})$ ***increases with*** γ***, where*** $\gamma^b = \frac{1}{2}$***.***

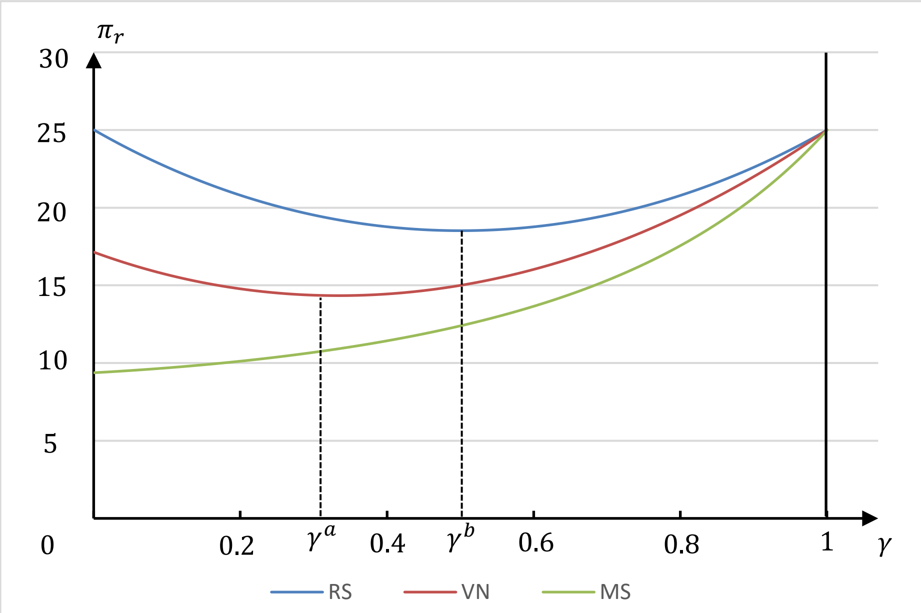

Figure 1. Effect of substitutability on retailer's profit ($a = 20,\ c = 10$)

Proposition 1 indicates that the profits of the manufacturers and the overall supply chain decrease with increasing substitutability. For manufacturers, a high product substitutability will push down wholesale prices due to intensified competition, but increased product homogeneity will influence the demand for each manufacturer's products, as denoted by Lemma 1. When substitutability is small ($0 \leq \gamma < \gamma^b$), both wholesale price and ordering quantities are a decreasing function of γ under all three power structures. When substitutability is high ($\gamma^b < \gamma \leq 1$), both wholesale price and ordering quantities are decreasing functions of γ in the VN model. However, in the MS and RS models, only wholesale price decreases; order quantities increase and the impact of decreasing wholesale prices on manufacturers' profits is greater than the impact of increasing ordering quantities. Combining these two effects, it can be observed that an increase in product substitutability harms manufacturers' profits under all three supply chain models, as observed in previous research (Zheng et al., 2020). Moreover, product homogeneity has a similar impact on retail prices and customer demand, and it decreases profits for the supply chain overall.

In contrast, and as illustrated in Figure 1, the influence of product substitutability on the retailer's profitability is more complex and varies for the diverse supply chain models considered. This finding contrasts with prior research (Choi, 1991), which has suggested that a retailer's profit consistently increases with product substitutability. This study shows that the retailer's profit can increase or decrease with the product substitutability under the VN and RS models. Moreover, critical thresholds in the substitutability define different ranges with contrasting effects, namely γ^a ($\gamma^a = \frac{1}{3}$) and γ^b ($\gamma^b = \frac{1}{2}$) for the VN and RS models, respectively. Retailer profits decrease with product substitutability if it is below the critical threshold, and it increases with substitutability if above.

The impact of product homogeneity on retail prices, wholesale prices, and ordering quantities is shown in Lemma 1. The trade-off between the negative and positive impacts on the retailer's profit determines the values of the varying critical thresholds in the three different supply chain models. The critical threshold in the VN model is lower than that in the RS model ($\gamma^a < \gamma^b$), as the retailer can gain a greater advantage when it is the Stackelberg leader. When the retailer leads the Stackelberg game, the effect of substitutability on the ordering quantities, which are determined by the retailer, is most significant in the effects on profits. Interestingly, the critical threshold in the MS model falls in the infeasible region ($\gamma < 0$) because the negative effect of substitutability on the manufacturers' wholesale prices has a dominant impact on the retailer's profit. This result indicates that the retailer can gain more financial benefits with increased product substitutability.

This result stands in clear contrast to Zheng et al. (2020), who perhaps neglected various supply chain structures, arguing that a retailer can benefit more from increasing substitutability in an MS triadic supply chain. Powerful retailers, such as Walmart, offer a wide range of products with varying degrees of substitutability. On small retailers' shelves, consumers can also find highly substitutable products from leading manufacturers, such as P&G and Unilever.

4.3. Effects of Substitutability on Social Welfare

In this section, social welfare is defined as comprising two parts: consumer surplus and the supply chain's profit, that is, $sw = cs + \sum_{i=1}^{2} \pi_i + \pi_r$. In our model, consumer surplus (cs) is the difference between the highest price a consumer is willing to pay for the consumption of two similar commodities and the equilibrium price of said commodities, which can be presented as $cs = a\sum_{i=1}^{2} q_i - \frac{1}{2}(\sum_{i=1}^{2} {q_i}^2 + \sum_{i=1}^{2} \gamma q_i q_{3-i}) - \sum_{i=1}^{2} p_i q_i$ (Chen et al., 2020; Hsiao et al., 2022), where p_i and q_i represent the unit retail price and the order quantity for product i, respectively. a represents the maximum retail price for the retailer, and γ denotes the extent of substitutability between products, as defined in Table 1. Having defined the terms, we turn to exploring product substitutability's impact on social welfare and derive the following corollary:

Corollary 1:

(1) If $0 \leq \gamma < \gamma^c$***, then*** sw^{MS} ***and*** sw^{RS} ***decrease with*** γ***; if*** $\gamma^c < \gamma \leq 1$***, then*** sw^{MS} ***and*** sw^{RS} ***increase with*** γ***, where*** $\gamma^c = \frac{1}{16}(17 - \sqrt{33})$***.***

(2) sw^{VN} ***decreases with*** γ***.***

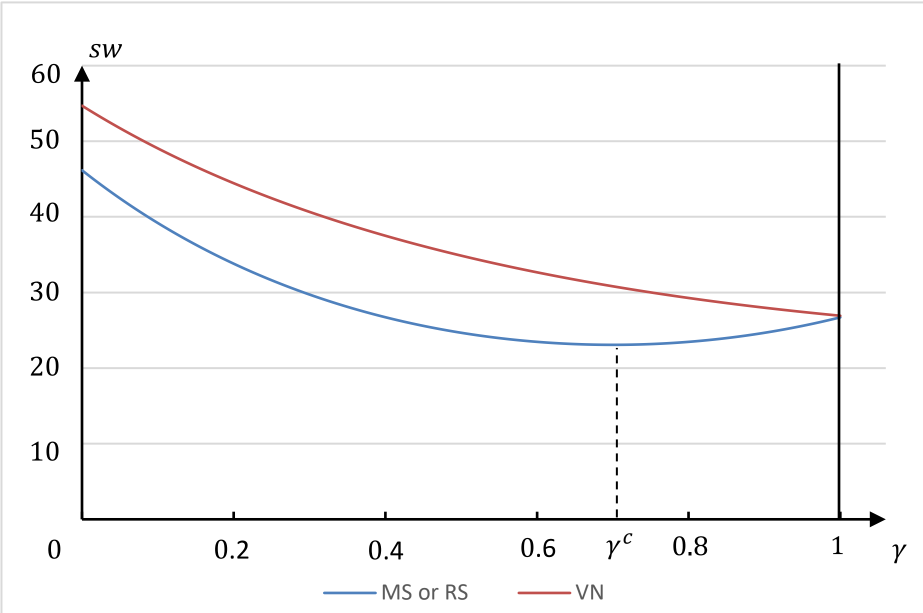

Figure 2. Effect of substitutability on social welfare in MS, VN, and RS models

$(a = 20,\ c = 10)$

As shown in Figure 2, Corollary 1 indicates that social welfare decreases as a function of the product substitutability under the VN model. In contrast, while the value of product substitutability is less than the critical threshold, social welfare declines. However, if the value surpasses the critical threshold, social welfare increases in both the MS and RS models. The impacts of product substitutability on social welfare are predominantly expressed in two aspects. On the one hand, high substitutability intensifies price competition, which consequently increases consumer surplus. Conversely, high substitutability decreases the total profits of the manufacturers and the retailer.

Consumer demand is affected by both product substitutability and retail prices, whereas retail prices are affected by product substitutability and consumer demand. High product substitutability has a direct negative impact on demand and simultaneously leads to lower retail prices, which indirectly increases demand. Under the VN model, with increasing product substitutability, the decrease in the overall supply chain's profit consistently outweighs the increase in consumer surplus. However, under the MS and RS models, the balance between the effects on consumer surplus and the firms' financial benefit is contingent upon the relationship between the product substitutability and the corresponding threshold. These findings partially corroborate those of Wang et al. (2019), who advised the government to encourage investments in reducing product substitutability. Additionally, our findings indicate that concentrating on the power structure and product substitutability levels is vital due to their influence on the advisability of investing in reducing product substitutability.

5. Effects of Power Structures on Supply Chain Decisions and Performance

This section examines how supply chain power structures affect optimal retail prices, the maximum profits of the manufacturers and the retailer, and social welfare.

5.1. Effect of Market Structures on Retail Price

First, we examined the effect of power structures on retail prices, and derived the following lemma:

Lemma 2: $\boldsymbol{p_i^{RS} = p_i^{MS} > p_i^{VN}}$ ***(*$\boldsymbol{i = 1, 2}$*).***

Lemma 2 indicates that the optimal retail price under the MS model is equivalent to that under the RS model, but higher than that under the VN model. This finding implies that an imbalanced power relationship can raise retail prices, as the dominant supply chain member often takes advantage of its power position to establish a price that maximizes its own profit margin. In contrast, a more balanced relationship creates a fairer and more competitive supply chain, resulting in lower retail prices. This result aligns with the findings of previous research (e.g., Luo et al., 2018).

5.2. Effect of Market Structures on Profits

Next, following Proposition 3, we examined the effect of supply chain power structures on the maximum profits of the manufacturers and the retailer.

Proposition 2:

(1) $\boldsymbol{\pi_i^{MS}(w_i^{MS}) > \pi_i^{VN}(w_i^{VN}) > \pi_i^{RS}(w_i^{RS})}$ *(* $\boldsymbol{i = 1, 2}$ *).*

(2) $\boldsymbol{\pi_r^{RS}(q_1^{RS}, q_2^{RS}) > \pi_r^{VN}(q_1^{VN}, q_2^{VN}) > \pi_r^{MS}(q_1^{MS}, q_2^{MS})}$ *(* $\boldsymbol{i = 1, 2}$ *).*

(3) $\boldsymbol{\pi^{VN} > \pi^{RS} = \pi^{MS}}$**.**

Proposition 2 implies that manufacturers can earn more profits under the MS model, but less under the RS model. However, the reverse is true for the retailer. The VN model generates the highest profit for the overall supply chain, meaning that a balanced power relationship between upstream and downstream supply chain participants can improve the financial performance of the overall supply chain. This finding aligns with previous research showing how power structures affect the profits of supply chain participants (Luo et al., 2018).

5.3. Effects of Power Structures on Social Welfare

Next, we examined the effects of power structures on social welfare with the following corollary:

Corollary 2: $\boldsymbol{sw^{VN} > sw^{RS} = sw^{MS}}$**.**

It is clear from Corollary 2 that the maximum social welfare is the same under the RS and MS models, but smaller under the VN model. The main contribution to social welfare comes from the profit of the overall supply chain and consumer surplus. On the one hand, a balanced power structure increases a supply chain's profits, as shown in Proposition 2. On the other hand, consumers can benefit from low retail prices, as shown in Lemma 2, which is consistent with previous studies demonstrating that consumer welfare suffers when either supplier dominates the supply chain (Shi et al., 2013). In sum, consumers and the overall supply chain benefit most under the VN model, which accords with previous studies showing that the social welfare contribution of the supply chain is maximized without a dominant firm (Chen et al., 2020).

6. Extended Models

In the previous sections, we assumed that wholesale prices are determined by the manufacturers. However, in actual supply chain practices, manufacturers and retailers often have respective bargaining power in the contract negotiation of wholesale prices. For example, Walmart and Costco negotiate wholesale prices with their upstream manufacturers (Shang and Cai, 2021). In the following extended model, we considered a scenario wherein wholesale prices are determined by the manufacturers and the retailer through the negotiation of wholesale contracts.

We assumed that each manufacturer's bargaining power is θ, where $0 \leq \theta \leq 1$. The decision-making sequence is as follows. First, manufacturers 1 and 2 negotiate their wholesale prices with the retailer both individually and simultaneously. As the members of the supply chain are required to keep the transaction information confidential, no information is exchanged between the two negotiations. Therefore, when negotiating with the retailer, manufacturer i must conjecture the wholesale price of manufacturer j $(j = 3 - i)$ in another negotiation. Second, the retailer decides on the order quantity of substitutable products from the two manufacturers. Eventually, depending on consumer demand, the retailer and the two manufacturers receive their revenue/profits as follows:

$$\max_{w_i} \Phi^B(w_i) \to \max_{q_1,q_2} \pi_r^B(q_1, q_2)$$

where the superscript B represents the Nash bargaining game.

In the two first-stage negotiations, the firms' negotiators deal with the other input price (i.e., the conjectured price) as a given throughout the bargaining (Davidson, 1988). The two-person Nash solution is used for each negotiation. The result is a set of wholesale prices that reflect the equilibrium in the Nash bargaining system (Iozzi and Valletti, 2014).

More specifically, we denoted the profit in the last stage for manufacturer i as $\pi_i^B(w_i, w_{3-i})$ and the profit of the retailer as $\pi_r^B(w_i, w_{3-i})$, where w_i is the wholesale price for manufacturer i, and w_{3-i} is the wholesale price for the other manufacturer. In such a triadic supply chain, we assumed that each manufacturer i cannot sell to any other alternative retailer, so their disagreement payoff is 0. Let $\pi_r^o = (p_{3-i} - w_{3-i})q_{3-i}$ be the retailer's disagreement payoff, which represents the profit the retailer can earn by serving manufacturer j $(j = 3 - i)$ when the negotiation between the retailer and manufacturer i breaks down. At stage one, the retailer and manufacturer i construct a separate negotiation unit and set w_i to obtain the optimization of the Nash bargaining, which is calculated accordingly:

$$\max_{w_i} \Phi^B(w_i) = \max_{w_i} [\pi_i^B(w_i)]^\theta [\pi_r^B(w_i) - \pi_r^o]^{1-\theta} \tag{3}$$

Lemma 3 provides the derivation of these optimum solutions:

Lemma 3: ***In the Nash bargaining model,*** $\boldsymbol{w_i^B = \frac{a(1-\gamma)\theta + c(2-\gamma-\theta+\gamma\theta)}{2-\gamma}}$**,** $\boldsymbol{q_i^B =}$

$\frac{(a-c)(2+\gamma(-1+\theta)-\theta)}{2(2-\gamma)(1+\gamma)}$, $p_i^B = \frac{c(2-\gamma-\theta+\gamma\theta)+a(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)}$, $\pi_i^B = \frac{(a-c)^2(1-\gamma)(2-\gamma(1-\theta)-\theta)\theta}{2(2-\gamma)^2(1+\gamma)}$,
$\pi_r^B(q_1^B, q_2^B) = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)^2}{2(2-\gamma)^2(1+\gamma)}$, *and* $\pi^B = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)^2(1+\gamma)}$ $(i = 1, 2)$.

6.1. Effect of Product Substitutability on Decision-Making

First, we examined the effect of substitutability on product decisions, including optimum retail prices, wholesale prices, and order quantities. Lemma 4 provides the following formulas:

Lemma 4: ***(a).*** p_i^B ***decreases with*** γ $(i = 1, 2)$.

(b). w_i^B ***decreases with*** γ $(i = 1, 2)$.

(c). If $\theta^a < \theta \leq 1$ ***and*** $\gamma^d < \gamma < 1$, ***then*** q_i^B ***increases with*** γ; ***if*** $0 \leq \theta < \theta^a$ ***and*** $0 \leq \gamma \leq 1$, $\theta^a < \theta \leq 1$ ***and*** $0 \leq \gamma < \gamma^d$, ***then*** q_i^B ***decreases with*** γ, ***where*** $\gamma^d = \frac{2-\theta-\sqrt{3\theta-2\theta^2}}{1-\theta}$, $\theta^a = \frac{1}{2}$.

It is evident from Lemma 4 that, under Nash bargaining, retail and wholesale prices are decreasing functions of the product substitutability. This relationship means that high product substitutability tends to push down retail and wholesale prices, which is consistent with the results of Lemma 1 in the setting of Stackelberg and Nash games. The findings corroborate previous research suggesting that people are less price-sensitive when paying for unique goods (Talluri and Van, 2005). The optimal ordering quantities may increase or decrease with respect to product substitutability, contingent upon the level of product substitutability and the manufacturers' bargaining power.

When manufacturers have more bargaining power ($\theta^a < \theta \leq 1$) and the substitutability is high ($\gamma^d < \gamma \leq 1$), although product homogeneity may directly reduce product demand, an increase in the degree of product substitutability results in reduced retail and wholesale prices, leading to an increase in order quantity. Moreover, the negative impact of product homogeneity on product demand is less than the positive impact of lower retail and wholesale prices on product demand, ultimately causing the order quantity to rise with increasing product substitutability. When this bargaining power is reduced, a decrease in the order quantity can be primarily explained by product homogeneity.

6.2. Effect of Product Substitutability on Profits

Next, we explored the effect of product substitutability on the maximum profits in the bargaining game in line with the following proposition:

Proposition 3:

(1) $\boldsymbol{\pi_i^B}$ ***decreases with*** $\boldsymbol{\gamma}$ ***(***$\boldsymbol{i=1,2}$***);*** $\boldsymbol{\pi^B}$ ***decreases with*** $\boldsymbol{\gamma}$***.***

(2) If $\boldsymbol{0 \le \theta < \theta^a}$ ***and*** $\boldsymbol{0 \le \gamma \le 1}$***, or if*** $\boldsymbol{\theta^a < \theta \le 1}$ ***and*** $\boldsymbol{0 \le \gamma < \gamma^e}$***, then*** $\boldsymbol{\pi_r^B(q_1^B, q_2^B)}$ ***decreases with*** $\boldsymbol{\gamma}$***; if*** $\boldsymbol{\theta^a < \theta \le 1}$ ***and*** $\boldsymbol{\gamma^e < \gamma \le 1}$***, then*** $\boldsymbol{\pi_r^B(q_1^B, q_2^B)}$ ***increases with*** $\boldsymbol{\gamma}$***, where*** $\boldsymbol{\gamma^e = \frac{4-\theta-\sqrt{3}\sqrt{\theta(8-5\theta)}}{2(1-\theta)}}$ ***and*** $\boldsymbol{\theta^a = \frac{1}{4}}$***.***

Proposition 3 shows that the profits of the two manufacturers and the overall supply chain consistently decrease with product substitutability, which is similar to the result described by Proposition 1. In contrast, the manufacturers' bargaining power impact's how product substitutability affects the retailer's profit, as shown in Figure 3.

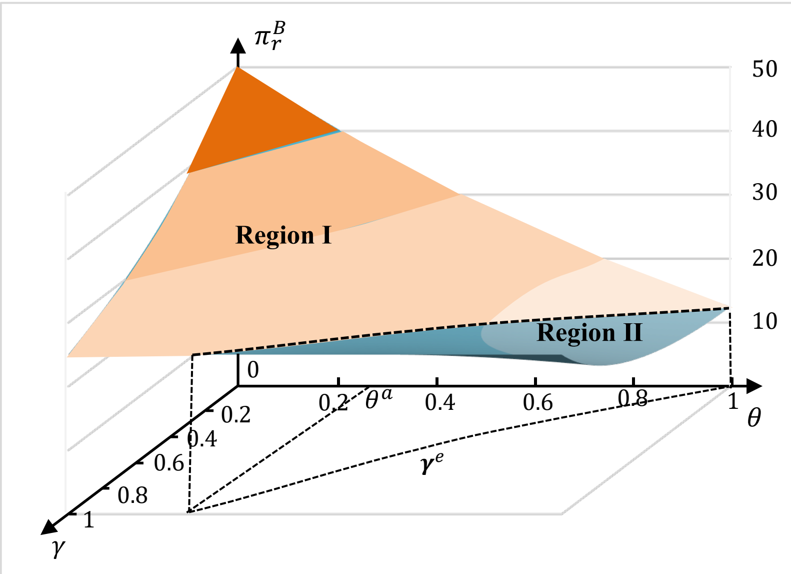

Figure 3. Effect of substitutability on retailer's maximum profit ($\boldsymbol{a=20,\ c=10}$)

In Figure 3, the dotted line γ^e divides the surface into two parts as Regions I and II, reflecting two trends in the impact of bargaining power (θ) and product substitutability (γ) on profits. In Region I, the common retailer attains high profits when possessing low bargaining power and dealing with products characterized by low substitutability. Conversely, in Region II, great profits for the common retailer result from significant bargaining power and highly-substitutable products.

Specifically, when the manufacturers' bargaining power (θ) is small, the maximum profit of the retailer consistently decreases with product substitutability. In contrast, when θ is large, the maximum profit of the retailer decreases with the product substitutability when substitutability falls below the critical threshold (γ^e), but increases with the product substitutability when the rate is greater than γ^e. In addition, γ^e decreases as the manufacturers' bargaining power increases. The latter can be attributed to the fact that the retailer's profit is mainly influenced by consumer demand. On the one hand, product homogeneity has a direct negative effect on demand. Conversely, increased competition between manufacturers has an indirect positive effect on demand via reduced wholesale and retail prices. When the manufacturers' bargaining power is small, the direct negative effect of product homogeneity on consumer demand consistently outweighs competition's positive effect on consumer demand. In contrast, when the manufacturers possess a significantly substantial bargaining power, enhanced competition's positive effect on demand can outweigh the negative effect of product homogeneity.

6.3. Effect of Substitutability on Social Welfare

Next, we examined the effect of product substitutability on social welfare and derived the following corollary:

Corollary 3:

(1) If $0 \le \theta < \theta^b$ ***and*** $0 \le \gamma \le 1$, ***or if*** $\theta^b < \theta \le 1$ ***and*** $0 \le \gamma < \gamma^f$, ***then*** sw^B ***decreases with*** γ.

(2) If $\theta^b < \theta \le 1$ ***and*** $\gamma^f < \gamma \le 1$, ***then*** sw^B ***increases with*** γ,

where $\theta^b = \frac{3}{4}$ ***and***

$$\gamma^f = \frac{1}{3(\theta^2+2\theta-3)}\left(2(\theta^2+4\theta-9) + \frac{\theta(54+61\theta-40\theta^2-11\theta^3)}{\left(+314\theta^3+132\theta^4+78\theta^5+17\theta^6-1053\theta^2+9\theta(3-2\theta-\theta^2)\sqrt{216+501\theta-634\theta^2+21\theta^3+132\theta^4+20\theta^5}\right)^{1/3}} + \left(314\theta^3+132\theta^4+78\theta^5+17\theta^6-1053\theta^2+9\theta(3-2\theta-\theta^2)\sqrt{216+501\theta-634\theta^2+21\theta^3+132\theta^4+20\theta^5}\right)^{1/3}\right).$$

Corollary 3 implies that the effect of product substitutability on social welfare is affected by the manufacturers' bargaining power (θ), as shown in Figure 4. In Region I, where product substitutability is low and bargaining power is relatively small, the former results in lower social welfare. On the other hand, in Region II, where product substitutability is high and bargaining power is large, the former leads to higher social welfare. Specifically, when θ is small, social welfare is consistently a decreasing function of product substitutability. When θ is sufficiently large, social welfare decreases with substitutability when the latter falls below the critical threshold (γ^f). Social welfare increases with substitutability when the substitutability is more than γ^f. In addition, γ^f decreases as the manufacturers' bargaining power increases. This result can be explained by an increase in product substitutability consistently reducing retail prices, which improves the consumer surplus.

Both consumer surplus and the profit of the overall supply chain contribute to social welfare. The profitability of the entire supply chain hinges on the retail prices and demand for the products from the two manufacturers. Although product substitutability impacts both product retail price and demand, the magnitude of its effect on the latter is subject to the manufacturers' bargaining power, as discussed in relation to Proposition 2.

In contrast to previous studies, which have predominantly researched the effect of substitutability on corporate financial performance (Zheng et al., 2020), we considered both substitutability and power relationships in the valuation of financial and social performance. Our findings can help governments to decide whether developing policies to encourage firms to invest in product substitutability is advisable in light of varying levels of bargaining power and product homogeneity.

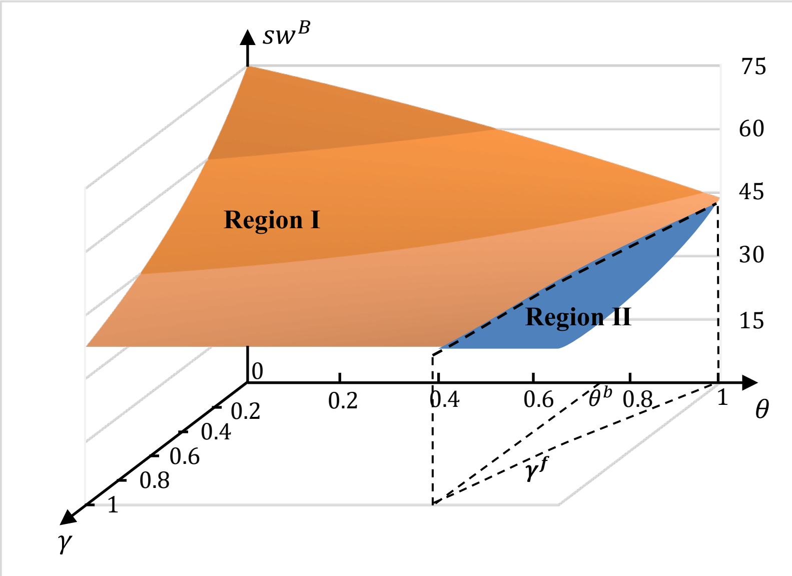

Figure 4. Effect of substitutability on social welfare ($a = 20,\ c = 10$)

6.4. Effect of Bargaining Power

This section explores the effect of bargaining power on the triadic supply chain under the Nash bargaining game. Proposition 4 provides the formulas:

Proposition 4: ***(1)*** $\boldsymbol{p_i^B}$ ***increases with*** $\boldsymbol{\theta}$***;*** $\boldsymbol{w_i^B}$ ***increases with*** $\boldsymbol{\theta}$***; and*** $\boldsymbol{q_i^B}$ ***decreases with*** $\boldsymbol{\theta}$ $\boldsymbol{(i = 1, 2)}$.

(2) $\boldsymbol{\pi_i^B}$ ***increases with*** $\boldsymbol{\theta}$ ***(*** $\boldsymbol{i = 1, 2}$ ***);*** $\boldsymbol{\pi_r^B(q_1^B, q_2^B)}$ ***decreases with*** $\boldsymbol{\theta}$ ***; and*** $\boldsymbol{\pi^B}$ ***decreases with*** $\boldsymbol{\theta}$***.***

(3) $\boldsymbol{sw^B}$ ***decreases with*** $\boldsymbol{\theta}$***.***

According to Proposition 4, both the wholesale and the retail price increase with the manufacturers' bargaining power, but the order quantity decreases. This outcome can be readily explained. When manufacturers have more bargaining power, they set higher wholesale prices in order to yield greater benefits. To make a profit at higher wholesale prices, retailers must raise retail prices, which decreases consumer demand. Accordingly, when the manufacturers' bargaining power is high, their profits increase, whereas the profits of the retailer and the whole supply chain decrease. This finding appears to be somewhat intuitive in that the party with high bargaining power optimizes its own profit by affecting wholesale prices, which damages the interests of the other party's financial benefit. For example, Microsoft and Intel can erode the profitability of PC retailers by raising wholesale prices, whereas Walmart can—and does—

drive down wholesale prices by negotiating with weaker manufacturers. Moreover, we found that social welfare decreases with the manufacturers' bargaining power. Considering the effect of bargaining power on decision-making and profits, we found that the decrease in the overall supply chain profit and lower consumer surplus caused by the combination of higher prices and fewer quantities leads to a reduction in social welfare.

7. Conclusions

In this research, we constructed a triadic supply chain model comprised of one common retailer and two rival manufacturers. The retailer provided substitutable products to consumers supplied by the two manufacturers. By comparing equilibriums under several representative power structures, we analyzed the impact of product substitutability on social welfare, optimal operational decisions, and firms' economic performance. Additionally, we explored the impact of power structures on these within a triadic supply chain. To enrich the research insights, we introduced bargaining power using the Nash bargaining game.

Our findings indicate that product substitutability can influence firm performance by enhancing competition and altering consumer demand. Interestingly, an increase in product substitutability has a complicated impact on consumer demand, despite a rise in product substitutability triggering intense competition between the two manufacturers, resulting in reduced prices. In particular, changes in retail prices and substitutability in different market structures affect demand in conjunction, causing order quantity to increase or decrease in relation to substitutability.

Under the three various power structures considered in this paper, increasing substitutability intensifies competition between the upstream manufacturers and detrimentally influences both firms' profitability. However, increasing substitutability also pushes down retail prices, which benefits consumers. The impact of product substitutability on retailers' profits is more complex due to its positive and negative effects on product demand. As manufacturers' dominance in the decision-making process declines, competition caused by product homogenization has less of an impact on retailers' performance, whereas the retailer's profit in the MS model consistently rises in line with substitutability.

In the RS and VN models, only when a product is sufficiently different does a decline in wholesale prices caused by competition between the manufacturers compensate for the negative impact on demand and increase the retailer's profit. The critical point at which the retailer's profit shifts from a decrease to an increase in the RS model is higher than that in the VN model. Nevertheless, the entire supply chain's profits consistently decrease with substitutability due to the profits of both manufacturers experiencing a greater drop than the increase to the retailer's profits.

In the MS model, social welfare consistently decreases as product substitutability increases, whereas in the VN and RS models, social welfare decreases with product substitutability when the substitutability is low, but increases when the substitutability is high. The critical point between a decrease and an increase in social welfare is the same for the two power structures.

The above findings were further confirmed by introducing bargaining power into the wholesale contract negotiation. We considered bargaining power in order to explore how substitutability influences optimal operational decisions, economic performance (at both individual and collective levels), and social welfare. The results of an analysis of equilibriums under the Nash bargaining game model show that a retailer's profit and social welfare both decrease with substitutability when the manufacturers' bargaining power (θ) and product substitutability (γ) are small. Conversely, these increase when substitutability when θ and γ are high.

In addition to the above theoretical contributions, our research has many significant managerial implications. We provide optimal solutions for retailers under various power structures, and for manufacturers under all three power structures, in a competitive market affected by power relationships and product substitutability. Manufacturers tend to produce highly-differentiated products regardless of the supply chain power structure. In contrast, retailers often prefer to sell low- or high-substitutable products subject to the different supply chain power structures. Retailers who follow manufacturer's decisions frequently benefit from increased substitutability, which runs counter to the manufacturers' preferences. Therefore, manufacturers may wish to highlight the differentiation of products through marketing. However, according to our findings, in the RS and VN market power structures, whether retailers advertise to highlight product differentiation or substitutability depends on the level of substitutability.

Concerning decision-making in production, pricing, and marketing, we have discussed how bargaining power and substitutability influence supply chain decisions when manufacturers and retailers negotiate wholesale prices. Furthermore, we explored substitutability's effect on social welfare. Our findings show that, although social welfare may increase or decrease as a function of product substitutability under the influence of market structure and bargaining power, the substitutability effect generally improves under a balanced power structure and low product substitutability. Accordingly, we conclude that competition between manufacturers and retailers, as well as product differentiation, should be encouraged from a social welfare standpoint.

In sum, this paper's central focus was to assess how product substitutability and inter-firm power relationships interactively influence social welfare and the performance of a common retail supply chain. This study sought to provide valuable managerial insights from the perspectives of manufacturers, retailers, and government stakeholders. In terms of theoretical contributions, we comprehensively analyzed the combined impact of product substitutability and power relationships on optimal operational decisions, and firms' economic and social performance. Our findings demonstrate that manufacturers tend to produce highly-differentiated products, while retailers' preference for selling low- or high-substitutable products depends on the power structure. Based on this finding, we introduced Nash bargaining power to further examine the impact of product substitutability and power relationships on the supply chain, yielding analytically-quantifiable results.

Furthermore, our research offers valuable practical insights into the perspectives of consumer supply chain operations and government policies. The findings highlight that the level of product substitutability can have a complex impact on consumer demand. This suggests that companies must carefully analyze consumer preferences and behaviors when introducing new products or altering existing ones.

Manufacturers are inclined to produce highly-differentiated products, which suggests that manufacturers can allocate research and development resources to enhance existing products or develop entirely new ones, thereby increasing their uniqueness and differentiation in the market. In contrast, common retailers' preferences for low- or high-substitutable products indicates that they can alter consumer perceptions of product substitutability based on market structure by product assortment, promotional activities, and personalized services. Understanding this preference can help manufacturers and retailers align their strategies to achieve mutually-beneficial outcomes.

In addition, our results show that social welfare generally improves under a balanced power structure and low product substitutability. Policymakers and industry stakeholders should thus consider these factors when designing regulations and policies to promote societal well-being. For example, policymakers can formulate relevant policies, such as fostering innovation and research and development, promoting education and public awareness, establishing standards and certifications, and enhancing market surveillance on product substitutability. Policymakers can also address the supply chain power relationship with the aim of increasing social welfare.

As with many models in the literature, our analysis was based on several assumptions. For instance, the production costs of the two manufacturers were assumed to be symmetrical, and the fixed cost was assumed to be zero. A future extension of this research would be to consider asymmetric manufacturers with different costs. Another limitation concerns our use of a deterministic demand function. An additional avenue for prospective investigation could incorporate stochastic demand in evaluating the effect of product substitutability on financial and social performance. We further assumed the cost for negotiation to be 0, whereas this cost is variable. Thus, the cost function for negotiation could be considered in future research. Finally, we only considered one common retailer in a triadic supply chain setting. Our research could be further extended by considering multiple producers and sellers. If the model used in this study were extended as described, the resulting analysis could be expected to yield new insights, but would become more challenging to conduct.

References

[1] Bayus, B. L., Putsis, W. P., 1999. Product proliferation: An empirical analysis of product line determinants and market outcomes. Mark. Sci. 18(2), 137-153.

[2] Beier, K., Ivan, P., 2020. China's "618" Shopping Festival 2020: Huawei and Xiaomi Dominate on Tmall. EqualOcean, Available at: https://equalocean.com/analysis/2020062314133 (Accessed: February 1, 2022).

[3] Berman, O., Wu, D., 2016. Bargaining within the supply chain and its implications in an industry. Decis. Sci. 47(2), 193-218.

[4] Besanko, D., Dubé, J. P., Gupta, S., 2005. Own-brand and cross-brand retail pass-through. Mark. Sci. 24(1), 123-137.

[5] Bester, H., Petrakis, E., 1993. The incentives for cost reduction in a differentiated industry. Int. J. Ind. Organ. 11(4), 519-534.

[6] Bian, J., Lai, K. K., Hua, Z., 2017. Service outsourcing under different supply chain power structures. Ann. Oper. Res. 248(1-2), 123-142.

[7] Cachon, G. P., Harker, P. T., 2002. Competition and outsourcing with scale economies. Manage. Sci. 48(10), 1314-1333.

[8] Cellini, R., Lambertini, L., Ottaviano, G. I. P., 2004. Welfare in a differentiated oligopoly with free entry: A cautionary note. Res. Econ. 58(2), 125-133.

[9] Chen, L. G., Ding, D., Ou, J., 2014. Power structure and profitability in assembly supply chains. Prod. Oper. Manag. 23(9), 1599-1616.

[10] Chen, X., Li, S., Wang, X., 2020. Evaluating the effects of quality regulations on the pharmaceutical supply chain. Int. J. Prod. Econ. 230, 107770.

[11] Chen, X., Luo, Z., Wang, X., 2019. Commission pricing strategy on online retail platforms: Power and dependence in triad. IEEE Trans. Eng. Manage. 69(6), 2769-2780.

[12] Chen, X., Wang, X., Chan, H. K., 2017. Manufacturer and retailer coordination for environmental and economic competitiveness: A power perspective. Transport. Res. E: Logist. Transport. Rev. 97, 268-281.

[13] Chen, X., Wang, X., Xia, Y., 2019. Production coopetition strategies for competing manufacturers that produce partially substitutable products. Prod. Oper. Manag., 28(6), 1446-1464.

[14] Chen, X., Wang, X., Xia, Y., 2022. Low-carbon technology transfer between rival firms under cap-and-trade policies. IISE Trans. 54(2), 105-121.

[15] Chen, Y., Nie, P., 2014. Duopoly innovation under product externalities. Ekon. Istraz. 27(1), 232-243.

[16] Choi, S., 1991. Price competition in a channel structure with a common retailer. Manage. Sci. 10(4), 271-296.

[17] Davidson, C., 1988. Multiunit bargaining in oligopolistic industries. J. Labor Econ. 6(3), 397-422.

[18] Deaton, A., John, M., 1980. Economics and consumer behavior. Cambridge University Press,

Cambridge.

[19] Dhingra, S., Morrow, J., 2019. Monopolistic competition and optimum product diversity under firm heterogeneity. J. Polit. Econ. 127(1), 196-232.

[20] Ertek, G., Griffin, P. M., 2002. Supplier-and buyer-driven channels in a two-stage supply chain. IISE Trans. 34(8), 691-700.

[21] Fang, F., Gurnani, H., Natarajan, H. P., 2018. Leadership, dominance, and preeminence in a channel structure with a common retailer. Decis. Sci. 49(1), 65-120.

[22] Feng, Q., Lu, L. X., 2012. The strategic perils of low cost outsourcing. Manage. Sci. 58(6), 1196-1210.

[23] Guo, L., Iyer, G., 2013. Multilateral bargaining and downstream competition. Mark. Sci. 32(3), 411-430.

[24] Horn, H., Wolinsky, A., 1988. Bilateral monopolies and incentives for merger. Rand J. Econ. 19(3), 408-419.

[25] Hotkar, P., Gilbert, S. M., 2021. Supplier encroachment in a nonexclusive reselling channel. Manage. Sci. 67(9), 5821-5837.

[26] Hsiao, L., Chen, Y. J., Xiong, H., Liu, H. Y., 2022. Incentives for disclosing the store brand supplier. Omega-Int. J. Manage. Sci. 109, 102590.

[27] Hsieh, C. C., Wu, C. H., 2009. Coordinated decisions for substitutable products in a common retailer supply chain. Eur. J. Oper. Res. 196(1), 273-288.

[28] Hsu, V. N., Lai, G., Niu, B., Xiao, W., 2017. Leader-based collective bargaining: Cooperation mechanism and incentive analysis. Manuf. Serv. Oper. Manag. 19(1), 72-83.

[29] Hu, Y., Qu, S., Li, G., Suresh, S., 2021. Power structure and channel integration strategy for online retailers. Eur. J. Oper. Res. 294(3), 951-964.

[30] Huang, H., Ke, H., Wang, L., 2016. Equilibrium analysis of pricing competition and cooperation in supply chain with one common manufacturer and duopoly retailers. Int. J. Prod. Econ. 178, 12-21.

[31] Iozzi, A., Valletti, T., 2014. Vertical bargaining and countervailing power. Am. Econ. J.-Microecon. 6(3), 106-35.

[32] Li, G., Li, L., Liu, M., Sethi, S. P., 2020. Impact of power structures in a subcontracting assembly system. Ann. Oper. Res. 291, 475-498.

[33] Li, G., Zhang, L., Guan, X., Zheng, J., 2016. Impact of decision sequence on reliability enhancement with supply disruption risks. Transport. Res. E: Logist. Transport. Rev. 90, 25-38.

[34] Liu, Y., Ren, W., Xu, Q., Liu, Z., 2021. Decision analysis of supply chains considering corporate social responsibility and government subsidy under different channel power structures. Ann. Oper. Res. 315(2), 1-29.

[35] Lovejoy, W. S., 2010. Bargaining chains. Manage. Sci. 56(12), 2282-2301.

[36] Luo, Z., Chen, X., Chen, J., Wang, X., 2017. Optimal pricing policies for differentiated brands under different supply chain power structures. Eur. J. Oper. Res. 259(2), 437-451.

[37] Luo, Z., Chen, X., Kai, M., 2018. The effect of customer value and power structure on retail supply chain product choice and pricing decisions. Omega-Int. J. Manage. Sci. 77, 115-126.

[38] Lus, B., Muriel, A., 2009. Measuring the impact of increased product substitution on pricing and capacity decisions under linear demand models. Prod. Oper. Manag. 18(1), 95-113.

[39] McGuire, T. W., Staelin, R., 1983. An industry equilibrium analysis of downstream vertical integration. Mark. Sci. 2(2), 161-191.

[40] Nash, J. F., 1950. The bargaining problem. Econometrica, 18(2),155-162.

[41] Qing, Q., Deng, T., Wang, H., 2017. Capacity allocation under downstream competition and bargaining. Eur. J. Oper. Res. 261(1), 97-107.

[42] Russell, G. J., Petersen, A., 2000. Analysis of cross category dependence in market basket selection. J. Retail. 76(3), 367-392.

[43] Scrimitore, M., 2011. Spatial discrimination, product substitutability and welfare. Bull Econ. Res. 63(3), 231-242.

[44] Shang, W., Cai, G., 2022. Implications of price matching in supply chain negotiation. Manuf. Serv. Oper. Manag. 24(2), 1074-1090.

[45] Shi, R., Zhang, J., Ru, J., 2013. Impacts of power structure on supply chains with uncertain demand. Prod. Oper. Manag. 22(5), 1232-1249.

[46] Shubik, M., Levitan, R., 2013. Market structure and behavior. Harvard University Press, Cambridge.

[47] Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. Rand J. Econ. 15(4), 546-554.

[48] Slade, M. E., 1995. Product rivalry with multiple strategic weapons: An analysis of price and advertising competition. J. Econ. Manage. Strategy. 4(3), 445-476.

[49] Sudhir, K., 2001. Structural analysis of manufacturer pricing in the presence of a strategic retailer. Mark. Sci. 20(3), 244-264.

[50] Taleizadeh, A. A., Babaei, M. S., Sana, S. S., Sarkar, B., 2019. Pricing decision within an inventory

model for complementary and substitutable products. Mathematics. 7(7), 568.

[51] Talluri, K. T., Van, R. G., 2004. The theory and practice of revenue management. Kluwer Academic Publishers, Boston.

[52] Wang, C., Nie, P., Cui, T., 2019. Endogenous product substitutability strategy under duopoly. Manag. Decis. Econ. 40(6), 623-632.

[53] Wang, Y. Y., Hua, Z., Wang, J. C., Lai, F., 2017. Equilibrium analysis of markup pricing strategies under power imbalance and supply chain competition. IEEE Trans. Eng. Manage. 64(4), 464-475.

[54] Wang, Y., Niu, B., Guo, P., 2013. On the advantage of quantity leadership when outsourcing production to a competitive contract manufacturer. Prod. Oper. Manag. 22(1), 104-119.

[55] Wu, C. H., Chen, C. W., Hsieh, C. C., 2012. Competitive pricing decisions in a two-echelon supply chain with horizontal and vertical competition. Int. J. Prod. Econ. 135(1), 265-274.

[56] Xia, J., Niu, W., Chen, X., Zhang, L., 2023. Investing in a shared supplier to encourage environmental responsibility under spillovers and demand uncertainty. Transport. Res. E: Logist. Transport. Rev. 174, 103134.

[57] Xia, Y., Xiao, T., Zhang, G. P., 2013. Distribution channel strategies for a manufacturer with complementary products. Decis. Sci. 44(1), 39-56.

[58] Xiao, T., Choi, T. M., Cheng, T. C. E., 2017. Pricing and benefit of decentralization for competing supply chains with fixed costs. IEEE Trans. Eng. Manage. 65(1), 99-112.

[59] Yang, S., Shi, V., Jackson, J. E., 2015. Manufacturers' channel structures when selling asymmetric competing products. Int. J. Prod. Econ. 170, 641-651.

[60] Zhang, L., Zhang, Y., Chutani, A., 2022. Riding the wave of fashion rental: The role of power structures and green advertising. Transport. Res. E: Logist. Transport. Rev. 168, 102946.

[61] Zhao, J., Tang, W., Zhao, R., Wei, J., 2012. Pricing decisions for substitutable products with a common retailer in fuzzy environments. Eur. J. Oper. Res. 216(2), 409-419.

[62] Zhao, J., Wei, J., Li, Y., 2014. Pricing decisions for substitutable products in a two-echelon supply chain with firms′ different channel powers. Int. J. Prod. Econ. 153, 243-252.

[63] Zheng, Q., Pan, X. A., Vakharia, A. J., 2020. Common retailer channel revisited: The role of supply network size. Prod. Oper. Manag. 29(9), 2175-2181.

Appendix

Proof of Table 2： (1) MS model: From formula (2), we obtain $\frac{\partial \pi_r^{MS}(q_1,q_2)}{\partial q_1} = a - 2q_1 - 2\gamma q_2 - w_1$ and $\frac{\partial \pi_r^{MS}(q_1,q_2)}{\partial q_2} = a - 2\gamma q_1 - 2q_2 - w_2$. Then, $\frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial {q_1}^2} = -2$, $\frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial {q_2}^2} = -2$, $\frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial q_2 \partial q_1} = -2\gamma$, and $\begin{vmatrix} \frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial {q_1}^2} & \frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_r^{MS}(q_1,q_2)}{\partial {q_2}^2} \end{vmatrix} = \begin{vmatrix} -2 & -2\gamma \\ -2\gamma & -2 \end{vmatrix} = 4 - 4\gamma^2 > 0$, that is, $\pi_r^{MS}(q_1,q_2)$ is joint concave in q_1 and q_2. Letting $\frac{\partial \pi_r^{MS}(q_1,q_2)}{\partial q_1} = \frac{\partial \pi_r^{MS}(q_1,q_2)}{\partial q_2} = 0$, we obtain $q_1(w_1,w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1,w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$. Replacing $q_1(w_1,w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1,w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$ into formula (1), we obtain $\frac{d\pi_1^{MS}(w_1)}{dw_1} = \frac{a+c-a\gamma-2w_1+\gamma w_2}{2-2\gamma^2}$ and $\frac{d\pi_2^{MS}(w_2)}{dw_2} = \frac{a+c-a\gamma+\gamma w_1-2w_2}{2-2\gamma^2}$. Then, $\frac{d^2\pi_1^{MS}(w_1)}{d{w_1}^2} = \frac{1}{-1+\gamma^2} < 0$, $\frac{d^2\pi_2^{MS}(w_2)}{d{w_2}^2} = \frac{1}{-1+\gamma^2} < 0$, that is, $\pi_i(w_i)$ is concave in w_i. Allowing $\frac{d\pi_1^{MS}(w_1)}{dw_1} = \frac{d\pi_2^{MS}(w_2)}{dw_2} = 0$, we obtain $w_i^{MS} = \frac{a+c-a\gamma}{2-\gamma}$, replacing $w_i^{MS} = \frac{a+c-a\gamma}{2-\gamma}$ into $q_1(w_1,w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1,w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$, we obtain $q_i^{MS} = \frac{a-c}{4+2\gamma-2\gamma^2}$, . Then, $\pi_i^{MS}(w_i^{MS}) = \frac{(a-c)^2(1-\gamma)}{2(2-\gamma)^2(1+\gamma)}$, $\pi_r^{MS}(q_1^{MS},q_2^{MS}) = \frac{(a-c)^2}{2(2-\gamma)^2(1+\gamma)}$, $\pi^{MS} = \frac{(a-c)^2(3-2\gamma)}{2(2-\gamma)^2(1+\gamma)}$.

(2) VN model: Let m_i denote the retailer's margin on product i, $m_i = p_i - w_i$. From formula (1), we obtain $\frac{d\pi_1^{VN}(w_1)}{dw_1} = \frac{a+c-a\gamma-(a-q_1-\gamma q_2-w_1)+\gamma(a-q_2-\gamma q_1)-2w_1}{1-\gamma^2}$ and $\frac{d\pi_2^{VN}(w_2)}{dw_2} = \frac{a+c-a\gamma-(a-q_2-\gamma q_1-w_2)+\gamma(a-q_1-\gamma q_2)-2w_2}{1-\gamma^2}$. Then $\frac{d^2\pi_1^{VN}(w_1)}{d{w_1}^2} = \frac{1}{-1+\gamma^2} < 0$ and $\frac{d^2\pi_2^{VN}(w_2)}{d{w_2}^2} = \frac{1}{-1+\gamma^2} < 0$, that is, $\pi_i^{VN}(w_i)$ is concave in w_i. From formula (2), we obtain $\frac{\partial \pi_r^{VN}(q_1,q_2)}{\partial q_1} = a - 2q_1 - 2\gamma q_2 - w_1$ and $\frac{\partial \pi_r^{VN}(q_1,q_2)}{\partial q_2} = a - 2\gamma q_1 - 2q_2 - w_2$. Then, $\frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial {q_1}^2} = -2$, $\frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial {q_2}^2} = -2$, $\frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial q_2 \partial q_1} = -2\gamma$, and $\begin{vmatrix} \frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial {q_1}^2} & \frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_r^{VN}(q_1,q_2)}{\partial {q_2}^2} \end{vmatrix} = \begin{vmatrix} -2 & -2\gamma \\ -2\gamma & -2 \end{vmatrix} = 4 - 4\gamma^2 > 0$, that is, $\pi_r^{VN}(q_1,q_2)$ is joint concave in q_1 and q_2. Letting $\frac{\partial \pi_r^{VN}(q_1,q_2)}{\partial q_1} = \frac{\partial \pi_r^{VN}(q_1,q_2)}{\partial q_2} = \frac{d\pi_1^{VN}(w_1)}{dw_1} = \frac{d\pi_2^{VN}(w_2)}{dw_2} = 0$, we obtain $w_i^{VN} = \frac{a-a\gamma+2c}{3-\gamma}$ and $q_i^{VN} = \frac{a-c}{(3-\gamma)(1+\gamma)}$, then $\pi_i^{VN}(w_i^{VN}) = \frac{(a-c)^2(1-\gamma)}{(3-\gamma)^2(1+\gamma)}$, $\pi_r^{VN}(q_1^{MS},q_2^{MS}) = \frac{2(a-c)^2}{(3-\gamma)^2(1+\gamma)}$, $\pi^{VN} = \frac{2(a-c)^2(2-\gamma)}{(3-\gamma)^2(1+\gamma)}$.

(3) RS model: Let m_i denote the retailer's margin on product i, $m_i = p_i - w_i$. From formula (1), we obtain $\frac{d\pi_1^{RS}(w_1)}{dw_1} = \frac{a+c-a\gamma-(a-q_1-\gamma q_2-w_1)+\gamma(a-q_2-\gamma q_1)-2w_1}{1-\gamma^2}$ and $\frac{d\pi_2^{RS}(w_2)}{dw_2} = \frac{a+c-a\gamma-(a-q_2-\gamma q_1-w_2)+\gamma(a-q_1-\gamma q_2)-2w_2}{1-\gamma^2}$. Then, $\frac{d^2\pi_1^{RS}(w_1)}{d{w_1}^2} = \frac{1}{-1+\gamma^2} < 0$ and $\frac{d^2\pi_2^{RS}(w_2)}{d{w_2}^2} = \frac{1}{-1+\gamma^2} < 0$, that is, $\pi_i^{RS}(w_i)$ is concave in w_i. sAllowing $\frac{d\pi_1^{RS}(w_1)}{dw_1} = \frac{d\pi_2^{RS}(w_2)}{dw_2} = 0$, we obtain $w_1(q_1,q_2) = a + c - a\gamma - (a -$

$q_1 - \gamma q_2) + \gamma(a - q_2 - \gamma q_1)$ and $w_2(q_1, q_2) = a + c - a\gamma - (a - q_2 - \gamma q_1) + \gamma(a - q_1 - \gamma q_2)$. Replacing $w_1(q_1, q_2) = a + c - a\gamma - (a - q_1 - \gamma q_2) + \gamma(a - q_2 - \gamma q_1)$ and $w_2(q_1, q_2) = a + c - a\gamma - (a - q_2 - \gamma q_1) + \gamma(a - q_1 - \gamma q_2)$ into formula (3), we obtain $\frac{\partial \pi_r^{RS}(q_1,q_2)}{\partial q_1} = a - c + 2(-2+\gamma^2)q_1 - 2\gamma q_2$, $\frac{\partial \pi_r^{RS}(q_1,q_2)}{\partial q_2} = a - c - 2\gamma q_1 + 2(-2+\gamma^2)q_2$. Accordingly, $\frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial {q_1}^2} = 2(-2+\gamma^2)$, $\frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial {q_2}^2} = 2(-2+\gamma^2)$, $\frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial q_2 \partial q_1} = -2\gamma$, and $\begin{vmatrix} \frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial {q_1}^2} & \frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_r^{RS}(q_1,q_2)}{\partial {q_2}^2} \end{vmatrix} = \begin{vmatrix} 2(-2+\gamma^2) & -2\gamma \\ -2\gamma & 2(-2+\gamma^2) \end{vmatrix} = 4(\gamma^2 - 1)(\gamma^2 - 4) > 0$, that is, $\pi_r^{RS}(q_1, q_2)$ is joint concave in q_1 and q_2. Letting $\frac{\partial \pi_r^{RS}(q_1,q_2)}{\partial q_1} = \frac{\partial \pi_r^{RS}(q_1,q_2)}{\partial q_2} = 0$, we obtain $q_i^{RS} = \frac{a-c}{4+2\gamma-2\gamma^2}$, replacing $q_i^{RS} = \frac{a-c}{4+2\gamma-2\gamma^2}$ into $w_1(q_1, q_2) = a + c - a\gamma - (a - q_1 - \gamma q_2) + \gamma(a - q_2 - \gamma q_1)$ and $w_2(q_1, q_2) = a + c - a\gamma - (a - q_2 - \gamma q_1) + \gamma(a - q_1 - \gamma q_2)$, we reach $w_i^{RS} = \frac{c(3-\gamma)+a(1-\gamma)}{2(2-\gamma)}$, then $\pi_i^{RS}(w_i^{RS}) = \frac{(a-c)^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)}$, $\pi_r^{RS}(q_1^{RS}, q_2^{RS}) = \frac{(a-c)^2}{2(2+\gamma-\gamma^2)}$, $\pi^{RS} = \frac{(a-c)^2(3-2\gamma)}{2(2-\gamma)^2(1+\gamma)}$.

Proof of Lemma 1: (a). From Table 2 and the inverse demand function, we get $\frac{dp_i^{MS}}{d\gamma} = -\frac{a-c}{2(2-\gamma)^2} < 0$, $\frac{dp_i^{VN}}{d\gamma} = -\frac{2(a-c)}{(3-\gamma)^2} < 0$, and $\frac{dp_i^{RS}}{d\gamma} = -\frac{a-c}{2(2-\gamma)^2} < 0$. Hence, p_i^{MS}, p_i^{VN} and p_i^{RS} decrease in γ.

(b) From Table 2 and the inverse demand function, we reach $\frac{dw_i^{MS}}{d\gamma} = -\frac{a-c}{(2-\gamma)^2} < 0$, $\frac{dw_i^{VN}}{d\gamma} = -\frac{2(a-c)}{(3-\gamma)^2} < 0$, and $\frac{dw_i^{RS}}{d\gamma} = -\frac{a-c}{2(2-\gamma)^2} < 0$. Hence, w_i^{MS}, w_i^{VN} and w_i^{RS} decrease in γ.

(c). From Table 2 and the inverse demand function, we get $\frac{dw_i^{MS}}{d\gamma} = \frac{(a-c)(2\gamma-1)}{2(2+\gamma-\gamma^2)^2}$, $\frac{dw_i^{VN}}{d\gamma} = \frac{2(a-c)(-1+\gamma)}{(3-\gamma)^2(1+\gamma)^2} < 0$, and $\frac{dw_i^{RS}}{d\gamma} = \frac{(a-c)(2\gamma-1)}{2(2+\gamma-\gamma^2)^2} < 0$. Hence, if $0 \le \gamma < \frac{1}{2}$, then q_i^{MS}, q_i^{RS} decrease in γ. If $\frac{1}{2} < \gamma \le 1$, then q_i^{MS}, q_i^{RS} increase in γ; if $0 \le \gamma \le 1$, q_i^{VN} decreases in γ, where $\gamma^b = \frac{1}{2}$.

Proof of Proposition 1: (1) From Table 2 and formula (1), we get $\pi_i^{MS}(w_i^{MS}) = \frac{(a-c)^2(1-\gamma)}{2(-2+\gamma)^2(1+\gamma)}$, $\pi_i^{VN}(w_i^{VN}) = \frac{(a-c)^2(1-\gamma)}{(-3+\gamma)^2(1+\gamma)}$ and $\pi_i^{RS}(w_i^{RS}) = \frac{(a-c)^2(1-\gamma)}{4(-2+\gamma)^2(1+\gamma)}$. Then, $\frac{d\pi_i^{MS}(w_i^{MS})}{d\gamma} = -\frac{(a-c)^2(1-\gamma+\gamma^2)}{(2-\gamma)^3(1+\gamma)^2} < 0$, $\frac{d\pi_i^{VN}(w_i^{VN})}{d\gamma} = \frac{2(a-c)^2(2-\gamma+\gamma^2)}{(-3+\gamma)^3(1+\gamma)^2} < 0$ and $\frac{d\pi_i^{RS}(w_i^{RS})}{d\gamma} = -\frac{(a-c)^2(1-\gamma+\gamma^2)}{2(2-\gamma)^3(1+\gamma)^2} < 0$. Hence, $\pi_i^{MS}(w_i^{MS})$, $\pi_i^{VN}(w_i^{VN})$ and $\pi_i^{RS}(w_i^{RS})$ decrease with γ.

(2) From Table 2 and formula (2), we reach $\pi_r^{MS}(q_1^{MS}, q_2^{MS}) = \frac{(a-c)^2}{2(-2+\gamma)^2(1+\gamma)}$, then $\frac{d\pi_r^{MS}(q_1^{MS},q_2^{MS})}{d\gamma} =$

$\frac{3(a-c)^2\gamma}{2(2-\gamma)^3(1+\gamma)^2} > 0$. From Table 2 and formula (2), we get $\pi_r^{VN}(q_1^{VN}, q_2^{VN}) = \frac{2(a-c)^2}{(-3+\gamma)^2(1+\gamma)}$, then $\frac{\partial\pi_r^{VN}(q_1^{VN},q_2^{VN})}{\partial\gamma} = -\frac{2(a-c)^2(-1+3\gamma)}{(-3+\gamma)^3(1+\gamma)^2}$. If $\gamma^a < \gamma \leq 1$, then $\frac{d\pi_r^{VN}(q_1^{VN},q_2^{VN})}{d\gamma} > 0$; if $0 \leq \gamma < \gamma^a$, then $\frac{d\pi_r^{VN}(q_1^{VN},q_2^{VN})}{d\gamma} > 0$. From Table 2 and formula (2), we obtain $\pi_r^{RS}(q_1^{RS}, q_2^{RS}) = \frac{(a-c)^2}{2(2+\gamma-\gamma^2)}$, then $\frac{d\pi_r^{RS}(q_1^{RS},q_2^{RS})}{d\gamma} = \frac{(a-c)^2(2\gamma-1)}{2(2+\gamma-\gamma^2)^2}$. If $\gamma^b < \gamma \leq 1$, then $\frac{d\pi_r^{RS}(q_1^{RS},q_2^{RS})}{d\gamma} > 0$; if $0 \leq \gamma < \gamma^b$, then $\frac{d\pi_r^{RS}(q_1^{RS},q_2^{RS})}{d\gamma} > 0$. Hence, a) $\pi_r^{MS}(q_1^{MS}, q_2^{MS})$ increases with γ; b) if $0 \leq \gamma < \gamma^a$, then $\pi_r^{VN}(q_1^{VN}, q_2^{VN})$ decreases with γ; if $\gamma^a < \gamma \leq 1$, then $\pi_r^{VN}(q_1^{VN}, q_2^{VN})$ increases with γ; and c) if $0 \leq \gamma < \gamma^b$, then $\pi_r^{RS}(q_1^{RS}, q_2^{RS})$ decreases with γ; if $\gamma^b < \gamma \leq 1$, then $\pi_r^{RS}(q_1^{RS}, q_2^{RS})$ increases with γ, where $\gamma^a = \frac{1}{3}$, $\gamma^b = \frac{1}{2}$.

(3) From Table 2, and formulas (1) and (2), we reach $\pi^{MS} = \sum_{i=1}^{2}\pi_i^{MS}(w_i^{MS}) + \pi_r^{MS}(q_1^{MS}, q_2^{MS}) = \frac{(a-c)^2(3-2\gamma)}{2(-2+\gamma)^2(1+\gamma)}$, $\pi^{VN} = \sum_{i=1}^{2}\pi_i^{VN}(w_i^{VN}) + \pi_r^{VN}(q_1^{VN}, q_2^{VN}) = \frac{2(a-c)^2(2-\gamma)}{(-3+\gamma)^2(1+\gamma)}$, and $\pi^{RS} = \sum_{i=1}^{2}\pi_i^{RS}(w_i^{RS}) + \pi_r^{RS}(q_1^{RS}, q_2^{RS}) = \frac{(a-c)^2(3-2\gamma)}{2(-2+\gamma)^2(1+\gamma)}$, then $\frac{d\pi^{MS}}{d\gamma} = -\frac{(a-c)^2(4-7\gamma+4\gamma^2)}{2(2-\gamma)^3(1+\gamma)^2} < 0$, $\frac{d\pi^{VN}}{d\gamma} = -\frac{2(a-c)^2(5-5\gamma+2\gamma^2)}{(3-\gamma)^3(1+\gamma)^2} < 0$ and $\frac{d\pi^{RS}}{d\gamma} = -\frac{(a-c)^2(4-7\gamma+4\gamma^2)}{2(2-\gamma)^3(1+\gamma)^2} < 0$. Hence, π^{MS}, π^{VN}, and π^{RS} decrease with γ.

Proof of Corollary 1: From Table 2, we obtain $sw^{MS} = \frac{(a-c)^2(7-4\gamma)}{4(2-\gamma)^2(1+\gamma)}$, $sw^{VN} = \frac{(a-c)^2(5-2\gamma)}{(3-\gamma)^2(1+\gamma)}$ and $sw^{RS} = \frac{(a-c)^2(7-4\gamma)}{4(2-\gamma)^2(1+\gamma)}$. Then, $\frac{dsw^{MS}}{d\gamma} = \frac{(a-c)^2(8-17\gamma+8\gamma^2)}{4(-2+\gamma)^3(1+\gamma)^2}$, if $0 \leq \gamma < \gamma^c$, then $\frac{dsw^{MS}}{d\gamma} < 0$, if $\gamma^c < \gamma \leq 1$, then $\frac{dsw^{MS}}{d\gamma} > 0$; $\frac{dsw^{VN}}{d\gamma} = \frac{(a-c)^2(11-13\gamma+4\gamma^2)}{(-3+\gamma)^3(1+\gamma)^2} < 0$; and $\frac{dsw^{RS}}{d\gamma} = \frac{(a-c)^2(8-17\gamma+8\gamma^2)}{4(-2+\gamma)^3(1+\gamma)^2}$, if $0 \leq \gamma < \gamma^c$, then $\frac{dsw^{RS}}{d\gamma} < 0$, if $\gamma^c < \gamma \leq 1$, then $\frac{dsw^{RS}}{d\gamma} > 0$. Hence, a) if $0 \leq \gamma < \gamma^c$, then sw^{MS} and sw^{RS} decrease with γ; if $\gamma^c < \gamma \leq 1$, then sw^{MS} and sw^{RS} increase with γ, where $\gamma^c = \frac{1}{16}\left(17 - \sqrt{33}\right)$; and b) sw^{VN} decreases with γ.

Proof of Lemma 2: From Table 2, $p_i^{\mathrm{RS}} = \frac{3a+c-2a\gamma}{4-2\gamma}$, $p_i^{\mathrm{MS}} = \frac{3a+c-2a\gamma}{4-2\gamma}$, and $p_i^{\mathrm{VN}} = \frac{c+a(2-\gamma)}{3-\gamma}$, $p_i^{\mathrm{MS}} - p_i^{\mathrm{VN}} = p_i^{\mathrm{RS}} - p_i^{\mathrm{VN}} = \frac{(a-c)(1-\gamma)}{2(3-\gamma)(2-\gamma)} > 0$, therefore, $p_i^{\mathrm{RS}} = p_i^{\mathrm{MS}} > p_i^{\mathrm{VN}}$.

Proof of Proposition 2: From Table 2, $\pi_i^{\mathrm{RS}} = \frac{(a-c)^2(1-\gamma)}{4(2-\gamma)^2(1+\gamma)}$, $\pi_i^{\mathrm{VN}} = \frac{(a-c)^2(1-\gamma)}{(3-\gamma)^2(1+\gamma)}$, $\pi_i^{\mathrm{MS}} = \frac{(a-c)^2(1-\gamma)}{2(2-\gamma)^2(1+\gamma)}$, $\pi_r^{\mathrm{RS}} = \frac{(a-c)^2}{2(2+\gamma-\gamma^2)}$, $\pi_r^{\mathrm{VN}} = \frac{2(a-c)^2}{(3-\gamma)^2(1+\gamma)}$, and $\pi_r^{\mathrm{MS}} = \frac{(a-c)^2}{2(2-\gamma)^2(1+\gamma)}$. Then, $\pi_i^{\mathrm{MS}} - \pi_i^{\mathrm{VN}} = \frac{(a-c)^2(-1+\gamma)\left(-1-2\gamma+\gamma^2\right)}{2(-3+\gamma)^2(-2+\gamma)^2(1+\gamma)} > 0$ and

$\pi_i^{\rm VN} - \pi_i^{\rm RS} = \frac{(a-c)^2(-1+\gamma)^2(7-3\gamma)}{4(-3+\gamma)^2(-2+\gamma)^2(1+\gamma)} > 0$. Therefore, $\pi_i^{\rm MS} > \pi_i^{\rm VN} > \pi_i^{\rm RS}$, $\pi_r^{\rm RS} - \pi_r^{\rm VN} = \frac{(a-c)^2(-1+\gamma)^2}{2(-3+\gamma)^2(2-\gamma)(1+\gamma)} > 0$, and $\pi_r^{\rm VN} - \pi_r^{\rm MS} = \frac{(a-c)^2(7-10\gamma+3\gamma^2)}{2(-3+\gamma)^2(-2+\gamma)^2(1+\gamma)} > 0$. Therefore, $\pi_r^{\rm RS} > \pi_r^{\rm VN} > \pi_r^{\rm MS}$.

Proof of Corollary 2: $sw^{RS} = \frac{(a-c)^2(7-4\gamma)}{4(2-\gamma)^2(1+\gamma)}$, $sw^{VN} = \frac{(a-c)^2(5-2\gamma)}{(3-\gamma)^2(1+\gamma)}$, and $sw^{MS} = \frac{(a-c)^2(7-4\gamma)}{4(2-\gamma)^2(1+\gamma)}$. $sw^{RS} - sw^{VN} = sw^{RS} - sw^{VN} = \frac{(a-c)^2(-17+34\gamma-21\gamma^2+4\gamma^3)}{4(3-\gamma)^2(2-\gamma)^2(1+\gamma)} < 0$. Hence, $sw^{VN} > sw^{RS} = sw^{MS}$.

Proof of Lemma 3: From formulas (1) and (2), we obtain $\frac{\partial \pi_r^B(q_1,q_2)}{\partial q_1} = a - 2q_1 - 2\gamma q_2 - w_1$ and $\frac{\partial \pi_r^B(q_1,q_2)}{\partial q_2} = a - 2\gamma q_1 - 2q_2 - w_2$. Then, $\frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial {q_1}^2} = -2$, $\frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial {q_2}^2} = -2$, $\frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial q_2 \partial q_1} = -2\gamma$, and $\begin{vmatrix} \frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial {q_1}^2} & \frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial q_1 \partial q_2} \\ \frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_r^B(q_1,q_2)}{\partial {q_2}^2} \end{vmatrix} = \begin{vmatrix} -2 & -2\gamma \\ -2\gamma & -2 \end{vmatrix} = 4 - 4\gamma^2 > 0$, that is, $\pi_r^B(q_1, q_2)$ is joint concave in q_1 and q_2. Allowing $\frac{\partial \pi_r^B(q_1,q_2)}{\partial q_1} = \frac{\partial \pi_r^B(q_1,q_2)}{\partial q_2} = 0$, we obtain $q_1(w_1, w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1, w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$. Replacing $q_1(w_1, w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1, w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$ into formula (3), we obtain $ln\, \Phi^B(w_i) = \theta\, ln\, \pi_i^B(w_i) + (1-\theta)\, ln(\pi_r^B(q_i(w_i)) - \pi_r^o)$. By derivation, we get $\frac{1}{\Phi^B(w_i)} \frac{\partial \Phi^B(w_i)}{\partial w_i} = \theta \frac{1}{\pi_i^B(w_i)} \frac{\partial \pi_i^B(w_i)}{\partial w_i} + (1-\theta) \frac{1}{(\pi_r^B(w_i)-\pi_r^o)} \frac{\partial(\pi_r^B(w_i)-\pi_r^o)}{\partial w_i}$. Then, $\frac{\partial \Phi^B(w_i)}{\partial w_i} = \Phi^B(w_i)\left[\theta \frac{1}{\pi_i^B(w_i)} \frac{\partial \pi_i^B(w_i)}{\partial w_i} + (1-\theta) \frac{1}{(\pi_r^B(w_i)-\pi_r^o)} \frac{\partial(\pi_r^B(w_i)-\pi_r^o)}{\partial w_i}\right]$. Letting $\frac{\partial \Phi^B(w_i)}{\partial w_i} = 0$, we obtain $w_i^B = \frac{a(1-\gamma)\theta+c(2-\gamma-\theta+\gamma\theta)}{2-\gamma}$. Replacing $w_i^B = \frac{a(1-\gamma)\theta+c(2-\gamma-\theta+\gamma\theta)}{2-\gamma}$ into $\frac{d^2\Phi(w_i)}{d{w_i}^2}$, we get $\frac{d^2\Phi(w_i^B)}{d{w_i^B}^2} = \frac{(-2+\gamma+\gamma\theta)^3\Phi(w_i^B)}{(a-c)^2(-1+\gamma)^2(-1+\theta)(-1+(-1+2\gamma)\theta)} < 0$; thus, w_i^B is the equilibrium solution of $\Phi^B(w_i)$. Then replacing $w_i^B = \frac{a(1-\gamma)\theta+c(2-\gamma-\theta+\gamma\theta)}{2-\gamma}$ into $q_1(w_1, w_2) = \frac{a-a\gamma-w_1+\gamma w_2}{2-2\gamma^2}$ and $q_2(w_1, w_2) = \frac{a-a\gamma+\gamma w_1-w_2}{2-2\gamma^2}$, we obtain $q_i^B = \frac{(a-c)(2+\gamma(-1+\theta)-\theta)}{2(2-\gamma)(1+\gamma)}$, then $\pi_i^B = \frac{(a-c)^2(1-\gamma)(2-\gamma(1-\theta)-\theta)\theta}{2(2-\gamma)^2(1+\gamma)}$, $\pi_r^B(q_1^B, q_2^B) = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)^2}{2(2-\gamma)^2(1+\gamma)}$, $and\ \pi^B = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)^2(1+\gamma)}$.

Proof of Lemma 4: (a) From Lemma 3 and the inverse demand function, we obtain $\frac{dp_i^B}{d\gamma} = \frac{(-a+c)\theta}{2(2-\gamma)^2} < 0$. Hence, $p_i^{\rm B}$ decreases with γ.

(b). From Lemma 3 and the inverse demand function, we obtain $\frac{dw_i^B}{d\gamma} = \frac{(-a+c)\theta}{(2-\gamma)^2} < 0$. Hence, $w_i^{\rm B}$ decreases with γ.

(c). From Lemma 3 and the inverse demand function, we obtain $\frac{dq_i^B}{d\gamma}=\frac{(a-c)(\gamma^2(\theta-1)+2\gamma(2-\theta)+3\theta-4)}{2(2-\gamma)^2(1+\gamma)^2}$. Hence, if $\frac{1}{2}<\theta\leq 1$ and $\gamma^d<\gamma\leq 1$, then q_i^B increases with γ; if $0\leq\theta<\frac{1}{2}$ and $0\leq\gamma\leq 1$, or $\frac{1}{2}<\theta\leq 1$ and $0\leq\gamma<\gamma^d$, then q_i^B decreases with γ, where $\gamma^d=\frac{2-\theta-\sqrt{3\theta-2\theta^2}}{1-\theta}$.

Proof of Proposition 3: (1) From Lemma 3, we obtain $\pi_i^B=\frac{(a-c)^2(1-\gamma)(2+\gamma(-1+\theta)-\theta)\theta}{2(2-\gamma)^2(1+\gamma)}$ and $\frac{d\pi_i^B}{d\gamma}=-\frac{\theta(6-2\gamma^2(-2+\theta)+\gamma^3(-1+\theta)-4\theta+\gamma(-7+5\theta))}{2(2-\gamma)^3(1+\gamma)^2}<0$. Hence, $\pi_i^B(w_i^B)$ decreases with γ.

(2) From Lemma 3, we obtain $\pi_r^B(q_1^B,q_2^B)=\frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)^2}{2(-2+\gamma)^2(1+\gamma)}$ and $\frac{d\pi_r^B(q_1^B,q_2^B)}{d\gamma}=\frac{(-\gamma(-4+\theta)+4(-1+\theta)+\gamma^2(-1+\theta))(2+\gamma(-1+\theta)-\theta)}{2(2-\gamma)^3(1+\gamma)^2}$. If $0\leq\theta<\theta^a$ and $0\leq\gamma\leq 1$, or if $\theta^a<\theta\leq 1$ and $0\leq\gamma<\gamma^e$, then $\frac{d\pi_r^B(q_1^B,q_2^B)}{d\gamma}<0$, that is, $\pi_r^B(q_1^B,q_2^B)$ decreases with γ, and if $\theta^a<\theta\leq 1$ and $\gamma^e<\gamma\leq 1$, then $\frac{d\pi_r^B(q_1^B,q_2^B)}{d\gamma}>0$, that is, $\pi_r^B(q_1^B,q_2^B)$ increases with γ, where $\gamma^e=\frac{4-\theta-\sqrt{3}\sqrt{\theta(8-5\theta)}}{2(1-\theta)}$ and $\theta^a=\frac{1}{4}$.

(3) From Lemma 3, we obtain $\pi^B=\pi_1^B(w_1^B)+\pi_2^B(w_2^B)+\pi_r^B(q_1^B,q_2^B)=\frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)^2(1+\gamma)}$ and $\frac{d\pi^B}{d\gamma}=-\frac{(a-c)^2(8-4\theta^2-2\gamma^2(-3+\theta^2)+\gamma^3(-1+\theta^2)+\gamma(-12+5\theta^2))}{2(2-\gamma)^3(1+\gamma)^2}<0$. Hence, π^B decreases with γ.

Proof of Corollary 3: From Lemma 3, we obtain $\frac{dsw^B}{d\gamma}=\frac{\gamma^3(-3+2\theta+\theta^2)-4(-6+3\theta+\theta^2)-2\gamma^2(-9+4\theta+\theta^2)+\gamma(-36+14\theta+5\theta^2)}{4(-2+\gamma)^3(1+\gamma)^2}$. If $0\leq\theta<\theta^b$ and $0\leq\gamma\leq 1$, or if $\theta^b<\theta\leq 1$ and $0\leq\gamma<\gamma^f$, then $\frac{dsw^B}{d\gamma}<0$, that is, sw^B decreases with γ, and if $\theta^b<\theta\leq 1$ and $\gamma^f<\gamma\leq 1$, then $\frac{dsw^B}{d\gamma}>0$, that is, sw^B increases with γ, where $\theta^b=\frac{3}{4}$ and $\gamma^f=\frac{1}{3(\theta^2+2\theta-3)}(2(\theta^2+4\theta-9)+\frac{\theta(54+61\theta-40\theta^2-11\theta^3)}{(+314\theta^3+132\theta^4+78\theta^5+17\theta^6-1053\theta^2+9\theta(3-2\theta-\theta^2)\sqrt{216+501\theta-634\theta^2+21\theta^3+132\theta^4+20\theta^5})^{1/3}}+(314\theta^3+132\theta^4+78\theta^5+17\theta^6-1053\theta^2+9\theta(3-2\theta-\theta^2)\sqrt{216+501\theta-634\theta^2+21\theta^3+132\theta^4+20\theta^5})^{1/3})$.

Proof of Proposition 4: (1) From Lemma 3 and the inverse demand function, we obtain $\frac{dp_i^B}{d\theta}=\frac{(a-c)(1-\gamma)}{2(2-\gamma)}>0$, $\frac{dw_i^B}{d\theta}=\frac{(a-c)(1-\gamma)}{2-\gamma}>0$, and $\frac{dq_i^B}{d\theta}=\frac{(a-c)(-1+\gamma)}{2(2-\gamma)(1+\gamma)}<0$. Hence, p_i^{B} increases with θ, w_i^{B} increases with θ, and q_i^B decreases with θ.

(2) From Lemma 3, we obtain $\pi_i^B = \frac{(a-c)^2(1-\gamma)(2+\gamma(-1+\theta)-\theta)\theta}{2(2-\gamma)^2(1+\gamma)}$ and $\frac{d\pi_i^B}{d\theta} = \frac{(a-c)^2(1-\gamma)(2-\gamma-2\theta+2\gamma\theta)}{2(2-\gamma)^2(1+\gamma)} > 0$; $\pi_r^B(q_1^B, q_2^B) = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)^2}{2(-2+\gamma)^2(1+\gamma)}$ and $\frac{d\pi_r^B(q_1^B,q_2^B)}{d\theta} = -\frac{(a-c)^2(1-\gamma)(2-\gamma-\theta+\gamma\theta)}{(2-\gamma)^2(1+\gamma)}$; $\pi^B = \pi_1^B(w_1^B) + \pi_2^B(w_2^B) + \pi_r^B(q_1^B, q_2^B) = \frac{(a-c)^2(2+\gamma(-1+\theta)-\theta)(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)^2(1+\gamma)}$ and $\frac{d\pi^B}{d\theta} = -\frac{(a-c)^2(1-\gamma)^2\theta}{(2-\gamma)^2(1+\gamma)} < 0$. Hence, $\pi_i^B(w_i^B)$ increases with θ, $\pi_r^B(q_1^B, q_2^B)$ decreases with θ, and π^B decreases with θ.

(3) From Lemma 3, we obtain $\frac{dsw^B}{d\theta} = -\frac{(1-\gamma)(2-\gamma+\theta-\gamma\theta)}{2(2-\gamma)^2(1+\gamma)} < 0$. Hence, sw^B decreases with θ.