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Models of rotating stars constrained by asteroseismic measurements of red giants

P. Eggenberger^{1,*}, N. Lagarde², A. Miglio², J. Montalbán³, S. Ekström¹, C. Georgy^{1,4}, S. Salmon⁵, G. Meynet¹, and A. Maeder¹

¹ Geneva Observatory, University of Geneva, Maillettes 51, CH-1290 Sauverny, Switzerland

² School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

³ Dipartimento di Fisica e Astronomia, Università di Padova, Vicolo dell'Osservatorio 3, I-35122 Padova, Italy

⁴ Astrophysics group, EPSAM, Keele University, Lennard-Jones Labs, Keele, Staffordshire ST5 5BG, UK

⁵ Laboratoire AIM, CEA/DSM-CNRS, Université Paris 7 Diderot, IRFU/SAP, Centre de Saclay, 91191, Gif-sur-Yvette, France

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Solar-like oscillations have now been characterized for a large number of stars, thanks to asteroseismic data obtained recently by space missions. This has led to the determination of the global and internal properties of these stars. In particular, core rotation rates have been obtained for red-giant stars, which is of prime importance to progress in the modelling of the dynamical processes at work in stellar interiors. In this presentation, we discuss which constraints can be brought by these asteroseismic measurements on stellar models that include rotational effects. Similarly to the solar case, we show that an efficient mechanism is required for the transport of angular momentum in the radiative zones of red giants. The efficiency of this transport process can be determined by asteroseismic observations of red-giant stars.

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1 Introduction: the internal rotation of the Sun

Rotation can play an important role in stellar evolution by changing all outputs of stellar models (see e.g. Maeder 2009). The inclusion of rotational effects in stellar evolution codes has to describe simultaneously the transport of angular momentum and the transport of chemical elements in stellar interiors. In current rotating models based on the assumption of shellular rotation (Zahn 1992), the change of the properties of rotating models, and in particular the quantitative impact of rotational mixing, are sensitive to the prescriptions used for the modelling of meridional currents and the transport by the shear instability (see e.g. Meynet et al. 2013). Observations of the rotational properties of stars of various masses and at different evolutionary stages are then required to progress in the modelling of rotational effects and their inclusion in stellar evolution codes. The determination of the internal rotation of stars is of course particularly valuable to constrain the transport of angular momentum in stellar radiative zones. Such constraints become now available thanks to asteroseismic data.

In the case of the Sun, helioseismic measurements show an approximately flat rotation profile in the radiative zone down to about $0.2 R_{\odot}$ (see e.g. Brown et al. 1989; Couvdat et al. 2003; Elsworth et al. 1995; García et al. 2007;

Kosovichev et al. 1997). Moreover, helioseismic data indicate that there is no latitudinal differential rotation in the solar radiative interior, while latitudinal differential rotation is present in the convective envelope.

The internal rotation of the Sun is a key observational constraint for rotating stellar models. Solar models computed with the assumption of shellular rotation (Zahn 1992), which take into account the internal transport of angular momentum and chemical elements by meridional circulation and the shear instability, predict a significant increase of the angular velocity when the distance to the solar center decreases (e.g. Chaboyer et al. 1995; Eggenberger et al. 2005; Pinsonneault et al. 1989; Turck-Chièze et al. 2010). This is a clear indication that meridional circulation and shear instability alone do not provide a sufficient coupling to correctly reproduce the helioseismic data. An additional process for the internal transport of angular momentum in the solar radiative zone is thus required.

The physical nature of this process for angular momentum transport in radiative zones is still a matter of debate. Magnetic fields could play a key role in explaining the uniform rotation of the Sun (e.g. Charbonneau & MacGregor 1993; Eggenberger et al. 2005; Gough & McIntyre 1998; Mestel & Weiss 1987; Rüdiger & Kitchatinov 1996; Spada et al. 2010), while internal gravity waves could be another possible efficient source of transport of angular momentum (e.g. Charbonnel & Talon 2005; Kumar & Quataert 1997;

* Corresponding author: patrick.eggenberger@unige.ch

Mathis et al. 2013; Schatzman 1993; Talon et al. 2002; Zahn et al. 1997).

An important question is to know whether such a process is also at work in the radiative interiors of other stars. In particular, one can wonder whether the uniform rotation of the solar radiative interior is also observed for main-sequence solar-type stars with different ages and different masses than the Sun. As discussed by Lund et al. (2014), it is difficult to obtain such an information using only asteroseismic data. Indeed, only pressure modes that mainly propagate in the external stellar layers can be observed for these main-sequence stars. A possibility to obtain some information about the radial differential rotation for these stars is to combine asteroseismic data with an independent determination of the surface rotation rate. The latter can be either deduced from spectroscopic measurements or from the analysis of luminosity variations induced by stellar spots. The idea is then to compare the mean rotation rate deduced from asteroseismic data (which is sensitive to the internal rotation of the star even if this mean value is strongly dominated by the rotation in the external layers) to the observed surface velocity. A difference between both values can then indicate that radial differential rotation is present.

First results show that the difference between the surface and the mean asteroseismic rotation rate is very small (Benomar et al. 2015). Using the hypothesis of solid-body rotation for both the convective and the radiative zone (but allowing of course these zones to rotate at different rates), Benomar et al. (2015) deduced that for most stars of their sample the difference of rotation rates between both zones is lower than a factor two. This suggests that main-sequence solar-type stars are characterized by a low degree of radial differential rotation and that an efficient transport of angular momentum is also at work in these stars. The next question is to determine whether angular momentum transport by meridional circulation and shear instability can produce a sufficient coupling to account for these observations, or if an additional transport process is required as is the case for the Sun. Preliminary results based on models including shellular rotation computed with the Geneva evolution code (Eggenberger et al. 2008) suggest that such an additional mechanism is required for solar-type stars with a deep convective envelope that experience a strong surface braking by magnetized winds on the main sequence. For more massive main-sequence with shallower convective envelopes that do not experience a significant surface braking, the low contrast between the core and surface rotation rates (and in particular the absence of strong radial differential rotation in the external parts of the radiative zone) predicted by rotating models may however be compatible with these observations.

2 Internal rotation of red-giant stars

With the launch of the CoRoT (Baglin et al. 2006) and *Kepler* (Borucki et al. 2010) spacecrafts, detection and charac-

terization of solar-like oscillations have been obtained for a large number of red giants. In contrast to non-evolved main-sequence solar-type stars, for which only pressure modes are detected, mixed oscillation modes can be observed for red-giant stars. These mixed modes are particularly interesting to study the internal stellar properties, because they behave similarly to gravity modes in the center of the star and similarly to pressure modes in the external layers. Being sensitive to the physical conditions in the deep stellar interiors, such modes can contain valuable information about the internal rotation rates of red-giant stars. Moreover, mixed modes are characterized by a different amount of pressure and gravity-mode character, which means that they are more or less sensitive to surface and core rotation rates. Observing rotational frequency splittings of mixed modes in red giants can thus bring important constraints about the internal rotation profiles of these stars. Rotational splittings of mixed modes being now observed for some red-giant stars (see Beck et al. 2012; Deheuvels et al. 2015, 2014, 2012; Mosser et al. 2012), it is interesting to discuss which constraints can be brought by these measurements on the modelling of angular momentum transport in stellar radiative zones.

2.1 Comparison with models including only rotational effects

Beck et al. (2012) reported the measurement of rotational splittings of dipole modes for the red giant KIC 8366239 based on observations obtained with the *Kepler* spacecraft. This red-giant branch star is characterized by a mass of about $1.5 M_{\odot}$ and a solar metallicity. These observations show that the values of the rotational splittings of modes that are more pressure-dominated are lower than the ones corresponding to modes that are more gravity-dominated. Assuming that the whole star rotates as a solid body, one would then predict higher values of rotational splittings for pressure-dominated modes compared to the gravity-dominated ones, in contradiction with observational constraints. Beck et al. (2012) conclude that radial differential rotation is present in the interior of this target with a rotation rate in the center at least ten times higher than at the surface of the star.

The observed rotational splittings can then be compared to the predictions of models including shellular rotation only. By computing such rotating models and the corresponding theoretical rotational splittings of dipole modes, Eggenberger et al. (2012) show that the theoretical values of the splittings are much higher than the observed ones. Even for a model with a very low initial velocity of only 1 km s^{-1} on the zero-age main sequence, the predicted values of the splittings are still one order of magnitude higher than the observed ones. This disagreement is a direct consequence of the large increase of the core rotation rate as a result of the central contraction after the main-sequence phase. This effect dominates all other transport processes included in shellular rotation modelling (e.g. Eggenberger et al. 2010; Palacios et al. 2006). Interestingly, the ratio of the values of rota-

tional splittings for gravity-dominated modes and pressure-dominated modes predicted by models including shellular rotation is also much higher than observed. This ratio is sensitive to the gradient of angular velocity in the stellar interior, which indicates that models with shellular rotation predict too steep rotation profiles compared to the moderate differential rotation deduced from the observed values of the rotational splittings. This shows that meridional circulation and the shear instability alone produce an insufficient coupling in the radiative zone to correctly reproduce the values of the rotational splittings obtained for KIC 8366239. For this target, an additional (in addition to meridional currents and shear instability) process for the internal transport of angular momentum is thus needed during the post-main sequence phase of evolution (Eggenberger et al. 2012; Marques et al. 2013).

While the need for an additional transport mechanism for the angular momentum has been found in the case of the red giant KIC 8366239 that is massive enough to have a convective core during the main-sequence phase, one can then investigate whether such a process is also at work in the interior of low-mass red giants that are characterized by radiative cores on the main sequence. For this purpose, measurements of rotational splittings of mixed modes for the *Kepler* target KIC 7341231 reported by Deheuvels et al. (2012) are particularly interesting; KIC 7341231 is indeed a red-giant star with a mass of about $0.8 M_{\odot}$. From these asteroseismic measurements, Deheuvels et al. (2012) have determined the rotation rate in the central layers of this star together with an upper limit on the surface velocity deduced from seismic data. Ceillier et al. (2012, 2013) have then computed rotating models of KIC 7341231 that only include the transport of angular momentum by meridional circulation and the shear instability. The comparison with the rotation rates deduced from rotational splittings clearly shows that models including shellular rotation predict higher rotation rates than observed. In the case of this low-mass red giant with a radiative core during the main sequence, one also finds that meridional currents and the shear instability alone produce an insufficient transport of angular momentum in the radiative zone and that an additional transport mechanism is required (see Ceillier et al. 2012, 2013, for more details).

Asteroseismic constraints on the internal rotation of red giants are not restricted to a small number of individual targets which have been studied in details. Indeed, mean core rotation rates have been determined by Mosser et al. (2012) from measurements of rotational splittings for a large sample of red-giant stars observed by the *Kepler* spacecraft. Contrary to individual targets subject to a detailed asteroseismic analysis, no asteroseismic information about surface rotation rates and the degree of radial differential rotation are available for the stars of this large sample. A preliminary comparison with rotating models of red giants computed with shellular rotation indicates that the core rotation rates deduced from asteroseismic data for this sample are much lower than predicted by the models. This sug-

gests that an additional mechanism for the internal transport of angular momentum is needed for red giants of different masses, in good agreement with the results discussed above for KIC 8366239 and KIC 7341231. Interestingly, Mosser et al. (2012) found a slight decrease of the core angular velocity when stars evolve on the red-giant branch. Such a trend cannot be reproduced by models with shellular rotation only, which predict an increase of the angular velocity in the core of a star that evolves along the red-giant branch. This also confirms the need for an efficient transport of angular momentum in the deep radiative interior of red giants.

In addition to the asteroseismic observations of red giants discussed above, Deheuvels et al. (2014) reported the measurement of rotational splittings of mixed modes for six subgiants and young red-giant stars. For these less evolved targets, a precise estimate of both core and surface rotation rates can be deduced from asteroseismic data. Deheuvels et al. (2014) find that the surface rotation rate decreases with the surface gravity of the star, while the core rotation rate increases. Such a trend can be qualitatively reproduced by models including shellular rotation, which predict an increase of the angular velocity in the stellar core due to the post-main sequence central contraction, and the simultaneous decrease of the surface rotation velocity due to the increase of the stellar radius. Preliminary results indicate that an additional mechanism for the transport of angular momentum is however needed for these subgiants and young red giants in order to correctly reproduce the core rotation rates and the degree of radial differential rotation deduced from asteroseismic data. Interestingly, the observation of an increase of the degree of differential rotation when the star evolves during the subgiant and early red-giant phase suggests that the undetermined additional process cannot be too efficient during the beginning of the post-main sequence phase, while this process must be efficient enough in order to reproduce the low values of core rotation rates observed for more evolved red giants.

2.2 Efficiency of the additional process for the internal transport of angular momentum

In the preceding section, the comparison of rotating models with asteroseismic constraints on the internal rotation of red giants has shown that an unknown mechanism is needed for the transport of angular momentum during the post-main sequence phase in addition to meridional circulation and the shear instability. The next step is to investigate how asteroseismic data can be used to characterize the properties of this unknown transport process and to quantify its efficiency. For this purpose, an additional constant viscosity ν_{add} corresponding to this undetermined transport mechanism is introduced in the equation describing the transport of angular momentum in radiative zones of rotating models computed with the hypothesis of shellular rotation. We then try to constrain the values of this additional viscosity (and hence the mean efficiency of the missing transport process) by confronting the predictions of rotating models computed

for different values of ν_{add} (and different initial velocities) to asteroseismic constraints.

Such rotating models have been computed in the case of the red-giant star KIC 8366239 (Eggenberger et al. 2012). An increase of the initial velocity of the model on the zero-age main sequence leads of course to a global increase of the internal rotation during the red-giant phase and hence to an increase of the theoretical values of the rotational splittings of the dipole modes. An increase of the additional viscosity ν_{add} results in a more efficient internal transport of angular momentum and hence to a lower degree of radial differential rotation. As briefly discussed in the preceding section, the ratio of rotational splittings for gravity-dominated modes and pressure-dominated modes is sensitive to the contrast between core and surface rotation rates. Increasing the value of the additional viscosity leads to a decrease of this ratio. Measurements of rotational splittings of mixed dipole modes obtained for KIC 8366239 can thus be used to simultaneously constrain the initial velocity on the zero-age main sequence of the model, and more importantly the value of the viscosity corresponding to the additional transport process. In the case of the $1.5 M_{\odot}$ red giant KIC 8366239 a value of $\nu_{\text{add}} = 3 \times 10^4 \text{ cm}^2 \text{ s}^{-1}$ is found for the efficiency of the unknown transport process, together with an initial velocity on the zero-age main sequence of 20 km s^{-1} (see Eggenberger et al. 2012, for more details).

To study how the efficiency of the additional transport mechanism changes with stellar properties, and in particular with the mass and the evolutionary stage, one can apply the same method to the other red giants for which asteroseismic constraints on the internal rotation rates are available. This can be done for the low-mass (a mass of about $0.8 M_{\odot}$) red-giant star KIC 7341231. Preliminary results indicate that the additional viscosity needed for this low-mass red giant is about one order of magnitude lower than the one found for the more massive target KIC 8366239. In contrast to the case of KIC 8366239, the initial velocity cannot be precisely determined for KIC 7341231. There is indeed a degeneracy between the value of the initial velocity on the zero-age main sequence and the adopted efficiency for the braking of the stellar surface by magnetized winds for this low-mass star with a deep convective envelope on the main sequence. However, the uncertainties related to the modelling of the braking of the stellar surface by magnetized winds during the main sequence have almost no impact on the determination of the efficiency of the additional transport process. The comparison of the values of ν_{add} found for KIC 7341231 and KIC 8366239 suggests that the efficiency of the additional transport process is lower when the mass of the star decreases.

The core and surface rotation rates deduced from the measurement of rotational splittings of mixed modes for six subgiants and young red-giant by Deheuvels et al. (2014) can also be used to determine the efficiency of the additional transport process. Preliminary results indicate that intermediate values between the ones found for

KIC 7341231 and KIC 8366239 are obtained for the additional viscosity ν_{add} of these stars. These six targets being characterized by masses in between the masses of KIC 8366239 and KIC 7341231, this seems compatible with the suggested increase with the mass of the additional transport process. Such a trend seems also to be in good agreement with the results obtained by Tayar & Pinsonneault (2013), who find that models computed with the assumption of solid-body rotation in the whole stellar interior are able to correctly reproduce the low core rotation rates deduced by Deheuvels et al. (2015) for red giants in the secondary clump. These results thus suggest that the additional transport process must be very efficient for these red giants that are massive enough to burn helium in non-degenerate conditions. Of course this simple preliminary link between the value of the additional viscosity and the mass of the red giant has to be studied in more details, since the evolutionary stage is also expected to play a key role. This is well illustrated by comparing the core rotation rates of models computed with a constant value of ν_{add} , with the core rotation rates determined by Mosser et al. (2012) along the red-giant branch. When the value of the additional viscosity ν_{add} is chosen in order to reproduce the asteroseismic core rotation rates obtained for stars at the base of the red-giant branch, one finds that a model with a constant additional viscosity then predicts an increase of the core rotation rate when the radius of the star becomes larger and larger. This is in contradiction with the asteroseismic observations and suggests that the efficiency of the additional transport mechanism increases during the evolution on the red-giant branch.

2.3 Physical nature of the additional process for the internal transport of angular momentum

While the properties of the additional process for the internal transport of angular momentum in red giants begin to be revealed thanks to asteroseismic data, its physical nature remains an open question. Magnetic fields could be the explanation of this undetermined additional transport process. Cantiello et al. (2014) have studied the effects of the Tayler-Spruit dynamo (Spruit 1999, 2002) on the internal rotation of red giants. They find that models computed with this dynamo are in better agreement with core rotation rates deduced from seismic data than models with rotational effects only, but that the efficiency of this process is not sufficient to correctly reproduce the asteroseismic constraints. As discussed by Rüdiger et al. (2014, 2015), it will also be interesting to study in more details the impact of the azimuthal magnetorotational instability on the internal rotation of red giants. In addition to the transport of angular momentum by magnetic instabilities, fossil magnetic fields could play a role for the internal rotation of red giants (see e.g. Maeder & Meynet 2014). Internal gravity waves are another possible candidate for the missing transport process. Fuller et al. (2014) made a study of the impact of internal gravity waves on the rotation profiles of red giants. They find that these

waves can transport angular momentum in the external part of the radiative zone, but are not able to extract angular momentum from the stellar core. Mixed oscillation modes have also been considered as a possible mechanism to redistribute angular momentum (Belkacem et al. 2015). As shown by Belkacem et al. (2015), such a mechanism seems to be negligible during the subgiant and the early red-giant phase, but may play a more important role for more evolved red giants.

3 Conclusion

We have seen that asteroseismic measurements of rotational splittings for red giants can help us characterize the efficiency of the needed additional transport process (in addition to the transport by the meridional circulation and the shear instability). Such constraints are crucial to progress in our understanding of the physical nature of this mechanism. It is interesting to note that the values of the additional viscosity deduced from asteroseismic observations of red giants are quite similar to the ones obtained in the solar case (e.g. Rüdiger & Kitchatinov 1996; Spada et al. 2010) or deduced from observations of the spin down of stars in open clusters (e.g. Denissenkov et al. 2010). Does this suggest that the same physical process is at work during the whole evolution of a solar-type star?

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